Pure Mathematics Algebra & Analytic Solid Geometry

Question Bank
& Practice Exams

EL-MOASSER

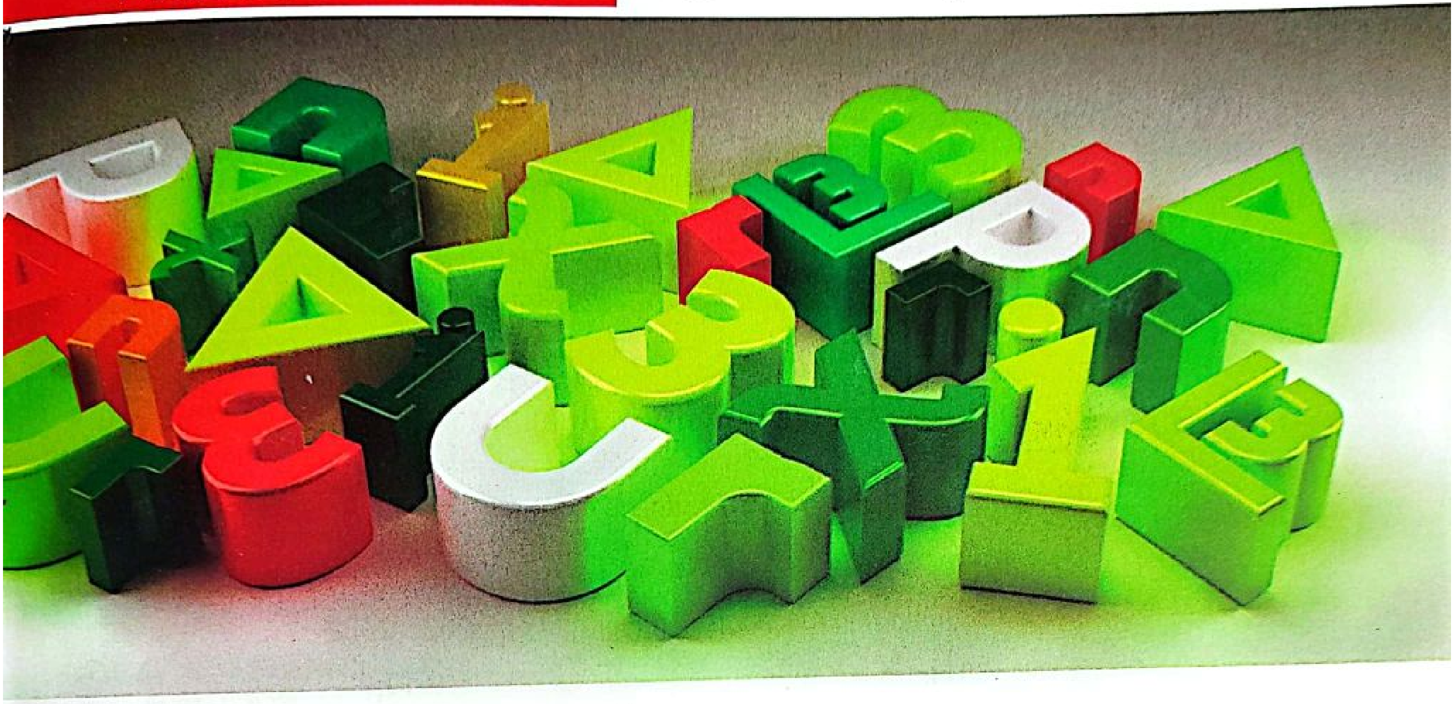
By a group of supervisors

3rd Sec
2022

EL-MOASSER

By a group of supervisors

PURE MATHEMATICS Algebra & Analytic Solid Geometry



Question Bank
& Practice Exams

3rd Sec

GPS

AL TALABA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
e-mail: info@elmoasserbooks.com
15014 www.elmoasserbooks.com

Egyptian National Library

Cataloging - in - Publishing

Prepared by Technical Affairs Department

El-Moasser question bank & practice exams /

By a group of supervisors. - 1st ed. - Cairo :

GPS for printing , publication and distribution , 2021-2022

2 vols ; 30 cm.

For 3rd Sec.

I.S.B.N. : 978 - 977- 475 - 866 - 3

1. Mathematics - study & teaching (secondary).

2. Education , secondary

510.7

Dep. No. 9507/2022

Preface

Thanks to God who helped us to introduce one of our famous series "El Moasser" in mathematics.

We introduce this book to our colleagues.

We also introduce it to our students to help them study mathematics.

In fact, this book is the outcome of more than thirty years experience in the field of teaching mathematics.

This book will make students aware of all types of questions.

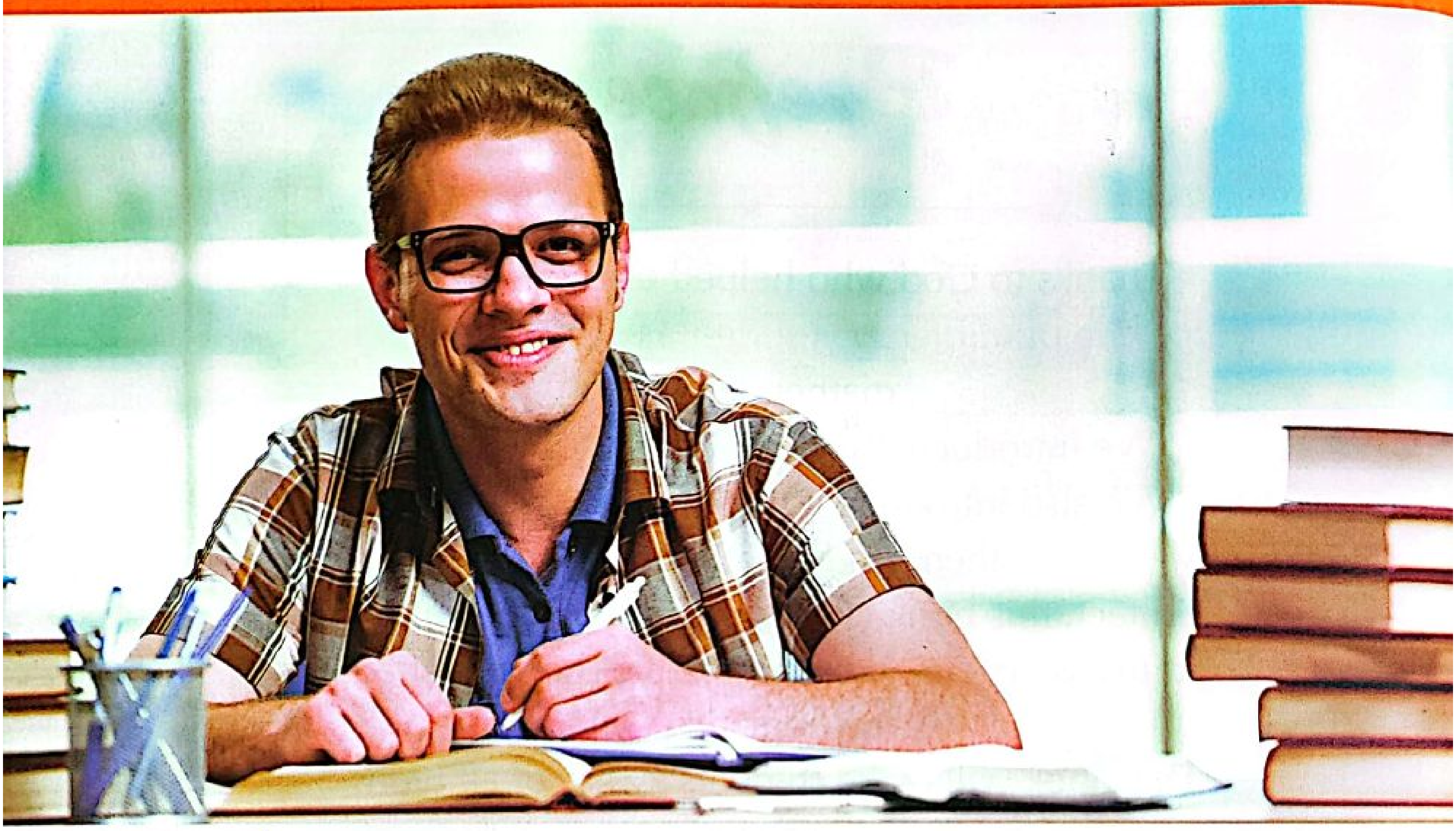
We would like to know your opinions about the book hoping that it will win your admiration.

We will be grateful if you send us your recommendations and your comments.

Love from us

The Author

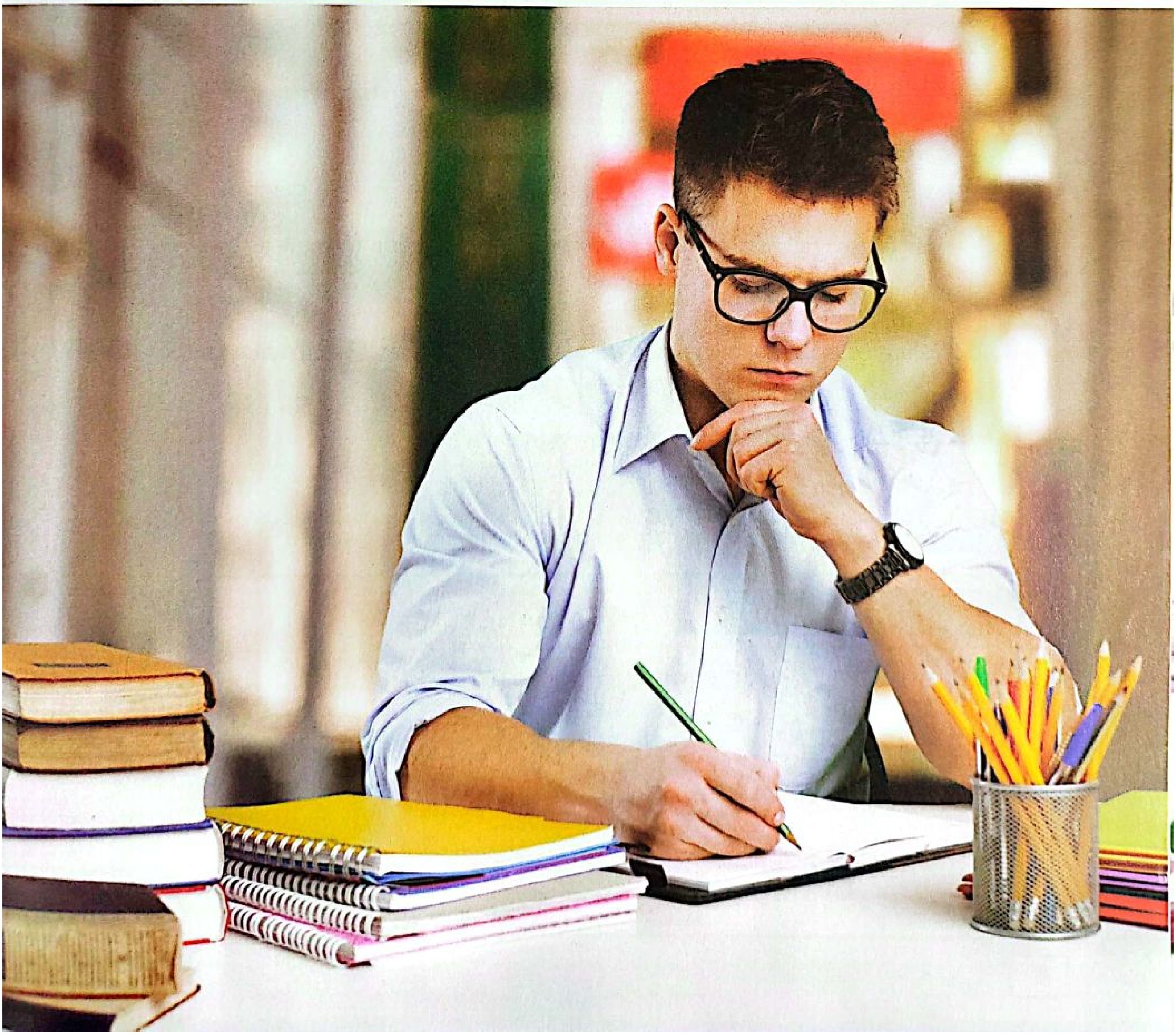
CONTENTS



Summary

for

Algebra & Analytic Solid Geometry



First: Summary of the important points in Algebra

Counting principle

If a certain act can be performed in m different ways and a second act can be performed in n different ways , then :

- The number of ways to perform the first act (and) the second act = m × n ways (multiplication rule)
- The number of ways to perform the first act (or) the second act = m + n ways (addition rule)

Laws of permutations and factorial of a number

If $n \in \mathbb{Z}^+$, $r \in \mathbb{Z}^+$, $n \geq r$, then :

1. ${}^nP_r = n(n-1)(n-2) \cdots (n-r+1)$ **2.** ${}^nP_n = \lfloor n = n(n-1)(n-2) \times \cdots \times 3 \times 2 \times 1$

3. $\lfloor n = n \lfloor n-1 = n(n-1) \lfloor n-2$ **4.** ${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$

Remarks

1. ${}^nP_0 = 1$, ${}^nP_1 = n$ **2.** $\lfloor 0 = \lfloor 1 = 1$ **3.** $\lfloor n \in \mathbb{Z}^+$, ${}^nP_r \in \mathbb{Z}^+$

4. Quotient of two factorials can be expressed in permutation formula as follow :

$\frac{\lfloor a}{\lfloor b} = {}^aP_{a-b}$ where $a \geq b$

5. The values of n and r which make ${}^nP_r$ have a value where ${}^nP_r \in \mathbb{Z}^+$ must satisfy the inequality $0 \leq r \leq n$ for every $n , r \in \mathbb{N}$

6. $\lfloor n = {}^nP_n = {}^nP_{n-1}$ so if ${}^nP_r = \lfloor n$, then $r = n$ or $r = n-1$

7. If ${}^aP_r = {}^bP_r$, then $a = b$ or $r = 0$

8. If : ${}^nP_a = {}^nP_b$, then $a = b$ or $a = n , b = n-1$ or $a = n-1 , b = n$

Laws of combinations

If $n \in \mathbb{Z}^+$, $r \in \mathbb{Z}^+$, $n \geq r$, then :

1. ${}^nC_r = \frac{{}^nP_r}{\lfloor r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}$ **2.** ${}^nC_r = {}^nC_{n-r}$ "simplifying rule"

3. If ${}^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$

4. $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$ **5.** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Remarks

1. * ${}^nC_1 = n$ * ${}^nC_n = {}^nC_0 = 1$

* ${}^nC_r \le {}^nP_r$ * ${}^nC_r \in \mathbb{Z}^+$

2. The maximum value of ${}^nC_r$ at a certain value of n

- is ${}^nC_{\frac{n}{2}}$ → Where n is even number
- is ${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$ → Where n is odd number

3. If ${}^aC_r = {}^bC_r$, then a = b or r = 0

4. If ${}^nC_r = {}^nP_r$, then $\lfloor r = 1$ **i.e.** r = 0 or r = 1

The number of ways to select r objects out of n objects

If the selection is

with replacement and arrangement	with replacement and without arrangement	without replacement and with arrangement	without replacement and without arrangement
, then the number of ways of selection			
n^r	${}^{n+r-1}c_r$	${}^nP_r$	${}^nc_r$
To form 2 digit number from $\{5, 3, 2, 1\}$	To distribute 4 balls in 5 baskets	To form 2-different digit number from $\{5, 2, 3, 4, 1\}$	To form a meal from 3 dishes out of 5 dishes
$4^2 = 16$	${}^{5+4-1}c_4 = 70$	${}^5P_2 = 5 \times 4 = 20$	${}^5C_3 = 10$

The number of diagonals in n-sided polygon = ${}^nC_2 - n$

The number of ways to arrange n distinct items

- In a row → $\lfloor n$
- In a circle → $\lfloor n - 1$

For example :

- The number of arrangements of 5 students in form of a row = $\underline{|5} = 120$ ways.
- The number of arrangements of 5 students in form of a circle = $\underline{|4} = 24$ ways.

✪ The number of ways to arrange r distinct elements in n places , where r < n if :

- The places are distinct and in form of a row → ${}^{n}P_{r}$
- The places are distinct and in form of a circle → ${}^{n}P_{r}$
- The places are identical and in form of a circle → $\frac{{}^{n}P_{r}}{n} = {}^{n-1}P_{r-1}$

For example :

- Number of ways to park 4 cars in a garage which has 6 distinct places in form of a row = ${}^{6}P_{4} = 360$
- Number of ways to park 4 cars in a garage which has 6 distinct places in form of a circle = ${}^{6}P_{4} = 360$
- Number of ways to park 4 cars in a garage which has 6 identical places in form of a circle = $\frac{{}^{6}P_{4}}{6} = {}^{5}P_{3} = 60$

✪ Number of ways to arrange r distinct items in n places such that items are adjacent to each other

- Places are distinct and in form of a row → $(n - r + 1)\,\underline{|r}$
- Places are distinct and in form of a circle → $n\,\underline{|r}$
- Places are identical and in form of a circle → $\underline{|r}$

For example :

- Number of ways to park 4 adjacent cars in a garage which has 6 distinct places in form of a row = $(6 - 4 + 1)\,\underline{|4} = 72$
- Number of ways to park 4 adjacent cars in a garage which has 6 distinct places in form of a circle = $6\,\underline{|4} = 144$
- Number of ways to park 4 adjacent cars in a garage which has 6 identical places in form of a circle = $\underline{|4} = 24$

Binomial theorem

If x , a $\in \mathbb{R}$, n is any positive integer , then :

1. $(x + a)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1}\, a + {}^nC_2\, x^{n-2}\, a^2 + \cdots + {}^nC_r\, x^{n-r}\, a^r + \cdots + {}^nC_n\, a^n$

2. $(x - a)^n = {}^nC_0\, x^n - {}^nC_1\, x^{n-1}\, a + {}^nC_2\, x^{n-2}\, a^2 - {}^nC_3\, x^{n-3}\, a^3 + \cdots + {}^nC_n\, (-a)^n$

Notice that

In the expansion of $(x + a)^n$ according to descending powers of x :

① Number of terms = (n + 1) terms

② In any term , the index of (x) + the index of (a) = n

③ The general term : $T_{r+1} = {}^nC_r\, a^r\, x^{n-r}$

i.e. The general term : $T_{r+1} = {}^nC_r \times (2^{nd}\text{ term})^r \times (\text{first term})^{n-r}$

④ $\frac{T_{r+1}}{T_r} = \frac{n - r + 1}{r} \times \frac{a}{x}$

⑤ The term order from the beginning

= number of terms of the expansion – the order from the end + 1

⑥ If n is even , then the order of the middle term = $\frac{n}{2} + 1$

⑦ If n is odd , then the order of the two middle terms are $\frac{n+1}{2}$, $\frac{n+3}{2}$

⑧ If we want to find the sum of the terms coefficient of expansion in a binomial expression , we have to put every value of variable in the binomial equals whole one without finding the expansion.

The sum of the term coefficients of expansion $(a\,x + b\,y)^n = (a + b)^n$

⑨ **If $x \in \mathbb{R}$, n is positive integer , then :**

1. $(1 + x)^n = 1 + {}^nC_1\, x + {}^nC_2\, x^2 + {}^nC_3\, x^3 + \cdots + x^n$

2. $(1 - x)^n = 1 - {}^nC_1\, x + {}^nC_2\, x^2 - {}^nC_3\, x^3 + \cdots + (-x)^n$

⑩ $(x + a)^n + (x - a)^n = 2\,(T_1 + T_3 + T_5 + \ldots)$ **i.e.** double the sum of the odd terms.

⑪ $(x + a)^n - (x - a)^n = 2\,(T_2 + T_4 + T_6 + \ldots)$ **i.e.** double the sum of the even terms.

The term including x^k in the binomial expansion

To find the term containing x^K in the expansion of $(x + a)^n$ according to the descending power of x , follow the following :

1. Find the term T_{r+1} in its simplest form to be able to determine the power of the variable x in terms of r
2. Equate the resultant power of x in the term T_{r+1} and the required power K to get the value of r and hence determine the term which contains x^K and it is T_{r+1}
3. Find the term which contains x^K by substituting by the value of r that we got in T_{r+1}

Remarks

1. If the value of r that we got doesn't belong to the set of natural numbers , then there is no term containing the required x^K in this expansion.
2. To find the term free of x , we consider that we need to get the term contains x^0

   i.e. Put the power of x in T_{r+1} equal zero , then get the value of r
3. In the expansion of $(1 + x)^n$

   ① If n is an even number , then: The greatest coefficient is the coefficient of the middle term $= {}^nC_{\frac{n}{2}}$

   ② If n is an odd number , then : The coefficients of the two middle terms are equal and each is the greatest coefficient in the expansion $= {}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$

   • In the expansion of $(1 - x)^n$ the greatest coefficient numerically (The absolute value) = the greatest coefficient in the expansion of $(1 + x)^n$
4. In the expansion of $(a\,x + b\,y)^n$ according to the descending power of x , given the values of x and y , the greatest term numerically T_{r+1} is the term which is greater than or equal to the previous terms and greater than or equal to the next terms

   i.e. The term which satisfies both conditions

   ① $\frac{T_{r+1}}{T_r} \geq 1$ "greater than or equal to the previous terms"

   i.e. $\frac{n-r+1}{r} \times \left|\frac{b\,y}{a\,x}\right| \geq 1$

   ② $\frac{T_{r+1}}{T_{r+2}} \geq 1$ "greater than or equal to the next terms"

   $\therefore \frac{T_{r+2}}{T_{r+1}} \leq 1$ $\quad\quad$ $\therefore \frac{n-(r+1)+1}{r+1} \times \left|\frac{b\,y}{a\,x}\right| \leq 1$

   By using these two conditions , you can find the greatest term and the greatest coefficient in the expansion.

✪ To evaluate the greatest coefficient in an expansion evaluate the greatest term when $x = y = 1$

Complex Numbers

The complex number

The algebraic form	The trigonometric form	The exponential form
$z = x + y\,i$	$z = r(\cos\theta + i\sin\theta)$	(Euler form) $z = r\,e^{\theta i}$

The algebraic form

- $z = x + y\,i$ where $x \in \mathbb{R}$, $y \in \mathbb{R}$, $i^2 = -1$
- $\bar{z} = x - y\,i$ "The conjugate number of Z"

  , $\left(\overline{z_1 \pm z_2}\right) = \bar{z}_1 \pm \bar{z}_2$, $\left(\overline{z_1 \times z_2}\right) = \bar{z}_1 \times \bar{z}_2$
- $z + \bar{z} = 2x$ "Pure real"

  , $z - \bar{z} = 2y\,i$ "Pure imaginary"

  , $z\,\bar{z} = x^2 + y^2 = r^2$ where r is the modulus of the complex number "$|z|$"
- If $z = x$, then $\bar{z} = x$

  , If $z = y\,i$, then $\bar{z} = -y\,i$
- If $z_1 = x_1 + y_1\,i$, $z_2 = x_2 + y_2\,i$, then :
  - $z_1 \pm z_2 = (x_1 \pm x_2) + (y_1 \pm y_2)\,i$
  - $z_1\,z_2 = (x_1\,x_2 - y_1\,y_2) + (x_1\,y_2 + x_2\,y_1)\,i$
  - $\dfrac{z_1}{z_2} = \dfrac{z_1 \times \bar{z}_2}{z_2 \times \bar{z}_2}$ "**i.e.** We multiply up and down by the conjugate of the denominator"
  - If $z_1 = z_2$, then : $x_1 = x_2$, $y_1 = y_2$

The trigonometric form

- If the complex number $z = x + y\,i$, then the trigonometric form of z is :

  $z = r(\cos\theta + i\sin\theta)$ where :
  - (r) is called modulus of the complex number $z = |z| = \sqrt{x^2 + y^2}$
  - (θ) is called the principle amplitude (argument) of the complex number z if $\theta \in]-\pi, \pi]$

  To identify the principle amplitude of the number z to change the algebraic form $x + y\,i$ to the trigonometric form we use the following diagram.

Notice that

If the complex number has modulus r and amplitude θ then :

$x = r\cos\theta$, $y = r\sin\theta$ and

$z = r\cos\theta + i\, r\sin\theta$

is the algebraic form.

If $x < 0$, $y > 0$ then the principle amplitude $\theta = \pi + \tan^{-1}\frac{y}{x}$

If $x > 0$, $y > 0$ then the principle amplitude $\theta = \tan^{-1}\frac{y}{x}$

If $x < 0$, $y < 0$ then the principle amplitude $\theta = -\pi + \tan^{-1}\frac{y}{x}$

If $x > 0$, $y < 0$ then the principle amplitude $\theta = \tan^{-1}\frac{y}{x}$

Remarks

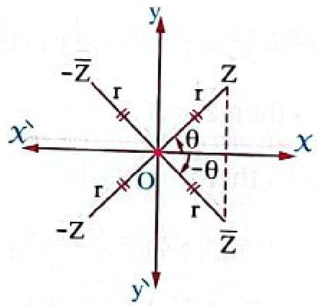
For each complex number $z = x + y\,i$ with argument (amplitude) θ :

1. $|z| \geq 0$, and $|z| = 0$ if $z = 0$
2. $|z| = |\bar{z}| = |-z| = |-\bar{z}|$
3. $z\bar{z} = |z|^2 = |\bar{z}|^2$
4. The amplitude of the complex number can be written in infinite forms by adding complete revolution (2π)

   i.e. The amplitude of the complex number $= \theta + 2\pi n$ where $n \in \mathbb{Z}$
5. The amplitude of the complex number doesnot change if the complex number is multiplied by positive real number.

   i.e. The amplitude of z = amplitude of $k\,z$ where $k \in \mathbb{R}^+$

From the graph :



- The number and its conjugate are symmetric about x-axis
- The number and its additive inverse are symmetric about origin.
- The number and its conjugate and its additive inverse and the conjugate of its additive inverse are all have the same modulus.

Convert the non-standard trigonometric form into standard form

- Determine the quadrant according to the sign in front of the two parts of the trigonometric functions , the real and the imaginary.

Second quadrant

* If $z = r(-\cos\theta + i\sin\theta)$ convert it to the form $z = r[\cos(180° - \theta) + i\sin(180° - \theta)]$

The trigonometric functions in their right positions.

* If $z = r(-\sin\theta + i\cos\theta)$ convert it to the form $z = r[\cos(90° + \theta) + i\sin(90° + \theta)]$

The trigonometric functions are reversed.

First quadrant

* If $z = r(\cos\theta + i\sin\theta)$ still as it is : $r(\cos\theta + i\sin\theta)$

The trigonometric functions in their right positions.

* If $z = r(\sin\theta + i\cos\theta)$ convert it to the form $z = r[\cos(90° - \theta) + i\sin(90° - \theta)]$

The trigonometric functions are reversed.

Third quadrant

* If $z = r(-\cos\theta - i\sin\theta)$ convert it to the form $z = r[\cos(-180° + \theta) + i\sin(-180° + \theta)]$

The trigonometric functions in their right positions.

* If $z = r(-\sin\theta - i\cos\theta)$ convert it to the form $z = r[\cos(-90° - \theta) + i\sin(-90° - \theta)]$

The trigonometric functions are reversed.

Fourth quadrant

* If $z = r(\cos\theta - i\sin\theta)$ convert it to the form $z = r[\cos(-\theta) + i\sin(-\theta)]$

The trigonometric functions in their right positions.

* If $z = r(\sin\theta - i\cos\theta)$ convert it to the form $z = r[\cos(-90° + \theta) + i\sin(-90° + \theta)]$

The trigonometric functions are reversed.

Notice that

* We use the previous way for $r > 0$, $\theta \in [0, 2\pi[$
* If the amplitude that we obtain $\in]-\pi, \pi]$, then it is a principle amplitude.
* If the amplitude that we obtain is not principle , then we add 360° to it or subtract 360° from it , to obtain the principle amplitude.

Summary

Exponential form of a complex number (Euler form)

Notice that

$$e^{ix} = 1 + \frac{ix}{\lfloor 1} + \frac{i^2 x^2}{\lfloor 2} + \frac{i^3 x^3}{\lfloor 3} + \frac{i^4 x^4}{\lfloor 4} + \cdots$$

$$= 1 + \frac{ix}{\lfloor 1} - \frac{x^2}{\lfloor 2} - \frac{ix^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - \cdots$$

$$= \left(1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \cdots\right) + i\left(\frac{x}{\lfloor 1} - \frac{x^3}{\lfloor 3} + \cdots\right)$$

$$\boxed{e^{ix} = \cos x + i \sin x}$$

The exponential form of a complex number is $\boxed{z = r\, e^{\theta i}}$ where r is the modulus of the number z , θ is the principle amplitude of the number z

Notice that

$i = \cos\frac{\pi}{2} + i \sin\frac{\pi}{2} = e^{\frac{\pi}{2} i}$, $-i = \cos -\frac{\pi}{2} + i \sin -\frac{\pi}{2} = e^{-\frac{\pi}{2} i}$

, $1 = \cos 0 + i \sin 0 = e^{\text{zero } i}$, $-1 = \cos \pi + i \sin \pi = e^{\pi i}$

Multiplying and dividing complex numbers

If z_1 , z_2 are two complex numbers

The algebraic form	The trigonometric form	The exponential form
$z_1 = x_1 + y_1 i$ $z_2 = x_2 + y_2 i$	$z_1 = r_1 (\cos \theta_1 + i \sin \theta_1)$ $z_2 = r_2 (\cos \theta_2 + i \sin \theta_2)$	$z_1 = r_1\, e^{\theta_1 i}$ $z_2 = r_2\, e^{\theta_2 i}$
$z_1 z_2 = (x_1 x_2 - y_1 y_2)$ $+ (x_1 y_2 + x_2 y_1) i$	$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2)$ $+ i \sin(\theta_1 + \theta_2)]$	$z_1 z_2 = r_1 r_2\, e^{(\theta_1 + \theta_2) i}$
$z_1 \div z_2 = \frac{x_1 + y_1 i}{x_2 + y_2 i} \times \frac{x_2 - y_2 i}{x_2 - y_2 i}$ **i.e.** Multiply both numerator and denominator by conjugate of the denominator	$z_1 \div z_2 = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2)$ $+ i \sin(\theta_1 - \theta_2)]$	$z_1 \div z_2 = \frac{r_1}{r_2}\, e^{(\theta_1 - \theta_2) i}$

Generalization

$z_1 z_2 \cdots z_n = r_1 r_2 \cdots r_n [\cos(\theta_1 + \theta_2 + \cdots + \theta_n) + i \sin(\theta_1 + \theta_2 + \cdots + \theta_n)]$

From the previous we can deduce that :

If $z = r(\cos\theta + i\sin\theta) = r e^{\theta i}$, then :

1. $z^n = r^n [\cos(n\theta) + i\sin(n\theta)] = r^n e^{n\theta i}$ where $n \in \mathbb{Z}^+$ and it is called De Moivre's theorem with positive integer power.

2. $z^{-1} = \frac{1}{z} = \frac{\cos 0 + i \sin 0}{r(\cos\theta + i\sin\theta)} = \frac{1}{r}[\cos(-\theta) + i\sin(-\theta)] = \frac{1}{r} e^{-\theta i}$

3. $\overline{z} = r\left(\cos(-\theta) + i\sin(-\theta)\right) = r e^{-\theta i}$

De Moivre's with positive rational power

It is used to find the n^{th} root of a complex number when it is written in trigonometric form

If $z = r(\cos\theta° + i\sin\theta°)$

, then $z^{\frac{1}{n}} = r^{\frac{1}{n}} \left[\cos\left(\frac{\theta + 360° k}{n}\right) + i\sin\left(\frac{\theta + 360° k}{n}\right)\right]$

for every $k = 0, 1, 2, \ldots, (k-1)$ where $k \in \mathbb{Z}^+$ and if the amplitude is not the principle amplitude it must be changed to the principle amplitude.

Remarks

The k^{th} roots of the complex number can be obtained directly such that the amplitude of each of them $\left(\frac{\theta + 2\pi n}{k}\right) \in]-\pi, \pi]$ putting $n = \cdots -2, -1, 0, 1, 2, \cdots$

Where if :

1. k is odd number , then $\frac{1-k}{2} \le n \le \frac{k-1}{2}$

i.e. $n = 0, 1, -1, 2, -2, \ldots$ to k values

2. k is even number , then

- $\frac{-k}{2} \le n < \frac{k}{2}$ when $\theta \in]0, \pi]$

  i.e. $n = 0, -1, 1, -2, 2, -3, \ldots$ to k values

  "**Notcie :** after zero we begin by the negative value"

- $\frac{-k}{2} < n \le \frac{k}{2}$ when $\theta \in]-\pi, 0]$

  i.e. $n = 0, 1, -1, 2, -2, 3, \ldots$ to k values

  "**Notice :** after zero we begin by the positive value"

For example :

1. To find the cubic roots of the complex number z , put $n = 0 , 1 , -1$ (Three values)

2. To find the 4^{th} roots of the complex number z

- If $\theta \in]0 , \pi]$, put

  $n = 0 , -1 , 1 , -2$ (4 values begin by the negative after zero)

- If $\theta \in]-\pi , 0]$, put

  $n = 0 , 1 , -1 , 2$ (4 values begin by the positive after zero)

The n^{th} roots

The equation $x^n = a$, where a is a complex number has n roots in the form $x = a^{\frac{1}{n}}$ and these roots lie on one circle where they are represented by Argand's diagram of radius $|a^{\frac{1}{n}}|$ and form vertices of regular polygon of n sides and the difference between the amplitude of each two consecutive roots $= \frac{360°}{n}$

Cubic roots of unity $(1 , \omega , \omega^2)$

The trigonometric form and the algebraic form of unity cubic roots :

- **The trigonometric form is :**

  $(\cos 0 + i \sin 0) , \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) , \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$

- **The algebraic form is :**

  $1 , -\frac{1}{2} + \frac{\sqrt{3}}{2} i , -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

  The cubic roots of the number one are complex numbers one of them is real "1" and the other roots are conjugate complex numbers , square of one of them is equal to the other.

- The sum of the cubic roots of unity = zero **i.e.** $1 + \omega + \omega^2 = 0$

$\omega + \omega^2 = -1$	$1 + \omega = -\omega^2$	$1 + \omega^2 = -\omega$

- product of the two not real cubic roots of unity = 1

  i.e. $\omega^3 = 1$ and so $\frac{1}{\omega} = \omega^2 , \frac{1}{\omega^2} = \omega$

- The difference between the two not real cubic roots of unity $= \pm\sqrt{3}\, i$

  i.e. $\boxed{\omega - \omega^2 = \pm\sqrt{3}\, i , \omega^2 - \omega = \pm\sqrt{3}\, i}$

Remarks

1. The conjugate of ω is ω^2 and so the conjugate of $(1 + \omega)$ is $(1 + \omega^2)$ and the conjugate of $(2018 + \omega)$ is $(2018 + \omega^2)$ and the conjugate of $(a\,\omega - b\,\omega^2)$ is $(a\,\omega^2 - b\,\omega)$ for every $a , b \in \mathbb{R}$

2. $\omega^{3n} = 1$, $\omega^{3n+1} = \omega$, $\omega^{3n+2} = \omega^2$ where $n \in \mathbb{Z}$

- **The n^{th} root of one : If $Z^n = 1$, then :**

$$z = (\cos 0° + i \sin 0°)^{\frac{1}{n}} = \cos \frac{2\pi k}{n} + i \sin \frac{2\pi k}{n} \text{ where } k \in \mathbb{Z} , \frac{2\pi k}{n} \in]-\pi , \pi]$$

- The n^{th} root of one on Argand's plane are represented by vertices of regular n-sided polygon and lies on a circle whose centre is the origin and its radius length is one and difference between each root amplitude and the next one is $\frac{360°}{n}$

Determinants

1. Second-order determinant $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

2. Third-order determinant $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Main properties of determinants

Properties

1. In any determinant if the rows are replaced by the columns in the same order , the value of the determinant is unchanged.

In other words :

The value of the determinant of a square matrix equals the value of the determinant of the transpose of this matrix.

2. The value of the determinant is unchanged by evaluating it in terms of the elements of any of its rows (columns).

3. The value of the determinant vanishes in each of the following cases.

Summary

1. If all the elements of any row (column) in any determinant equal zero.
2. If all the corresponding elements of any two rows (columns) in any determinant are equal.
3. If the elements in any row (column) are multiplies of elements in another row (column) in the determinant.

4. If there is a common factor in all elements of any row (column) in a determinant , then this factor can be taken outside the determinant.

5. In any determinant , if the positions of two rows (columns) are interchanged , the value of the resulting determinant equals the additive inverse of the value of the original determinant.

6. In any determinant if all the elements of any row (column) are written as a sum of two elements , then the value of the determinant can be written as a sum of two determinants.

7. If all the elements of any row (column) are added to the multiples of the elements of another row (column) , the value of the determinant is unchanged.

8. In any determinant , if the elements of any row (column) are multiplied by the cofactors of the corresponding elements of another row (column) and added their products then the result is equal to zero.

9. The value of the determinant in triangular form equals the product of the elements of its main diagonal.

i.e. The value of the determinant in triangular form = $a_{11} \times a_{22} \times a_{33}$

Notice that

The determinant whose all elements under or above the main diagonal are zeroes , the determinant is called the triangular form as in the following

The main diagonal

$$\begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Lower triangular form

The main diagonal

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix}$$

Upper triangular form

And the elements a_{11} , a_{22} , a_{33} are called the elements of main diagonal.

Remark

If $(x - a)$ is one of the factors of a determinant , then the value of the determinant at $x = a$ equals zero.

Matrices

The multiplicative inverse of matrix "A^{-1}"

A square matrix $A_{m \times m}$ has multiplicative inverse when the determinant of the matrix $\neq 0$

i.e. $\Delta \neq 0$ where $\Delta = |A|$

Notice that

The matrix which has no multiplicative inverse is known as singular (non invertible) matrix and the matrix that has multiplicative inverse known as non singular matrix.

First The multiplicative inverse of matrix of order 2 × 2

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

, then $A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Notice that

The adjoint matrix of the matrix $A_{2 \times 2}$ produced by interchanging the two elements of the main diagonal and change the signs of the two elements of the other diagonal.

Second The multiplicative inverse of matrix of order 3 × 3

If : A is a non-singular matrix. **i.e.** $|A| \neq 0$, then its multiplicative inverse.

$$A^{-1} = \frac{1}{\text{The determinant of the matrix}} \times \text{adjoint matrix}$$

$A^{-1} = \frac{1}{|A|} \times \text{adj}(A)$, where adj (A) is the transprose of the cofactor matrix.

- **How to find the cofactors matrix :**

If a_{ij} is an element of matrix A , then a_{ij} has cofactor denoted by $\overline{a_{ij}} = (-1)^{i+j} \times$ the determinant produced by eleminating the i row and the j column.

i.e. If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

, then the cofactors matrix of matrix A is

$$C = \begin{pmatrix} (-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & (-1)^{1+3}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ (-1)^{2+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & (-1)^{2+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (-1)^{2+3}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ (-1)^{3+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & (-1)^{3+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & (-1)^{3+3}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

Notice that

You can determine the sign of the cofactor of each element by using the sign rule with no need to use multiplying by $(-1)^{i+j}$ the sign rule : $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

Remarks

1. Let A be a matrix of order 2×2 say $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $\text{Adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

i.e. The adjoint matrix of 2×2 square matrix produced by interchanging the main diagonal elements and changing the sign of the elements of the secondary diagonal.

2. $|AB| = |A| \times |B|$ where A and B are square matrices.

3. If A is a square matrix of order $m \times m$, then :

① $|KA| = K^m |A|$ ② $|A^{-1}| = \frac{1}{|A|}$

③ $|\text{Adj } A| = |A|^{m-1}$

4. For any non singular square matrix A

$A.\text{adj}(A) = \text{adj}(A).A = \Delta I$ where $\Delta = |A|$

5. In the identity matrix I , the cofactors elements of the main diagonal elements , each = 1 and the cofactors elements of the remaining elements = 0 and so $\boxed{\text{Adj}(I) = I}$

i.e. The adjoint matrix of the identity matrix is the same identity matrix.

Some properties of the multiplicative inverse of a matrix

If A and B are two non singular matrices , then

1. $AA^{-1} = A^{-1}A = I$

2. $(AB)^{-1} = B^{-1}A^{-1}$

3. $(A^{-1})^{-1} = A$

4. $(A^{-1})^t = (A^t)^{-1}$

5. $(A^{-1})^2 = (A^2)^{-1}$

6. $(I)^{-1} = I$

The matrix equation

The system of linear equations formed from m equations and contains n variables could be expressed by matrix $\boxed{AX = B}$

Where A is the coefficients matrix , X is the variable matrix (unknowns) , B is the constant matrix.

Notice that

- If B = $\boxed{}$, then the system form system of homogeneous equations.
- If B ≠ $\boxed{}$, then the system form system of non-homogeneous equations.

Solving system of linear equations by using the multiplicative inverse of matrix

If The matrix form of system of linear equation is $AX = B$ **and** The cofactors matrix A is non singular matrix of order 2×2 or 3×3 **then** Elements of matrix $X = A^{-1} B$ are the values of the required variables (solution of the system of equations)

Rank of a matrix

- The rank of the non-zero matrix is the greatest order of determinant or minor determinant of the matrix whose value does not vanish.
- If A is a non-zero matrix of order $m \times n$, then the rank of the matrix A is denoted by RK (A) and
- $1 \le RK(A) \le n$ if $m \ge n$
- $1 \le RK(A) \le m$ if $m \le n$
- The rank of zero matrix = zero

i.e. If $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, then $Rk(A) = 0$

For Example

- If $A_{2 \times 2}$, then $1 \le Rk(A) \le 2$
- If $A_{4 \times 3}$, then $1 \le Rk(A) \le 3$

Summary

Remarks

1. The rank of the matrix A = 2 that means that two things are verified.

   ① There is determinant or minor determinant at least from the 2^{nd} order such that its value $\neq 0$

   ② The value of all minor determinants of order greater than 2 = zero

2. If A is a row (or column) non-zero matrix , then $R\,k\,(A) = 1$
3. If A is an identity matrix of order $n \times n$, then $R\,k\,(A) = n$
4. $R\,k\,(A) = R\,k\,(A^t)$
5. If a row (or a column) of zeroes is added to or removed from matrix A , then its rank does not change.
6. If a row (or column) formed from adding rows (columns) is added to or removed from the matrix , then its rank does not change.

The augmented matrix

If we have (m) linear equations in (n) variables , then we write $AX = B$ and we define the augmented matrix as $A^* = (A \,\vdots\, B)$ of the order $m \times (n + 1)$

For example : If system of equations

$$\boxed{\begin{array}{l} 3x - 4y + z = 7 \\ 7x + y - 3z = 1 \end{array}}$$, then the augmented matrix $A^* = \left(\begin{array}{ccc:c} 3 & -4 & 1 & 7 \\ 7 & 1 & -3 & 1 \end{array}\right)$

Investigation of possibility of solving system of n linear equations in n variables

① Write the matrix equation $\boxed{AX = B}$

② Find A^*

③ Find Rk (A) and Rk (A^*)

There are two cases

- **The non-homogeneous equations** i.e. $B \neq \square$
  - **The system has**
    - **unique solution** If $Rk(A) = Rk(A^*) = n$
    - **infinite solutions** If $Rk(A) = Rk(A^*) = k < n$
    - **No solution** If $Rk(A) \neq Rk(A^*)$
- **The homogeneous equations** i.e. $B = \square$

  In this case $Rk(A) = Rk(A^*)$ Because adding column of zeroes to the coefficient matrix does not change its rank and the system has
    - **unique solution is called zero solution "trivial solution"** If $Rk(A) = n$
    - **Infinite solution containing the zero solution** If $Rk(A) < n$

- If $A(x_1, y_1, z_1)$ is a point in the space where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in direction of $\overrightarrow{ox}, \overrightarrow{oy}, \overrightarrow{oz}$ respectively, "O" is the origin point, then :

1. Position vector of a point A with respect to O : $\overline{OA} = \overrightarrow{A} = (x_1, y_1, z_1)$

2. $\overrightarrow{A} = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$

3. $\| \overrightarrow{A} \|$ = length of $\overline{OA} = \sqrt{x^2 + y^2 + z^2}$

4. The unit vector in direction of $\overrightarrow{A} = \dfrac{\overrightarrow{A}}{\| \overrightarrow{A} \|}$

5. If $\theta_x, \theta_y, \theta_z$ are the angles that vector $\overrightarrow{A}$ makes with the positive directions of cartesian coordinates (direction angles of vector $\overrightarrow{A}$), then $\cos\theta_x, \cos\theta_y, \cos\theta_z$ are the direction cosines of vector $\overrightarrow{A}$ where $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

6. The vector $(-\overrightarrow{A}) = (-x_1, -y_1, -z_1)$ is the additive inverse of the vector $\overrightarrow{A}$ where $\overrightarrow{A} + (-\overrightarrow{A}) = \overrightarrow{O}$ and its direction angles are $\pi - \theta_x, \pi - \theta_y, \pi - \theta_z$

7. • **Generalization :**

If $(\theta_x, \theta_y, \theta_z)$ are the direction angles of the vector $\overrightarrow{A}$, then :

- $(\theta_x, \theta_y, \theta_z)$ are the direction angles of the vector $k\overrightarrow{A}$ where $k > 0$
- $(\pi - \theta_x, \pi - \theta_y, \pi - \theta_z)$ are the direction angles of the vector $k\overrightarrow{A}$ where $k < 0$

8. Unit vector in direction of $\overrightarrow{A}$

i.e. $\overrightarrow{U_A} = \dfrac{\overrightarrow{A}}{\| \overrightarrow{A} \|} = (\cos\theta_x, \cos\theta_y, \cos\theta_z)$ where $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

Notice that

direction angles of vectors in directions of positive directions of x, y and z axes are $(0, 90°, 90°)$, $(90°, 0, 90°)$, $(90°, 90°, 0)$ respectively and hence the direction cosines of them are $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$

9. If the vector $\overrightarrow{A}$ makes equal angles with the positive coordinate axes.

i.e. $\theta_x = \theta_y = \theta_z = \theta$, then $\cos\theta_x = \cos\theta_y = \cos\theta_z = \cos\theta$

, $\because \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$ $\quad \therefore 3\cos^2\theta = 1$ $\quad \therefore \cos^2\theta = \frac{1}{3}$

$\therefore \cos\theta = \frac{1}{\sqrt{3}}$ $\therefore \theta \simeq 54^\circ\ 44'\ 8''$

Or $\cos\theta = -\frac{1}{\sqrt{3}}$ $\therefore \theta \simeq 125^\circ\ 15'\ 52''$

10. * The sum of measures of two direction angles is greater than or equal to 90°

* If the measure sum of two directions angles is 90° , then the measure of the third angle is 90°

11. If $k \in \mathbb{R}^*$, then $k\vec{A} = k(x_1, y_1, z_1) = (kx_1, ky_1, kz_1)$ where $\vec{A} \parallel k\vec{A}$ and in the same direction if $k > 0$ and in opposite direction if $k < 0$

If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ are three points in 3-dimensional space , then :

1. Coordinates of midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

2. The directed line segment $\overrightarrow{AB} = \vec{B} - \vec{A} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

3. $\|\overrightarrow{AB}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

4. The direction angles of a vector $\overrightarrow{AB}$ (not passing through the origin) in the space are the measures of the direction angles made by another vector passing through the origin parallel to $\overrightarrow{AB}$ and its direction cosines are $\left(\frac{x_2 - x_1}{\|\overrightarrow{AB}\|}, \frac{y_2 - y_1}{\|\overrightarrow{AB}\|}, \frac{z_2 - z_1}{\|\overrightarrow{AB}\|}\right)$

5. $\vec{A} + \vec{B} = \vec{B} + \vec{A} = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

6. $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}) = \vec{A} + \vec{B} + \vec{C}$

7. $\vec{A} + \vec{O} = \vec{O} + \vec{A} = \vec{A}$ where $\vec{O} = (0, 0, 0)$

8. If $\vec{A} + \vec{B} = \vec{A} + \vec{C}$, then $\vec{B} = \vec{C}$

9. If $k\vec{A} = k\vec{B}$, then $\vec{A} = \vec{B}$

10. • $k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$ • $(k + \ell)\vec{A} = k\vec{A} + \ell\vec{A}$

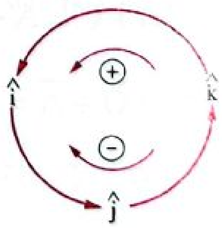
11. $\vec{A} = \vec{B}$ if and only if : $x_1 = x_2$, $y_1 = y_2$, $z_1 = z_2$

12. $\|\vec{B}\| = \|m\vec{A}\|$, then $\|\vec{B}\| = |m| . \|\vec{A}\|$

13. $\|\vec{A} + \vec{B}\| < \|\vec{A}\| + \|\vec{B}\|$

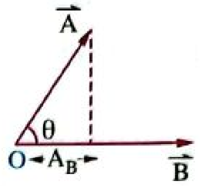
Summary

Comparison between the scalar product and the vector product

The scalar product	The vector product
$\overrightarrow{A} \cdot \overrightarrow{B} = \lVert\overrightarrow{A}\rVert \lVert\overrightarrow{B}\rVert \cos\theta$ (scalar quantity)	$\overrightarrow{A} \times \overrightarrow{B} = \left(\lVert\overrightarrow{A}\rVert \lVert\overrightarrow{B}\rVert \sin\theta\right) \overrightarrow{C}$ (vector quantity) where $\overrightarrow{C}$ is a perpendicular unit vector to the plane containing $\overrightarrow{A}$ and $\overrightarrow{B}$ and in direction shown by the right hand rule.
$\overrightarrow{A} \cdot \overrightarrow{B} = A_x B_x + A_y B_y + A_z B_z$	$\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$
$\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{B} \cdot \overrightarrow{A}$	$\overrightarrow{A} \times \overrightarrow{B} = -(\overrightarrow{B} \times \overrightarrow{A})$
$\overrightarrow{A} \cdot \overrightarrow{B} =$ zero and $\overrightarrow{A}$ and $\overrightarrow{B}$ are non-zero vectors , then $\overrightarrow{A}$ and $\overrightarrow{B}$ are perpendicular.	$\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{O}$ and $\overrightarrow{A}$ and $\overrightarrow{B}$ are non-zero vectors , then $\overrightarrow{A}$ and $\overrightarrow{B}$ are parallel.
$\overrightarrow{A} \cdot \overrightarrow{A} = \lVert\overrightarrow{A}\rVert^2$	$\overrightarrow{A} \times \overrightarrow{A} = \overrightarrow{O}$
• $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ • $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$	• $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \overrightarrow{O}$ • $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$ • $\hat{j} \times \hat{i} = -\hat{k}$ • $\hat{i} \times \hat{k} = -\hat{j}$ • $\hat{k} \times \hat{j} = -\hat{i}$ 
$\overrightarrow{A} \cdot \overrightarrow{O} = \overrightarrow{O} \cdot \overrightarrow{A} = \overrightarrow{O} \cdot \overrightarrow{O} = 0$	$\overrightarrow{A} \times \overrightarrow{O} = \overrightarrow{O} \times \overrightarrow{A} = \overrightarrow{O} \times \overrightarrow{O} = \overrightarrow{O}$
$(m\overrightarrow{A}) \cdot \overrightarrow{B} = \overrightarrow{A} \cdot (m\overrightarrow{B}) = m(\overrightarrow{A} \cdot \overrightarrow{B})$	$(m\overrightarrow{A}) \times \overrightarrow{B} = \overrightarrow{A} \times (m\overrightarrow{B}) = m(\overrightarrow{A} \times \overrightarrow{B})$
$(\overrightarrow{A} + \overrightarrow{B}) \cdot \overrightarrow{C} = \overrightarrow{A} \cdot \overrightarrow{C} + \overrightarrow{B} \cdot \overrightarrow{C}$	$(\overrightarrow{A} + \overrightarrow{B}) \times \overrightarrow{C} = \overrightarrow{A} \times \overrightarrow{C} + \overrightarrow{B} \times \overrightarrow{C}$

Remarks on scalar product

1. Algebraic component (algebraic projection) of vector $\overrightarrow{A}$ in direction of vector $\overrightarrow{B}$



$$A_B = \|\overrightarrow{A}\| \cos\theta = \frac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{B}\|}$$

2. Vector component of vector $\overrightarrow{A}$ in direction of vector $\overrightarrow{B} = \frac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{B}\|} \cdot \frac{\overrightarrow{B}}{\|\overrightarrow{B}\|} = \left(\frac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{B}\|^2}\right) \overrightarrow{B}$

3. $\|\overrightarrow{A} + \overrightarrow{B}\|^2 = \|\overrightarrow{A}\|^2 + 2(\overrightarrow{A} \cdot \overrightarrow{B}) + \|\overrightarrow{B}\|^2 = \|\overrightarrow{A}\|^2 + \|\overrightarrow{B}\|^2 + 2\|\overrightarrow{A}\|\|\overrightarrow{B}\| \cos\theta$

Remarks on vector product

- $\|\overrightarrow{A} \times \overrightarrow{B}\| = \|\overrightarrow{A}\|\|\overrightarrow{B}\| \sin\theta$, $\sin\theta = \frac{\|\overrightarrow{A} \times \overrightarrow{B}\|}{\|\overrightarrow{A}\|\|\overrightarrow{B}\|}$
- The unit vector in direction of $\overrightarrow{A} \times \overrightarrow{B}$ is $\overrightarrow{C} = \frac{\overrightarrow{A} \times \overrightarrow{B}}{\|\overrightarrow{A} \times \overrightarrow{B}\|} = \frac{\overrightarrow{A} \times \overrightarrow{B}}{\|\overrightarrow{A}\|\|\overrightarrow{B}\| \sin\theta}$
- If $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are three non-zero vectors and $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{A} \times \overrightarrow{C}$, then it is not necessary that $\overrightarrow{B} = \overrightarrow{C}$

The scalar triple product

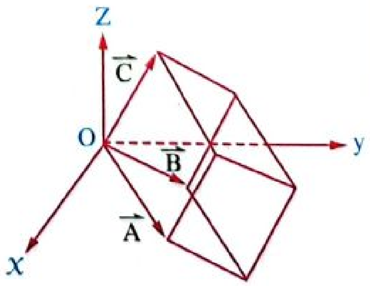
$$\overrightarrow{A} . \overrightarrow{B} \times \overrightarrow{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

Remember

The geometrical meaning of the norm of vector product and scalar triple product :

① $\|\overrightarrow{A} \times \overrightarrow{B}\|$ = area of parallelogram in which $\overrightarrow{A}$ and $\overrightarrow{B}$ are two adjacent sides = double the area of triangle in which $\overrightarrow{A}$ and $\overrightarrow{B}$ are two adjacent sides.

② If $\overrightarrow{A}$, $\overrightarrow{B}$ and $\overrightarrow{C}$ are three vectors represent three not parallel sides in parallelepiped , then its volume $= |\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C}|$



Important remarks related to vectors

If A , B , C , D are four points in the space , then :

1. To find measures of the smaller angle θ between two vectors $\overrightarrow{A}$, $\overrightarrow{B}$ use the formula

$$\cos\theta = \frac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{A}\| \|\overrightarrow{B}\|}$$

2. To prove that two non-zero vectors $\overrightarrow{A}$, $\overrightarrow{B}$ are parallel , prove that :

• $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{O}$ or $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$ or $\cos\theta = \pm 1$

Also the opposite is always true :

If $\overrightarrow{A} // \overrightarrow{B}$, then $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{O}$, $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$, $\cos\theta = \pm 1$

3. To prove that two non-zero vectors $\overrightarrow{A}$, $\overrightarrow{B}$ are perpendicular prove that :

• $\overrightarrow{A} \cdot \overrightarrow{B} = 0$ **i.e.** $A_x B_x + A_y B_y + A_z B_z = 0$ or $\cos\theta = 0$

Also the opposite is always true :

If $\overrightarrow{A} \perp \overrightarrow{B}$, then $\overrightarrow{A} \cdot \overrightarrow{B} = 0$ and $\cos\theta = 0$

4. To prove that the points A , B and C are collinear prove that : $\overrightarrow{AB} \times \overrightarrow{BC} = \overrightarrow{O}$

5. To prove that the points A , B and C lie in the same plane that passes through the origin , prove that :

$\overrightarrow{A} \cdot (\overrightarrow{B} \times \overrightarrow{C}) =$ zero which also prove that $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are coplanar position vectors.

6. To prove that the points A , B , C and D are coplanar points prove that

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} =$ zero

Equation of the sphere in space

1. Standard form : $\boxed{(x-\ell)^2 + (y-k)^2 + (z-n)^2 = r^2}$

and hence : centre of the sphere M (ℓ , k , n) and its radius length r

2. General form : $\boxed{x^2 + y^2 + z^2 + 2\ell x + 2ky + 2nz + d = 0}$

and hence : centre of the sphere M ($-\ell$, $-k$, $-n$) and its radius length

$r = \sqrt{\ell^2 + k^2 + n^2 - d}$

Remarks

1. In the general equation of the sphere it must be :

- Coefficient of x^2 = coefficient of y^2 = coefficient of $z^2 \neq$ zero
- $\ell^2 + k^2 + n^2 - d > 0$
- The equation has no term including xy , yz , xz or xyz

2. • The surface area of the sphere $= 4\pi r^2$ • The volume of the sphere $= \frac{4}{3}\pi r^3$

3. The sphere which touches the positive cartesian planes and its radius length is r
Its centre is the point (r , r , r)

4. The sphere whose centre is the origin and the point (a , b , c) lies on it :

- The radius length $r = \sqrt{a^2 + b^2 + c^2}$
- Its standard form is : $x^2 + y^2 + z^2 = a^2 + b^2 + c^2$

5. If a sphere has centre (a , b , c) and :

- Touches the xy plane then $r = |c|$
- Touches the xz plane then $r = |b|$
- Touches the yz plane then $r = |a|$

6. If a sphere has centre on

- The x-axis and touche the yz-plane then Centre (a , 0 , 0) , $r = |a|$
- The y-axis and touches the xz-plane then Centre (0 , b , 0) , $r = |b|$
- The z-axis and touches the xy-plane then Centre (0 , 0 , c) , $r = |c|$

7. Let M and N be two spheres with radii r_1 , r_2 respectively $(r_1 > r_2)$:

If the two spheres are	Then
(1) Distant	$MN > r_1 + r_2$
(2) Touching externally	$MN = r_1 + r_2$
(3) Intersecting	$r_1 - r_2 < MN < r_1 + r_2$
(4) Touching internally	$MN = r_1 - r_2$
(5) One inside the other	$MN < r_1 - r_2$
(6) Concentric	$MN =$ zero

Different forms of equation of the straight line in space

If L is a straight line in space where $A(x_1, y_1, z_1)$ is a given point on it and $\vec{d} = (a, b, c)$ is a direction vector of it , then :

1. $\vec{r} = \vec{A} + t\,\vec{d}$ "vector form of the equation of the straight line"

- $\vec{r}$: position vector of any point on the straight line
- $\vec{A}$: position vector of a given point on the straight line
- t : Real number
- $\vec{d}$: Direction vector of the straight line

2. $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$ "Cartesian equation of the straight line"

3. $x = x_1 + t\,a$, $y = y_1 + t\,b$, $z = z_1 + t\,c$ "Parametric equations of the straight line"

Remarks

1. If θ_x , θ_y , θ_z are directed angles of straight line ℓ , then $(\cos\theta_x , \cos\theta_y , \cos\theta_z)$ is a unit vector in direction of the straight line and direction vector for it.

2. In the cartesian equation of the straight line $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$

where (x_1 , y_1 , z_1) is a point on it and (a , b , c) is a direction vector of it :

If

- $a = 0$ The equation is $x = x_1$, $\dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$
- $b = 0$ The equation is $y = y_1$, $\dfrac{x - x_1}{a} = \dfrac{z - z_1}{c}$
- $c = 0$ The equation is $z = z_1$, $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b}$

3. Equation of

- x-axis is $y = 0$, $z = 0$ and its direction vector is $(k , 0 , 0)$
- y-axis is $x = 0$, $z = 0$ and its direction vector is $(0 , k , 0)$
- z-axis is $x = 0$, $y = 0$ and its direction vector is $(0 , 0 , k)$

where $k \neq 0$

4. Equation of the straight line passing through the point (x_1, y_1, z_1) and parallel to

- x-axis is → $y = y_1, z = z_1$
- y-axis is → $x = x_1, z = z_1$
- z-axis is → $x = x_1, y = y_1$

5. Different forms of the equation of the straight line passing through the origin and the vector (a , b , c) is direction vector are :

* $\vec{r} = t(a, b, c)$ "vector form"

* $x = ta$, $y = tb$, $z = tc$ "parametric equations"

* $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ "cartesian form"

6. If A and B are two points on the straight line , then :

- Direction vector of the straight line $\simeq \overrightarrow{AB} = \vec{B} - \vec{A}$
- $k(\overrightarrow{AB})$ where $k \in \mathbb{R}^*$ is also direction vector of the same straight line.

7. If a straight line passing through the origin and the point $A(x_1, y_1, z_1)$, then $\vec{A} = (x_1, y_1, z_1)$ is direction vector of the straight line.

8. The straight line whose direction vector $\vec{d} = (a, b, 0)$ lies in a plane parallel to the plane xy , similarly the straight line whose direction vector $\vec{d} = (a, 0, c)$ lies in a plane parallel to the plane xz and the straight line whose direction vector $\vec{d} = (0, b, c)$ lies in a plane parallel to the plane yz

- Notice the difference between directed cosines of a straight line and its direction vector :

  If ℓ , m , n are directed cosines of the straight line where (ℓ, m, n) is unit vector in the direction of the straight line and $\ell^2 + m^2 + n^2 = 1$

  "because $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$"

  If a , b , c are the direction ratios of the same straight line , then (a , b , c) is direction vector of the straight line , $(a, b, c) = k(\ell, m, n)$, $k \neq 0$

- $(\ell, m, n) = \frac{(a, b, c)}{\sqrt{a^2 + b^2 + c^2}}$

Equation of a plane in space

Let A (x_1 , y_1 , z_1) be a given point on the plane P its position vector $\vec{A}$ and $\vec{n}$ = (a , b , c) be the normal direction vector to the plane :

- $\boxed{\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}}$ "The vector form of the equation of the plane"
- $\boxed{a(x - x_1) + b(y - y_1) + c(z - z_1) = 0}$ "The standard form of the equation of the plane"
- $\boxed{a x + b y + c z + d = 0}$ "The general form of the equation of the plane"

Also the equation of the plane can be obtained in the following cases :

1. Equation of the plane intersects the coordinate axes at the points (x_1 , 0 , 0) , (0 , y_1 , 0) , (0 , 0 , z_1) is : $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$

2. If three known points A (x_1 , y_1 , z_1) , B (x_2 , y_2 , z_2) and C (x_3 , y_3 , z_3) are lying on a plane but they are not collinear , then the equation of this plane can be obtained using the following steps.

- Find the vector product $\overrightarrow{AB} \times \overrightarrow{BC}$ to find the normal vector to the plane.
- Use one of the three points.
- State the vector equation of the plane $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

also it can be obtained directly from the determinant $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

Remarks

From the general equation of the plane P : a x + b y + c z + d = 0 , we deduce that :

- (a , b , c) is normal direction vector to the plane P , d = $-\vec{A} \cdot \vec{n}$ where $\vec{A}$ is the position vector of a point $\in$ the plane and $\vec{n}$ is a normal direction vector.
- (a , b , c) is normal direction vector to any other plane parallel to the plane P
- If d = 0 , then the origin point is contained in the plane.
- If
  - a = 0 → then the equation is → b y + c z + d = 0 → and it is equation of plane → parallel to x-axis → and perpendicular to → y z
  - b = 0 → then the equation is → a x + c z + d = 0 → and it is equation of plane → parallel to y-axis → and perpendicular to → x z
  - c = 0 → then the equation is → a x + b y + d = 0 → and it is equation of plane → parallel to z-axis → and perpendicular to → x y

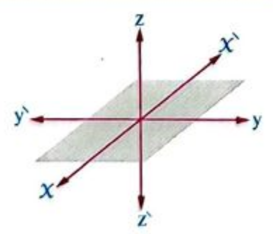
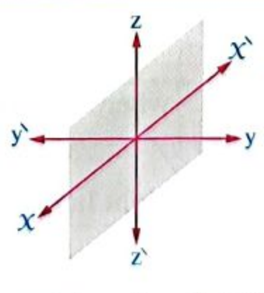
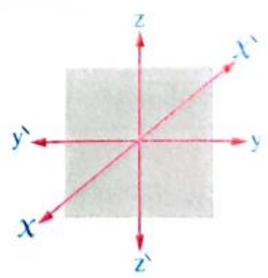
- If $a = d = 0$ → then the equation is → $b y + c z = 0$ → and it is equation of plane → contains x-axis → and perpendicular to → $y z$
- If $b = d = 0$ → then the equation is → $a x + c z = 0$ → and it is equation of plane → contains y-axis → and perpendicular to → $x z$
- If $c = d = 0$ → then the equation is → $a x + b y = 0$ → and it is equation of plane → contains z-axis → and perpendicular to → $x y$

- The equation of the plane $x y$ is $\boxed{z = 0}$, the equation of the plane parallel to x y-plane and passes through the point (x_1 , y_1 , z_1) is $\boxed{z = z_1}$
- The equation of the plane y z is $\boxed{x = 0}$, the equation of the plane parallel to y z and passes through the point (x_1 , y_1 , z_1) is $\boxed{x = x_1}$
- The equation of the plane x z is $\boxed{y = 0}$, the equation of the plane parallel to x z and passes through the point (x_1 , y_1 , z_1) is $\boxed{y = y_1}$
- If $E (x_1 , y_1 , z_1)$, $F (x_2 , y_2 , z_2)$, $G (x_3 , y_3 , z_3)$ are three points in the space and substituting by each of them in the plane equation gives :

- $a x_1 + b y_1 + c z_1 + d = 0$
- $a x_2 + b y_2 + c z_2 + d > 0$
- $a x_3 + b y_3 + c z_3 + d < 0$

That means $E (x_1 , y_1 , z_1)$ belonges to plane P and each of $F (x_2 , y_2 , z_2)$ and $G (x_3 , y_3 , z_3)$ does not belong to the plane and each of them lies in different side from , the plane P

- **Cartesian planes :**

x y plane	x z plane	y z plane
		
It contains all points in the space whose coordinates are $(x , y , 0)$ and its equation is $z = 0$	It contains all points in the space whose coordinates are $(x , 0 , z)$ and its equation is $y = 0$	It contains all points in the space whose coordinates are $(0 , y , z)$ and its equation is $x = 0$

Remarks

Equation of the plane contains all the points in the form.

- (x , y , a) → is the equation of the plane parallel to the plane → xy – which is → $\boxed{z = a}$
- (x , a , y) → is the equation of the plane parallel to the plane → xz – which is → $\boxed{y = a}$
- (a , y , z) → is the equation of the plane parallel to the plane → yz – which is → $\boxed{x = a}$

→ Where a is constant $\in \mathbb{R}$

✪ The angle between (two vectors - two straight lines - two planes - a plane and a straight line).

1. Let $\overrightarrow{A}$ and $\overrightarrow{B}$ be two non-zero vectors , including the smaller angle θ

Two vectors: $\boxed{\cos\theta = \dfrac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{A}\|\|\overrightarrow{B}\|}}$ where $0° \le \theta \le 180°$

2. The angle θ between two straight lines ℓ_1 , ℓ_2 in the space where their direction vectors are $\overrightarrow{d_1} , \overrightarrow{d_2}$ can be found from the relation :

Two straight lines: $\boxed{\cos\theta = \dfrac{|\overrightarrow{d_1} \cdot \overrightarrow{d_2}|}{\|\overrightarrow{d_1}\| \; \|\overrightarrow{d_2}\|}}$ where $0° \le \theta \le 90°$

and if (ℓ_1 , m_1 , n_1) and (ℓ_2 , m_2 , n_2) are the directed cosines of the two straight lines , then $\cos\theta = |\ell_1 \ell_2 + m_1 m_2 + n_1 n_2|$

3. The included angle θ between two planes where $\overrightarrow{n_1}$ is the normal direction vector on first plane and $\overrightarrow{n_2}$ is the normal direction vector on second plane can be calculated using the relation

Two planes: $\boxed{\cos\theta = \dfrac{|\overrightarrow{n_1} \cdot \overrightarrow{n_2}|}{\|\overrightarrow{n_1}\|\|\overrightarrow{n_2}\|}}$ where $0° \le \theta \le 90°$

4. The measure of the angle between a straight line has direction vector $\overrightarrow{d}$ and a plane has normal vector $\overrightarrow{n}$ is $(90° - \theta)$ where : $\cos\theta = \dfrac{|\overrightarrow{n} \cdot \overrightarrow{d}|}{\|\overrightarrow{n}\| \; \|\overrightarrow{d}\|}$

- **Condition of parallelism of two (straight lines - planes) in space :**

1. Two straight lines L_1 , L_2 in space are parallel if their direction vectors are parallel.

i.e. $\boxed{\overrightarrow{d_1} \parallel \overrightarrow{d_2}}$

2. Two planes in space are parallel if their normal direction vectors are parallel.

i.e. $\boxed{\overrightarrow{n_1} \parallel \overrightarrow{n_2}}$

- **Condition of perpendicularity of two (straight lines - planes) in space :**

1. Two straight lines are perpendicular if their direction vectors are perpendicular.

i.e. $\boxed{\overrightarrow{d_1} \perp \overrightarrow{d_2}}$

2. Two planes are perpendicular if their normal direction vectors are perpendicular.

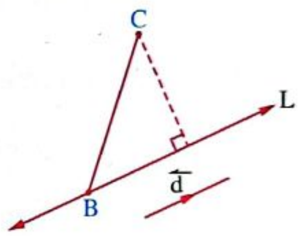
i.e. $\boxed{\overrightarrow{n_1} \perp \overrightarrow{n_2}}$

- **The length of the perpendicular drawn from a point to a straight line in space :**

Let L be a straight line in space , B is a point on it.

The direction vector of the straight line L is $\vec{d}$

, then the distance between C and the straight line L $= \dfrac{\| \overrightarrow{CB} \times \vec{d} \|}{\| \vec{d} \|}$



- **Length of the perpendicular drawn from a point to a plane :**

If the general equation of the plane is : $a\,x + b\,y + c\,z + d = 0$

, then length of the perpendicular drawn from a point B (x_1 , y_1 , z_1)

to the plane is $\boxed{\ell = \dfrac{|a\,x_1 + b\,y_1 + c\,z_1 + d|}{\sqrt{a^2 + b^2 + c^2}}}$

- The length of the perpendicular drawn from the point (x_1 , y_1 , z_1) to ... or the distance between the point (x_1 , y_1 , z_1) and ...
  - the x-axis $= \sqrt{y_1^2 + z_1^2}$
  - the y-axis $= \sqrt{x_1^2 + z_1^2}$
  - the z-axis $= \sqrt{x_1^2 + y_1^2}$

- The length of the perpendicular drawn from the point (x_1 , y_1 , z_1) to ... or the distance between the point (x_1 , y_1 , z_1) and ...
  - The plane y z $= |x_1|$
  - The plane x z $= |y_1|$
  - The plane x y $= |z_1|$

Summary

1. As solving two straight lines equations simultaneously and →

(1) The S.S. = $\varnothing$, then the two lines are skew or parallel.

(2) The S.S. = one point , then the two lines are intersecting and contained in one plane.

- If the two lines intersect in more than one point , then they are coincident.

2. As solving the equations of a straight line and a plane simultaneously and →

(1) The S.S. = $\varnothing$, then the straight line is parallel to the plane.

(2) The S.S. = one point , then the straight line intersects the plane in this point.

- If the straight line and the plane are intersecting in more than one point , then the plane contains this line.

Equation of intersection line of two planes :

If $P_1 : a_1 x + b_1 y + c_1 z + d_1 = 0$, $P_2 : a_2 x + b_2 y + c_2 z + d_2 = 0$

are equations of two distinct planes in the space and the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ one of them is different at least , then the two planes are intersecting and equation of the intersection line between them can be found by eliminating variable z and find the value of x in terms of y and eliminating variable y and find the value of x in terms of z , then deduce the equation of the intersection line.

Determine a point on a straight line :

For example : to determine a point on a straight line $\vec{r} = (5 , -4 , 2) + t (3 , 6 , -5)$, replace the parameter t with any value $k \in \mathbb{R}$

i.e. Replace t with 1 , then $(8 , 2 , -3)$ belongs to this straight line.

Determine a point belongs to a plane :

For example : to determine a point belongs to the plane : $2x + 3y - 5z + 9 = 0$ replace two variables with two real numbers and calculate the third.

i.e. Put $x = 0$, $y = 2$ in the equation so $2(0) + 3(2) - 5z + 9 = 0$

$\therefore z = 3$ so the point $(0 , 2 , 3)$ belongs to the plane.

Determine a point belongs to the line of intersection of two planes :

For example : to determine a point belongs to the line of intersection of the two planes $x + 4y - z = 5$, $3x - y + 2z = 4$ replace any variable with a real number.

i.e. Put $x = 2$ in the two equations

$\therefore 4y - z = 3$, $-y + 2z = -2$ and solve the two equations simultaneously.

$\therefore y = \frac{4}{7}$, $z = \frac{-5}{7}$

$\therefore$ The point $\left(2 , \frac{4}{7} , \frac{-5}{7}\right)$ belongs to the line of intersection of the two planes.

Remarks

1. The two parallel straight lines are contained in the same plane.

2. The two intersecting straight lines are contained in the same plane.

3. The two perpendicular straight lines : either are intersecting orthogonally and hence contained in the same plane or are skew and hence they are not contained in one plane.

4. If the two straight lines are parallel and there is a point on one of them satisfying the equation of the other , then the two straight lines are coincident.

5. In the two planes P_1 , P_2

$P_1 : a_1 x + b_1 y + c_1 z + d_1 = 0$

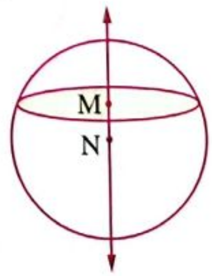
$P_2 : a_2 x + b_2 y + c_2 z + d_2 = 0$

① If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \neq \frac{d_1}{d_2}$, then the two planes are parallel and not coincident.

② If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$, then the two planes are coincident.

6. To find the distance between two parallel planes in space, find a point on one of them , then find the length of the perpendicular from this point to the other plane.

7. The straight line passing through the centre of the sphere and centre of the circle resulted from the intersection of a plane and this sphere is perpendicular to the plane of the circle.



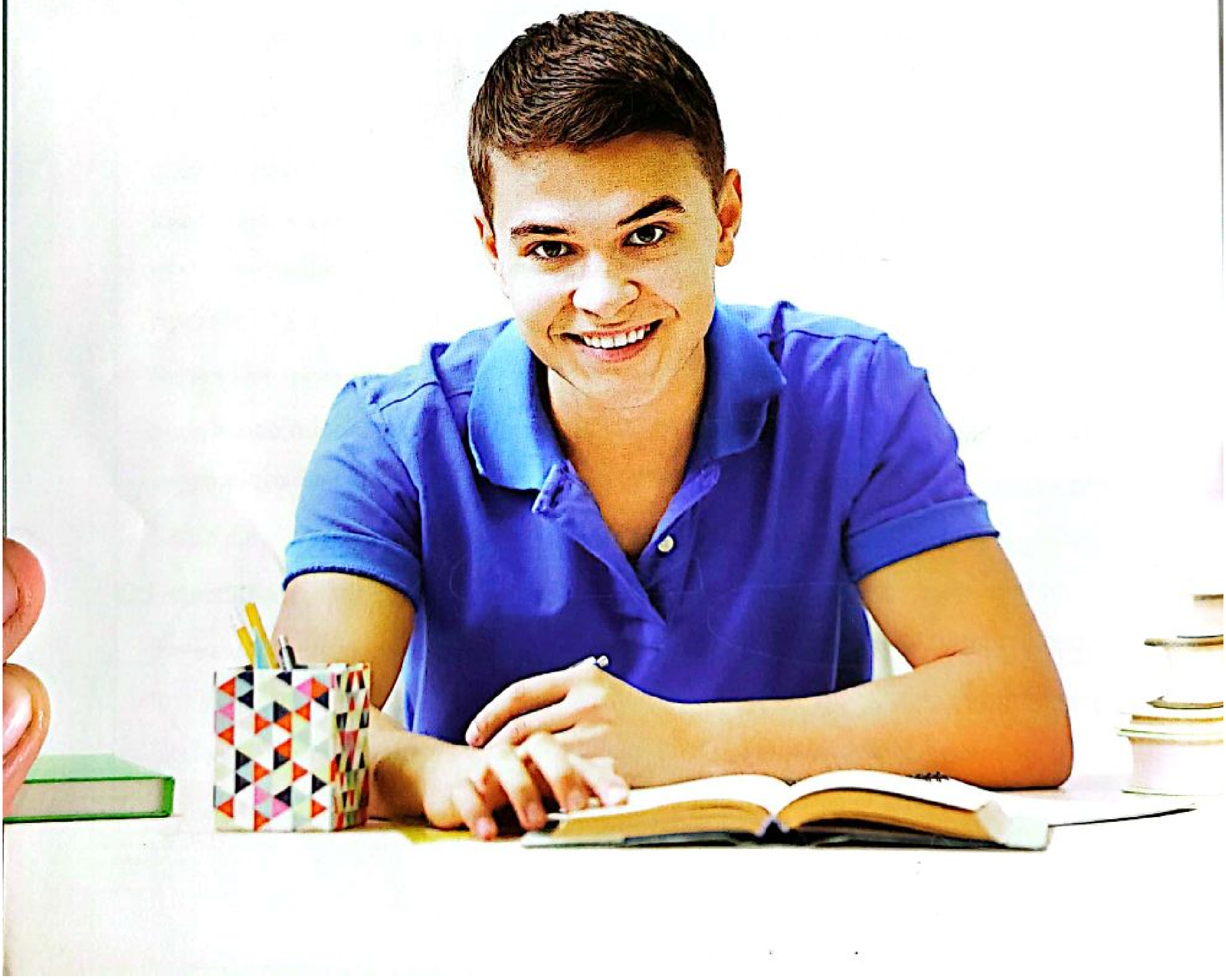
For example :

If a plane intersects a sphere whose centre N in a circle whose centre M , then $\overleftrightarrow{MN}$ is perpendicular to the plane of the circle.

Multiple Choice Question Bank

In

Algebra & Analytic Solid Geometry



18 The number of ways to choose 4-different letters together at least from the elements of the set $\{a , b , c , d , e\}$ is ....

(a) ${}^5C_4 + {}^5C_5$ (b) ${}^5C_4 \times {}^5C_5$ (c) ${}^5P_4 + {}^5P_5$ (d) ${}^5P_4 \times {}^5P_5$

19 Number of ways to select a team from 4 persones from same gender out of 9 boys and 6 girls equals ....

(a) ${}^{15}C_4$ (b) ${}^9C_4$ (c) ${}^9C_4 \times {}^6C_4$ (d) ${}^9C_4 + {}^6C_4$

20 The number of ways of selecting a team of 7 members out of 9 girls and 5 boys , if the team has 3 boys only equals ....

(a) 136 (b) 3084 (c) 1260 (d) 1287

21 The number of ways of choosing 3 persons together out of a set of 5 men and 3 women , if two persons from the three are of the same gender equals ....

(a) ${}^5C_2 + {}^3C_1$ (b) ${}^5C_1 + {}^3C_2$

(c) ${}^5C_2 \times {}^3C_1$ (d) ${}^5C_2 \times {}^3C_1 + {}^5C_1 \times {}^3C_2$

22 The number of ways of distributing 3 identical balls on 4 different boxes = ...

(a) ${}^4C_3$ (b) ${}^5C_3$ (c) ${}^6C_3$ (d) ${}^4P_3$

23 The number of ways can 3 identical balls be put in 5 places in shape of a row if the place can hold only one ball is ....

(a) 5^3 (b) ${}^5P_3$ (c) ${}^7C_5$ (d) ${}^5C_3$

24 The number of ways of distributing 8 prizes equally among 4 students ....

(a) ${}^8C_4$ (b) ${}^8C_3$

(c) ${}^8C_2 + {}^6C_2 + {}^4C_2 + 1$ (d) ${}^8C_2 \times {}^6C_2 \times {}^4C_2$

25 The number of ways can 8 identical balls be distributed on 3 distincts boxes non being empty ?

(a) 21 (b) 28 (c) 42 (d) 56

26 The number of ways of distributing 15 identicals cards on 4 persons such that every one get at least 2 cards equals ....

(a) ${}^{15}C_4$ (b) ${}^{7}C_4$ (c) ${}^{10}C_4$ (d) ${}^{10}C_7$

27 In a football league , each two teams play together once , if the number of matches in this league is 153 matches , then the number of the competing teams equals ....

(a) 9 (b) 13 (c) 18 (d) 19

28 *(Trial 2021)* The number of ways a person can choose to play at least 3 games of (football , handball , volleyball and basketball) equals ....

(a) ${}^{4}C_3 + {}^{4}C_4$ (b) ${}^{4}C_3 \times {}^{4}C_4$ (c) ${}^{4}P_3 + {}^{4}P_4$ (d) ${}^{4}P_3 \times {}^{4}P_4$

29 The number of ways to choose a team consists of 11 players from 22 players but 4 of them are always excluded and two are always included equals ....

(a) ${}^{16}C_{11}$ (b) ${}^{16}C_9$ (c) ${}^{20}C_9$ (d) ${}^{18}C_{11}$

30 If the number of ways to select 3 elements together from a set equals the number of ways to select 5 elements together from the same set , then the number of elements of this set equals ....

(a) ${}^{5}C_3$ (b) ${}^{5}P_3$ (c) 8 (d) 15

31 If X is non empty set contains (n) elements and $Y = \{(a , b) , a \in X , b \in X , a \neq b\}$ and has 72 elements , $Z = \{\{a , b\} , a \in X , b \in Y\}$, then the number of elements of Z = ...

(a) 12 (b) 18 (c) 24 (d) 36

32 A password consists of 4 digits from the set of digits $\{1 , 2 , 5 , 7 , 8\}$ What is the greatest number of wrong trials can be done before you got the right password ?

(a) 1023 (b) 624 (c) 124 (d) 120

33 *(1st Session 2021)* An office has 9 men and 6 women. It is required to form a committee of 5 persons and the majority of them should be women and contains the two genders , then the number of committees equals ....

(a) 11880 (b) 2871 (c) 3003 (d) 855

34 If the secret number of a lock consists of 3 different digits from the digits $\{1, 2, 3, ..., 9\}$ in how many ways a secret number can be formed including 6 ?

(a) 168 (b) 126 (c) 336 (d) 224

35 In one of the governorates , the license plates of cars consist of 3 different alphabets followed by 4 different digits. If the number of alphabets used is 26 and the digits used are $(1, 2, 3, ..., 9)$, then the number of plates that can be created in this governorate is equal to ....

(a) ${}^{26}P_3 \times {}^{9}P_4$ (b) ${}^{26}P_3 + {}^{9}P_4$ (c) ${}^{26}C_3 \times {}^{9}C_4$ (d) ${}^{26}C_3 + {}^{9}C_4$

36 A comitee of 12 members , by how many ways a president , vice president and two assistants can be selected for this comitee ....

(a) 480 (b) 495 (c) 11880 (d) 5940

37 Number of different ways which a person can invite a friend or more from 6 friends equals ....

(a) 15 (b) 30 (c) 63 (d) 120

38 In a car-park , if 5 cars get in one after another and there were 7 places in a form of a row , then the number of ways of occupying these places equals ....

(a) ${}^{7}C_5$ (b) ${}^{7}P_5$ (c) $\underline{|5}$ (d) $\underline{|7}$

39 The student should answer 10 questions out of 13 questions on condition that he should answer 4 questions at least from the first 5 questions. How many ways can he answer ?

(a) 140 (b) 196 (c) 280 (d) 346

40 The number of ways of acceptance decision by majority for a committee consists of 5 persons equals ....

(a) 16 (b) 50 (c) 300 (d) 120

41 The number of ways of selection of two committees each of them consists of 3 persons out of 12 persons , in condition that no person can participate in the two committees is ....

(a) ${}^{12}C_3 \times {}^{9}C_3$ (b) ${}^{12}C_3 + {}^{9}C_3$ (c) ${}^{12}P_3 + {}^{9}C_3$ (d) ${}^{12}P_3 + {}^{9}P_3$

42 The number of diagonals of m-sided polygon is ....

ⓐ $^{m}C_2$ ⓑ $^{m}C_2 - 2$ ⓒ $^{m}C_2 - m$ ⓓ $^{m}P_2 - m$

43 Number of diagonals of a polygon which the number of its sides is 8 equals .... diagonal.

ⓐ 20 ⓑ 8 ⓒ 28 ⓓ 10

44 The polygon which has 44 diagonals the number of its sides is ....

ⓐ 11 ⓑ 10 ⓒ 12 ⓓ 13

45 The number of triangles can be drawn such that its vertices are vertices of an octagon is ....

ⓐ 8^3 ⓑ $^{8}P_3$ ⓒ $^{8}C_3$ ⓓ $^{10}C_8$

46 If a , b , c , d , e are points on a circle , then the number of polygons can be drawn from these points equals ....

ⓐ 5 ⓑ 50 ⓒ 16 ⓓ 9

47 The number of parallelograms can be formed from 10 parallel horizontal lines and 6 parallel vertical lines lie on the same plane = ...

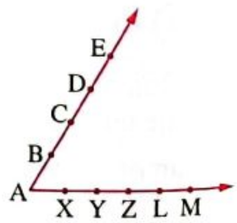
ⓐ $^{10}P_2 \times {}^{6}P_2$ ⓑ $^{10}C_2 \times {}^{6}C_2$ ⓒ $^{10}C_2 + {}^{6}C_2$ ⓓ $^{10}P_2 + {}^{6}P_2$

48 By using 10 coplanar points , such that there are no 3 points of them are on the same straight line , if the number of line segments can be drawn = m , and the number of the directed line segments can be drawn = n , then n + m = ...

ⓐ 90 ⓑ 135 ⓒ 180 ⓓ 210

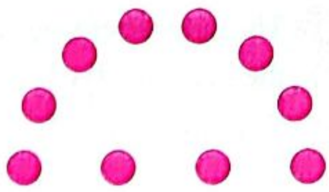
49 *(2^{nd} session 2021)* **In the opposite figure :**

The ten points lie on the two rays starting from the point A , then the number of different straight lines that can be drawn using these points equals ....



ⓐ 22 ⓑ 45 ⓒ 90 ⓓ 30

50 **In the opposite figure :**



10 points are coplanar , 4 of them are collinear and the other points positioned such that there are no 3 points on the same straight line , then

First : Number of straight lines can be drawn equals ....

ⓐ 40 ⓑ 45 ⓒ 50 ⓓ 90

Second : Number of triangles can be formed equals ....

ⓐ 56 ⓑ 80 ⓒ 96 ⓓ 116

Third : Number of quadrilaterals can be formed equals ....

ⓐ 95 ⓑ 170 ⓒ 185 ⓓ 195

---

51 If you have the numbers 1 , 2 , 3 , 4 , 5 and you can repeat any of them , how many even numbers more than 300 and smaller than 100000 can be formed ?

ⓐ 111 ⓑ 812 ⓒ 1530 ⓓ 2540

---

52 If you have the numbers 1 , 2 , 3 , 4 , 5 and you can not repeat any of them , how many numbers more than 300 and smaller than 100000 can be formed ?

ⓐ 111 ⓑ 812 ⓒ 1530 ⓓ 2540

---

53 The number of ways to form an even number consists of 3 different digits and greater than 500 out of $\{0 , 2 , 3 , 6 , 7\}$ equals ....

ⓐ 15 ⓑ 18 ⓒ 30 ⓓ 75

---

54 4 males , 3 females and 2 children are to be seated in a circle , then the number of their arrangements = ...

ⓐ $\underline{|8}$ ⓑ $\underline{|9}$ ⓒ $\frac{1}{2}\underline{|8}$ ⓓ $\frac{1}{2}\underline{|9}$

---

55 Number of ways to park 4 cars next to each other in a parking area with 10 places to park if the parking area in a shape of a circle ....

ⓐ $10\underline{|4}$ ⓑ $7\underline{|4}$ ⓒ $9\underline{|4}$ ⓓ $9\underline{|4}$

---

56 Number of ways to park 4 cars next to each other in a parking area with 10 places to park if the parking area in shape of a row ....

ⓐ $7\underline{|3}$ ⓑ $7\underline{|4}$ ⓒ $9\underline{|3}$ ⓓ $9\underline{|4}$

57 The number of ways of forming a 5-digit number of different digits divisible by 3 from the digits 0 , 1 , 2 , 3 , 4 , 5 equals ....

(a) 216 (b) 240 (c) 600 (d) 3125

58 The number of diagonals in a regular polygon is double the number of its sides , then the number of its sides equals ....

(a) 5 (b) 6 (c) 7 (d) 8

59 Number of ways to arrange 4 men and 3 women in row where the women are consecutive is ....

(a) $\lfloor 4 \lfloor 3$ (b) $\lfloor 5$ (c) $5 \lfloor 3$ (d) $\lfloor 5 \lfloor 3$

60 The number of ways that 4 boys and 3 girls can be seated in a row of 7 seats such that boys sit beside each other and girls also sit beside each other equals ....

(a) $\lfloor 7$ (b) $\lfloor 4 \lfloor 3$ (c) ${}^7P_3 \times {}^7P_4$ (d) $2 \lfloor 4 \lfloor 3$

Second | Questions on permutations and combinations rules

Choose the correct answer from the given ones :

1 If ${}^nC_3 = 4$, then n = ... ${}^nP_3$ $\frac{\lfloor n}{\lfloor 3 \lfloor n-3} = 4$

(a) 3 (b) 4 (c) 5 $4 \lfloor 3 \lfloor n-3 = \lfloor n$ (d) 6

$(n-3)(n-2)(n-1)$

2 If ${}^6C_r = {}^5C_r$, then r = ...

(a) 5 (b) 6 (c) zero (d) 11

3 If $\lfloor n = 720$, then ${}^nP_2 =$ ....

(a) 30 (b) 20 (c) 15 (d) 12

4 If ${}^nC_5 = {}^nC_{n-2}$, then n = ...

(a) 6 (b) 7 (c) 8 (d) 10

5 If ${}^{17}P_{m+5} = {}^{29}P_{m+5}$, then m = ...

(a) zero (b) – 5 (c) 5 (d) 46

6 If ${}^nP_4 = 504\ n$, then n = ...

(a) 6 (b) 7 (c) 9 (d) 10

7 If ${}^nP_r = n(n-1)(n-2) \times \cdots \times 5 \times 4 \times 3$, then r = ...

(a) n (b) n – 2 (c) n – 1 (d) n – 3

8 If $(x-2) \times {}^nC_3 = {}^nP_3$, then the value of x may be ....

(a) 5 (b) 6 (c) 8 (d) 3 n

9 ${}^{15}P_{r+1} \div {}^{14}P_r =$ ....

(a) $\frac{r}{r+1}$ (b) $\frac{10}{r+1}$ (c) 15 (d) 14

10 ${}^nP_r \div {}^nC_r =$ ....

(a) $\underline{|r-1}$ (b) $\underline{|r}$ (c) $\underline{|n}$ (d) 1

11 ${}^nP_r = 720\ {}^nC_r$, then r = ...

(a) 6 (b) 5 (c) 4 (d) 7

12 If ${}^nP_n = 5040$, then n = ...

(a) – 1 (b) 5 (c) 7 (d) 3

13 Which of the following values could be equal to ${}^nP_2$?

(a) 24 (b) 25 (c) 27 (d) 30

14 If ${}^nC_{n-3} = 20$, then n = ...

(a) 3 (b) 4 (c) 5 (d) 6

15 If ${}^{n-3}P_7 = \underline{|8}$, then n = ...

(a) 8 (b) 10 (c) 11 (d) 15

16 If ${}^{n+1}P_n = 24$, then n = ...

(a) 2 (b) 3 (c) 4 (d) 5

17 If ${}^{n+1}P_4 = 10\ {}^nP_3$, then n = ...

(a) 8 (b) 9 (c) 10 (d) 4

18 ${}^nP_2 = {}^nC_2 +$ ....

(a) ${}^nP_2$ (b) ${}^nC_2$ (c) $\underline{|2}$ (d) n^2

19 If ${}^nP_r = a\ {}^nC_r$, then a could be equal ....

(a) 4 (b) 5 (c) 6 (d) 8

20 $\dfrac{\lfloor r-2}{\lfloor r-3} =$ ....

(a) ${}^rP_3$ (b) ${}^{r-2}P_3$ (c) ${}^{r-2}P_1$ (d) ${}^{r-2}P_{r-3}$

21 If $a = {}^nC_r$, $b = {}^nC_{r-1}$, then $\dfrac{a+b}{b} =$ ....

(a) $\dfrac{n-1}{r}$ (b) $\dfrac{n+1}{r}$ (c) $\dfrac{r}{n+1}$ (d) $\dfrac{n}{r+1}$

22 If $2\ {}^nP_3 = {}^{n+1}P_3$, then n = ...

(a) 4 (b) 5 (c) 6 (d) 7

23 If $n \in \mathbb{Z}^+$, $n \geq 3$, then $n(n^2-1) =$ ....

(a) ${}^{n+1}P_2$ (b) ${}^{n+1}P_3$ (c) ${}^{n-1}P_2$ (d) ${}^{n-1}P_3$

24 If ${}^nP_r = a$, ${}^nC_r = b$, then which of the following statements is not true ?

(a) $a \geq b$ (b) $\dfrac{a}{b} \in \mathbb{Z}^+$ (c) $b - a \in \mathbb{Z}^+$ (d) $a - b \in \mathbb{N}$

25 If $\dfrac{\lfloor 8}{\lfloor 8-r} : \dfrac{\lfloor 9}{\lfloor 9-r} = 2 : 3$, then r = ...

(a) 1 (b) 2 (c) 3 (d) 4

26 If $\lfloor x-3 \times {}^xP_3 = 20 \lfloor x-2$, then $x =$ ....

(a) 3 (b) 4 (c) 5 (d) 6

27 If $\lfloor r = n$ and $\lfloor n = r$, then r + n = ...

(a) zero or 2 (b) 1 or zero (c) 2 or 1 (d) 2 or 4

28 If $n \in \mathbb{Z}^+$, then $n \lfloor n =$ ....

(a) $\lfloor n^2$ (b) $\lfloor n+1$ (c) $n^2 \lfloor n-1$ (d) $\lfloor 2n$

29 If ${}^nP_5 = {}^nP_5$, then n could be equal to ....

(a) 1 (b) 4 (c) 3 (d) 9

30 If ${}^{n}P_4 = {}^{n}P_5$, then n = ...

(a) 1 (b) 4 (c) 5 (d) 9

31 If ${}^{24}C_{2r+1} = {}^{24}C_{r+4}$, then r = ...

(a) 3 (b) 4 (c) 5 (d) 6

32 If ${}^{29}C_{r} = {}^{29}C_{r-1}$, then the value of ${}^{r}C_{12}$ = ...

(a) 450 (b) 455 (c) 460 (d) 465

33 If ${}^{14}C_{r^2} = {}^{14}C_{r+2}$, then the positive value of r = ...

(a) 2 (b) 4 (c) 3 (d) 2 or 3

34 If ${}^{n+2}C_4 = n^2 - 3$, then n = ...

(a) 2 (b) 4 (c) 6 (d) 10

35 If ${}^{n}C_5 : {}^{n}C_4 = 3 : 1$, then n = ...

(a) 7 (b) 9 (c) 17 (d) 19

36 If ${}^{n}C_9 : {}^{n}C_7 = 7 : 9$, then n = ...

(a) 7 (b) 15 (c) 16 (d) 9

37 If ${}^{n}C_3 : {}^{n-1}C_4 = 8 : 5$, then n = ...

(a) 5 (b) 7 (c) 8 (d) 9

38 If ${}^{10}C_{r+1} : {}^{10}C_{r-1} = 21 : 10$, then r = ...

(a) 3 (b) 4 (c) 5 (d) 6

39 If ${}^{n+1}C_3 : {}^{n}C_4 = 2 : 3$, then n = ...

(a) 2 (b) 3 (c) 5 (d) 11

40 If $36\ {}^{2n-1}P_{n-1} = 9\ {}^{2n}P_n$, then n = ...

(a) 1 (b) 2 (c) 3 (d) 4

41 If $^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then n = ...

(a) 19 (b) 15 (c) 16 (d) 21

42 $^nP_r \div {}^nP_{r-1}$ = ...

(a) n − r (b) n − r − 1 (c) n − r + 1 (d) n + r

43 If $\frac{^9P_{r-1}}{^8P_r} = \frac{1}{8}$, then the value of r = ...

(a) 1 (b) 2 (c) 3 (d) 4

44 *(2nd session 2021)* If $^nC_r : {}^{n-1}C_r = 3 : 1$, then $\underline{|4n \over r}$ = ...

(a) 24 (b) 120 (c) 720 (d) 5040

45 The number of solutions for the equation : $x = \underline{|x}$ in $\mathbb{Z}$ equals ....

(a) zero (b) 1
(c) 2 (d) infinite solutions.

46 Which of the following equations has no solution in $\mathbb{N}$?

(a) $^nC_3 = 165$ (b) $^6C_n = 15$ (c) $^4C_n = 6$ (d) $^4C_n = 5$

47 If $^nC_{\cos\theta} = {}^nC_{\sin 2\theta}$, then θ = ...

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) π

48 If $^nP_5 = \underline{|5}$, then n = ...

(a) 5 or 6 (b) 6 (c) 5 (d) 6 or 7

49 The highest common factor of the numbers $\underline{|n}$, $\underline{|n+1}$, $\underline{|n+2}$ is ....

(a) $\underline{|n}$ (b) $\underline{|n+2}$ (c) n (d) n + 2

50 The lowest common multiple of the numbers $\underline{|n}$, $\underline{|n+1}$, $\underline{|n+2}$ is ....

(a) $\underline{|n}$ (b) $\underline{|n+2}$ (c) n (d) n + 2

51 The expression : $^nC_r + {}^nC_{r+1}$ = ...

(a) $^{n+1}C_r$ (b) $^nC_{r+1}$ (c) $^{n+1}C_{r+1}$ (d) $^{n+1}C_{r+2}$

52 If ${}^{n}C_6 + {}^{n}C_5 = 84$, then n = ...

(a) 2 (b) 4 (c) 6 (d) 8

53 *(1st Session 2021)* If $\frac{{}^{n}C_4 + {}^{n}C_3}{{}^{n+1}C_3} = 1$, then $\lfloor n-6 =$ ....

(a) 6 (b) 1 (c) zero (d) 24

54 ${}^{n+1}C_3 + 2\ {}^{n+1}C_2 + {}^{n+1}C_1 =$ ....

(a) ${}^{n+1}C_3$ (b) ${}^{n+1}C_2$ (c) ${}^{n+1}C_1$ (d) ${}^{n+3}C_3$

55 If ${}^{n}C_{10} + 2\ {}^{n}C_{11} + {}^{n}C_{12} = {}^{25}C_{12}$, then n = ...

(a) 25 (b) 24 (c) 23 (d) 22

56 Each of the following equals ${}^{n}C_r$ except ....

(a) $\frac{n}{r}\ {}^{n-1}C_{r-1}$ (b) ${}^{n}C_{n-r}$ (c) $\frac{{}^{n}P_r}{\lfloor r}$ (d) $\frac{\lfloor n}{\lfloor n-r}$

57 ${}^{n}C_r \times {}^{r}C_k = {}^{n}C_k \times$ ....

(a) ${}^{n}P_k$ (b) ${}^{n}C_1$ (c) ${}^{n-k}C_{r-k}$ (d) ${}^{n}C_{r-k}$

58 ${}^{n}C_r = {}^{n}P_r$ If r = ...

(a) n (b) $\frac{n}{2}$ (c) 1 or 2 (d) 0 or 1

59 If ${}^{n}C_{10} > {}^{n}C_9$, then n ....

(a) = 19 (b) > 19 (c) < 19 (d) ≤ 19

60 If ${}^{7}C_r > 1$, ${}^{r}C_5 > 1$, then the value of $\lfloor 6-r =$ ....

(a) zero (b) 1 (c) 720 (d) 6

61 If ${}^{n}C_7 \times {}^{n}C_5 \geq {}^{n}C_6 \times {}^{n}C_4$, $n \in \mathbb{Z}^+$, then n ≥ ....

(a) 11 (b) 10 (c) 12 (d) 13

62 If ${}^{n}C_r > {}^{n}C_{r-1}$, then ....

(a) $r > \frac{n-1}{2}$ (b) $r < \frac{n-1}{2}$ (c) $r > \frac{n+1}{2}$ (d) $r < \frac{n+1}{2}$

63 *(Trial 2021)* If ${}^{n+1}P_r > {}^{n+1}P_{r-1}$, then n > ....

(a) $r-1$ (b) $r-3$ (c) $r+1$ (d) $1-r$

64 If ${}^{10}C_{x-1} > 2 \times {}^{10}C_x$, then $x \in$ ....

(a) $\{8, 9, 10\}$ (b) $\{8, 9, 10, ...\}$
(c) $\{3, 2, 1, ...\}$ (d) $\{9, 10, 11\}$

65 If $(x^2+3x+2)\,\underline{|x} = 120$, ${}^{x+y}P_4 = 840$, then ${}^7C_x + {}^7C_y =$ ....

(a) ${}^7C_3$ (b) ${}^7C_5$ (c) ${}^8C_4$ (d) ${}^{11}C_4$

66 *(Trial 2021)* If $2\,{}^{n+1}C_r = {}^{n+1}P_r$, $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{5}{3}$, then ${}^nC_r + {}^nP_r =$ ....

(a) 63 (b) 33 (c) 60 (d) 36

67 If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then n + r = ...

(a) 3 (b) 4 (c) 5 (d) 6

68 If ${}^{x+y}P_2 = 210$, ${}^{y-3}C_3 = 35$, then $\underline{|2x-y} =$ ....

(a) 5 (b) 10 (c) 2 (d) 1

69 If ${}^{x+y}P_4 = 360$, $\underline{|2x+y} = 5040$, then ${}^yC_{2x} =$ ....

(a) 1 (b) 5 (c) 10 (d) 15

70 If ${}^{2n+1}C_{2n-1} - 2\,{}^{n+2}C_n = 46$, then n = ...

(a) 5 (b) 6 (c) 7 (d) 8

71 If $\frac{\underline{|n-1}}{\underline{|n}} + \frac{\underline{|n+1}}{\underline{|n+2}} = \frac{6}{n^2+2n}$, then n = ...

(a) 2 (b) 4 (c) 8 (d) 9

72 Solution set of the equation $\frac{\underline{|x+1}}{\underline{|x-9}} \div {}^xP_9 = 11$ is ....

(a) $\{10\}$ (b) $\{9\}$ (c) $\{11\}$ (d) $\{9, 11\}$

73 If ${}^{30}C_r = {}^{30}C_{r+10}$, ${}^{n}P_7 = 90 \times {}^{n-2}P_5$, then $\underline{|n-r}$ = ...

(a) zero (b) 1 (c) 10 (d) $\underline{|20}$

74 ${}^{n-1}C_6 + {}^{n-1}C_7 > {}^{n}C_6$ If ....

(a) n > 4 (b) n > 12 (c) n > 13 (d) n ≥ 13

75 If ${}^{4}C_{r+2} = {}^{4}C_{2-r}$, then r ∈ ....

(a) {0} (b) {– 1 , 4} (c) {– 2 , – 1 , 0 , 1 , 2} (d) {0 , 3}

76 If ${}^{5}P_{n-3} = \underline{|5}$, then n = ...

(a) 8 (b) 5 (c) 3 or 5 (d) 7 or 8

77 If ${}^{8}C_r - {}^{7}C_3 = {}^{7}C_2$, then r = ...

(a) 3 or 5 (b) 2 or 5 (c) 3 or 4 (d) 4 or 5

78 If $\frac{{}^{7}C_r}{{}^{8}C_{r-2}} \times \frac{{}^{8}C_{r-1}}{{}^{7}C_{r-1}} = 2$, then r = ...

(a) 3 (b) 4 (c) 15 (d) 20

79 If ${}^{n+1}P_{r+1} = 7 \times {}^{n}P_r$, $4 \times {}^{n}C_r = 3 \times {}^{n}C_{r-1}$, then $\underline{|n-r-2}$ = ...

(a) zero (b) 1 (c) 6 (d) 24

80 If ${}^{9}P_{n-4} = \underline{|9}$, then the sum of values of n equals ....

(a) 5 (b) 12 (c) 13 (d) 25

81 If ${}^{8}P_{n-3} = {}^{8}P_{n-3}$, then n must be an integer ∈ ....

(a) [0 , 3] (b) [3 , 11] (c) [0 , 8] (d) [0 , 11]

82 If ${}^{n-4}P_9 = {}^{n-4}P_9$, then n must be an integer ∈ ....

(a) [0 , 4] (b) [4 , 13] (c) [0 , 13] (d) [13 , ∞[

83 If n is an even constant , then the value of r which makes ${}^{n}C_r$ is maximum is ....

(a) n – 1 (b) $\frac{n}{2} - 1$ (c) $\frac{n}{2}$ (d) $\frac{n}{2} + 1$

84 If $\underline{|2n-3} = 2n-3$, then the sum of all possible values of $n =$ ....

ⓐ 2 ⓑ $\frac{5}{2}$ ⓒ 5 ⓓ $\frac{9}{2}$

85 If ${}^{n+1}C_3 - {}^{n}C_3 = 21$, then $n =$ ....

ⓐ 7 ⓑ 6 ⓒ 8 ⓓ 9

86 If $\frac{1}{{}^{5}C_r} + \frac{1}{{}^{6}C_r} = \frac{1}{{}^{4}C_r}$, then $r =$ ....

ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 5

87 If $\underline{|10-\underline{|n}} = 24$, then $n =$ ....

ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 5

88 If ${}^{n}P_r = 210$, then $(n , r) =$ ....

ⓐ (210 , 1) ⓑ (15 , 2)

ⓒ (7 , 3) ⓓ All the previous

89 If $4k\,\underline{|2k-1} = \frac{32}{9} \times \frac{{}^{11}C_3 + {}^{11}C_4}{{}^{12}C_3} + 40$, then $k =$ ....

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

90 If $\underline{|\sin x} = 1$ where $0 \le x < 2\pi$, then $x \in$ ....

ⓐ $\{0 , \pi\}$ ⓑ $\{\frac{\pi}{2}\}$ ⓒ $\{0 , \frac{\pi}{2}\}$ ⓓ $\{0 , \pi , \frac{\pi}{2}\}$

91 Solution set of the equation : $\underline{|4-|x|} = 1$ is ....

ⓐ $\{3 , -3\}$ ⓑ $\{4 , -4\}$ ⓒ $\{3 , -3 , 4 , -4\}$ ⓓ $\{3 , 4\}$

92 If $\underline{|2n} = \underline{|n}\,\underline{|n+2}$, then $n =$ ....

ⓐ zero ⓑ 2 ⓒ 3 ⓓ 4

93 If ${}^{n}P_x + {}^{x}P_n = 1440$, then ${}^{n+4}P_{x-5} =$ ....

ⓐ 4 ⓑ 5 ⓒ 9 ⓓ 10

94 If ${}^{n}P_2$, ${}^{n}P_3$, ${}^{n+1}P_3$ form an arithmetic sequence , then n = ...

(a) 6 (b) 7 (c) 8 (d) 9

---

95 If ${}^{n+2}C_3$, ${}^{n}P_2$, ${}^{n}C_2$ form a geometric sequence , then n = ...

(a) 2 or 3 (b) 3 or 7 (c) 2 or 7 (d) 7 or 9

---

96 The arithmetic mean of the values : ${}^{n}C_0$, ${}^{n}C_1$, ${}^{n}C_2$, ${}^{n}C_3$,... , ${}^{n}C_n$ is ....

(a) 2^n (b) $\frac{2^n}{n}$ (c) $\frac{2^{n+1}}{n}$ (d) $\frac{2^n}{n+1}$

---

97 If $\lfloor n$, $3\lfloor n$, $\lfloor n+1$ form a geometric sequence , then ....

(a) $\lfloor n$, $4\lfloor n$, $\lfloor n+1$ form a geometric sequence.

(b) $\lfloor n$, $4\lfloor n$, $\lfloor n+1$ form an arithmetic sequence.

(c) $\lfloor n$, $5\lfloor n$, $\lfloor n+1$ form a geometric sequence.

(d) $\lfloor n$, $5\lfloor n$, $\lfloor n+1$ form an arithmetic sequence.

---

98 If $\lfloor \log_5 n$ is defined , then n ∈ ....

(a) $\mathbb{N}$ (b) $\{5 , 10 , 15 , 20 , ...\}$

(c) $\mathbb{Z}^+$ (d) $\{1 , 5 , 25 , 125 , ...\}$

---

99 If $\lfloor 13 = 2^a \times b$ where b doesn't divisible by 2 , then a = ...

(a) 8 (b) 9 (c) 10 (d) 11

---

100 If r increased , then the value of ${}^{10}C_r$ ....

(a) is always increasing. (b) is always decreasing.

(c) is increasing , then decreasing. (d) is decreasing , then increasing.

---

101 If ${}^{30}P_x = {}^{30}P_{x+2y}$, then (x , y) could be any of the following except ....

(a) $\left(29 , \frac{1}{2}\right)$ (b) $\left(30 , -\frac{1}{2}\right)$ (c) $(17 , 0)$ (d) $\left(29 , -\frac{1}{2}\right)$

---

102 The value of : ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ equals ....

(a) ${}^{56}C_4$ (b) ${}^{56}C_2$ (c) ${}^{55}C_4$ (d) ${}^{55}C_3$

103 The solution set of the equation : $\underline{|1 + \log x} = 1$ is ....

(a) $\left\{\frac{1}{10}\right\}$ (b) $\{1\}$ (c) $\{0 , -1\}$ (d) $\left\{\frac{1}{10} , 1\right\}$

104 If $\dfrac{{}^{13}C_r + {}^{13}C_{r-1}}{{}^{13}C_{r+1} + {}^{13}C_r} = 2$, then r = ...

(a) 3 (b) 6 (c) 9 (d) 12

105 If ${}^{a+b}P_3 = x$, ${}^{a-b}P_2 = y$, then the least value of the expression $\underline{|x + y} =$ ....

(a) $\underline{|5}$ (b) $\underline{|6}$ (c) $\underline{|7}$ (d) $\underline{|8}$

106 If $\dfrac{\underline{|a + b}}{\underline{|b}} = 24$, then the possible values of ${}^aP_b$ are ....

(a) 3 , 4 (b) 3 , 1 (c) 23 , 3 , 1 (d) 23 , 1

107 If $\frac{1}{2}\underline{|n}$, $\underline{|n - 2}$, $\underline{|2 - n}$ are the sides lengths of a triangle in cm., then the numerical value of the area of the triangle = ... cm^2.

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{\sqrt{3}}{4}$

108 If $\frac{1}{2}\underline{|n}$, $\underline{|n - 3}$, n $\underline{|3 - n}$ are the side lengths of a triangle , then the numerical value of the perimeter of the triangle equals ....

(a) 3 (b) 6 (c) 7 (d) 8

109 If ${}^nC_r : {}^nC_{r+2} : {}^nC_{r+4} = 3 : 14 : 14$, then ${}^nP_r =$ ....

(a) 56 (b) 72 (c) 720 (d) 90

110 If ${}^nC_4 + {}^nC_5 + {}^nC_6 + \dots {}^nC_{10} = 848$, then ${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 =$ ....

(a) 55 (b) 120 (c) 166 (d) 176

111 $1 + {}^6C_5 + {}^7C_5 + {}^8C_5 + \dots + {}^{49}C_5 =$ ....

(a) ${}^{100}C_5$ (b) ${}^{99}C_6$ (c) ${}^{50}C_6$ (d) ${}^{50}C_5$

112 $1 + {}^nC_1 + {}^{n+1}C_2 + {}^{n+2}C_3 + \dots + {}^{n+r-1}C_r =$ ....

(a) ${}^{n+r}C_r$ (b) ${}^{n+r-1}C_r$ (c) ${}^{n+r}C_{r-1}$ (d) ${}^{n+r}C_{r+1}$

113 If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$, ${}^nC_{r+1} = 126$, then one of the values of r is ....

(a) 1 (b) 2 (c) 3 (d) nothing of these.

114 If $n = {}^mC_2$, then the value of ${}^nC_2$ is given by ....

(a) ${}^{m+1}C_4$ (b) ${}^{m-1}C_4$ (c) $3\ {}^{m+1}C_4$ (d) ${}^{m+2}C_4$

115 If ${}^{x+1}C_4 + \sum_{r=x}^{x+5} {}^{r+1}C_3 = 1365$, then $x =$ ....

(a) 8 (b) 9 (c) 10 (d) 11

116 If $k = n (n^2 - 1) (n^2 - 4)$ .... $(n^2 - r^2)$ where n , $r \in \mathbb{Z}^+$, n > r , then k is divisible by ....

(a) $\underline{|2r+1}$ (b) $\underline{|2r+2}$ (c) $\underline{|2r+4}$ (d) $\underline{|2r+6}$

117 The unit digit in the summation $\sum_{r=0}^{2023} \underline{|r}$ equals ....

(a) zero (b) 2 (c) 3 (d) 4

Third Questions on the binomial theorem

Choose the correct answer from the given ones :

1 The fourth term in the expansion of : $\left(x + \frac{1}{x}\right)^4$ according to the descending order of the power of x equals ....

(a) $4x^2$ (b) $\left(\frac{1}{x}\right)^4$ (c) $\frac{1}{x^2}$ (d) $\frac{4}{x^2}$

2 In the expansion of : $(1 + bx)^9$ according to the ascending powers of x , the coefficient of the sixth term is ....

(a) ${}^9C_5$ (b) ${}^9C_6$ (c) ${}^9C_5\, b^5$ (d) ${}^9C_6\, b^6$

3 The fifth term from the end of the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^{12}$ according to descending power of x equals ....

(a) $\frac{7920}{x^4}$ (b) $\frac{-7920}{x^4}$ (c) $7220\, x^{-4}$ (d) $-7520\, x^4$

4 The coefficient of the middle term in the expansion of : $\left(3x - \frac{1}{6}\right)^{10}$ equals ....

(a) $\frac{-63}{8}$ (b) $\frac{-67}{8}$ (c) $\frac{63}{8}$ (d) $\frac{67}{8}$

5 The last term in the expansion of : $(2 - x)^5 (2 + x)^5$ is ....

(a) x^5 (b) $-x^5$ (c) $-x^{10}$ (d) x^{10}

6 The coefficient of the term containing x^3 in the expansion of : $(1 + x)^{10}$ according to the ascending powers of x = ...

(a) ${}^{10}C_2$ (b) ${}^{10}C_3$ (c) ${}^{10}C_4$ (d) 3

7 The coefficient of x^5 in the expansion of $(3 - 2x)^7$ equals ....

(a) 6048 (b) -6048 (c) 1520 (d) -1520

8 The term free of x in the expansion of : $\left(x^2 + \frac{1}{x^2}\right)^8$ equals ....

(a) 70 (b) -70 (c) 56 (d) -56

9 The term free of x in the expansion of : $\left(x - \frac{1}{x}\right)^{10}$ according to the descending powers of x is ....

(a) T_5 (b) T_7 (c) T_6 (d) T_4

10 The coefficient of : x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is ....

(a) $\frac{405}{256}$ (b) $\frac{405}{259}$

(c) $\frac{450}{263}$ (d) Nothing of the previous

11 ***(1st Session 2021)*** In the expansion of $\left(x^2 + 2 + \frac{1}{x^2}\right)^6$ the coefficient of the term containing x^2 is ....

(a) ${}^{12}C_5$ (b) ${}^{12}C_6$ (c) ${}^{12}C_2$ (d) ${}^{6}C_5$

12 From the expansion of : $x^3 (1 + x)^8$ the coefficient of the term containing x^5 is ....

(a) 8 (b) 28 (c) 56 (d) 70

13 The term free of x in the expansion of $\left(1 - \frac{1}{x}\right)^8 (x + 1)^8$ is ....

(a) $-{}^{8}C_6$ (b) ${}^{8}C_5$ (c) $-{}^{8}C_4$ (d) ${}^{8}C_4$

14 If the middle term in the expansion of : $\left(\frac{2a}{3}+\frac{b}{a^2}\right)^{8n}$ is the ninth term , then n = ...

(a) 1 (b) 2 (c) 3 (d) 4

15 If the middle term in the expansion of : $\left(x^2-\frac{1}{x}\right)^{n+5}$ is T_{12} , then n = ...

(a) 19 (b) 18 (c) 20 (d) 17

16 If the orders of the two middle terms in the expansion $(x+y)^n$ are 7 , 8 , then n = ...

(a) 14 (b) 13 (c) 16 (d) 56

17 If the number of terms in the expansion of : $(x+y)^{2n-1}$ equals 12 terms , then n = ...

(a) 5 (b) 6 (c) 7 (d) 12

18 The number of terms in the expansion of $\left((a+4b)^3\,(a-4b)^3\right)^2$ equals ....

(a) 6 (b) 7 (c) 8 (d) 32

19 The number of terms in the expansion of $(1+2x+x^2)^{50}$ equals ....

(a) 101 (b) 50 (c) 51 (d) 100

20 The sum of the coefficients of the terms of the expansion of : $\left(2x^2-\frac{3}{x}\right)^{15}$ equals ....

(a) 1 (b) zero (c) −1 (d) −15

21 The sum of coefficients of the terms of the expansion : $(1+x-3x^2)^{2021}$ = ...

(a) − 1 (b) 1 (c) 0 (d) 2017

22 The sum of the coefficients of the terms of the expansion : $(4x+3y-5z)^n$ equals 64 , then n = ...

(a) 2 (b) 3 (c) 6 (d) 8

23 If the sum of the terms coefficients in the expansion of $(a^2x^2-2ax+1)^9$ equals zero , then a = ...

(a) 2 (b) − 2 (c) 1 (d) − 1

24 The product of coefficients of terms of the expansion of : $(1 + x)^5$ equals ....

(a) zero (b) 2 (c) 250 (d) 2500

25 In the expansion of a binomial , it has 7 terms their coefficients are positive and 6 terms their coefficients are negative , then the expression is in the form of ....

(a) $(a-b)^{12}$ (b) $(a+b)^{13}$ (c) $(a+b)^{12}$ (d) $(a-b)^{13}$

26 In the expansion of $(1 + x)^n$, if the coefficient of T_b = coefficient of T_c where $b \neq c$, then $b + c =$ ....

(a) n (b) n + 2 (c) n − 2 (d) 2 n

27 If the coefficients of the sixth and sixteenth terms in the expansion of : $(x + y)^n$ are equals , then : n = ...

(a) 19 (b) 20 (c) 21 (d) 22

28 If the two middle terms in the expansion of : $(a + 3b)^{2n+1}$ are equal , then ....

(a) $a = 3b$ (b) $3a = b$ (c) $a = 9b$ (d) $\frac{a}{b} = \frac{1}{9}$

29 In the expansion of : $(a x + b)^{2n+1}$, if the two middle terms are equal at $x = 2$, then ....

(a) $a = 2b$ (b) $b = 2a$ (c) $ab = 2$ (d) $ab = \frac{1}{2}$

30 In the expansion of $(1 + x)^n$ according to ascending powers of x , if the coefficient of x^5 equals the coefficient of T_{10} , then : n = ...

(a) 9 (b) 5 (c) 14 (d) 4

31 In the expansion of $(a - b)^n$ according to the descending powers of a where $n \geq 5$, if T_5 , T_6 each is additive inverse of the other then $\frac{a}{b} =$ ....

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

32 If the two middle terms in the expansion $\left(x^2+\frac{1}{x}\right)^{2n+1}$ are equal then $x =$ ....

(a) 1 (b) -1 (c) ± 1 (d) 2

---

33 The ratio between the two middle terms in the expansion of $\left(x+\frac{1}{x}\right)^{2n-1}$ according to the descending power of x respectively = ...

(a) x^2 (b) $\frac{x^2}{2n}$ (c) $\frac{x^2}{n}$ (d) $\frac{2n+1}{2n+3}$

---

34 ***(Trial 2021)*** In the expansion of $(x+a)^n$ according to the descending powers of x if T_4 is the same as T_{15} from the end , then $n =$ ....

(a) 16 (b) 17 (c) 18 (d) 19

---

35 The coefficients of x^n , x^m in the expansion of $(1+x)^{m+n}$ ....

(a) are equal.

(b) are equal but they have different sign.

(c) multiplicative inverse of each other.

(d) one of them half the other.

---

36 If a and b are the coefficient of x^n and x^{n+1} respectively in the expansion of $(1+x)^{2n+1}$, then ....

(a) $a = 2b$

(b) $b = 2a$

(c) $a = b$

(d) $a + b = \underline{|2n+1}$

---

37 The order of the middle term in the expansion $\left(1+x+\frac{x^2}{4}\right)^n$ equals ....

(a) $\frac{n}{2}+1$ (b) $\frac{n+1}{2}$ (c) $n+1$ (d) $\frac{n+3}{2}$

---

38 The middle term in the expansion $(1+x)^{2n}$ is ....

(a) ${}^{2n}C_n$

(b) ${}^{2n}C_{n+1}\, x^{n+1}$

(c) ${}^{2n}C_{n-1}\, x^{n-1}$

(d) $\frac{1 \times 3 \times 5 \times \cdots (2n-1)}{\underline{|n}} \times 2^n \times x^n$

---

39 In the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, the middle term $\neq$ .... for every $n > 1$

(a) ${}^{2n}C_n$

(b) $\frac{{}^{2n}P_n}{\underline{|n}}$

(c) ${}^{2n}P_n$

(d) $\frac{1 \times 3 \times 5 \times \cdots \times (2n-1) \times 2^n}{\underline{|n}}$

40 If a is the sum of the coefficients of the odd-order terms in the expansion of $\left(2\,x - \frac{3}{x^2}\right)^{19}$ and b is the sum of the coefficients of the even-order terms in the same expansion, then a + b = ...

(a) -1 (b) zero (c) 1 (d) 5

---

41 If the number of terms in the expansion of $(a + b)^n$ equals x and the number of terms in the expansion of $(a + b)^m$ equals y, then the number of terms in the expansion of $(a + b)^{n+m}$ equals ....

(a) $x + y$ (b) $x + y + 1$ (c) $x + y + 2$ (d) $x + y - 1$

---

42 *(trial 2021)* In the expansion of $\left(a\,x^2 - \frac{b}{x}\right)^{12}$ according to the descending powers of x T_7 is ....

(a) The term containing x^6 (b) the term free of x

(c) the term before the last (d) the term containing x^7

---

43 *(1st Session 2021)* In the expansion of $\left(x^5 - \frac{k}{x^2}\right)^{7n}$ according to the descending powers of x, the term free of x is ...., where k, n $\in \mathbb{Z}^+$

(a) T_{5n} (b) T_{5n+1} (c) T_{6n+1} (d) T_{6n-1}

---

44 The term free of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^{5n}$ where n $\in \mathbb{Z}^+$ equals ....

(a) $\frac{\lfloor 5n}{\lfloor 2n \lfloor 3n}$ (b) $\frac{\lfloor 5n}{\lfloor 2n}$ (c) $\frac{\lfloor 5n}{\lfloor 3n}$ (d) $\frac{\lfloor 5n}{\lfloor n \lfloor 4n}$

---

45 In the expansion of : $(1 + a\,x)^7$ according to the ascending powers of x, if the coefficient of $T_5 = 560$, then a = ...

(a) 2 (b) 4 (c) ± 2 (d) ± 4

---

46 If the middle term in the expansion of : $\left(3\,x^2 + \frac{2}{3\,x}\right)^8$ equals 17920, then x = ...

(a) ± 2 (b) 3 (c) ± 4 (d) 5

---

47 If the coefficient of the ninth term in the expansion of $\left(a\sqrt{x} - \frac{1}{a\sqrt{x}}\right)^{12}$ according to the descending powers of x equals 7920, then a = ...

(a) $\pm \frac{1}{2}$ (b) ± 2 (c) $\pm \frac{1}{4}$ (d) ± 4

48 If the term free of x in the expansion $\left(x+\frac{1}{x}\right)^n$ is T_7 , then n = ...

ⓐ 6 ⓑ 10 ⓒ 12 ⓓ 8

49 If the term free of x in the expansion of $\left(\frac{a}{x^2}+x\right)^9$ is 672 , then a = ...

ⓐ 8 ⓑ 2 ⓒ 3 ⓓ 4

50 ***(1st Session 2021)*** If the coefficient of T_6 in the expansion of $\left(a\,x+\frac{1}{b\,x}\right)^{10}$ according to the descending powers of x equals ${}^{10}C_5$, then $\frac{a}{b}$ = ... where $a \in \mathbb{R}^*$, $b \in \mathbb{R}^*$

ⓐ -1 ⓑ 1 ⓒ 10 ⓓ $\frac{1}{10}$

51 If the absolute term in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ equals 405 , then k = ...

ⓐ ± 1 ⓑ ± 2

ⓒ ± 3 ⓓ Nothing of the previous

52 In the expansion of $\left(2+\frac{x}{3}\right)^n$ if the coefficients of x^7 and x^8 are equal , then n = ...

ⓐ 56 ⓑ 55 ⓒ 45 ⓓ 15

53 If the coefficients of the two terms have x^2 , x^3 in the expansion of $(3+a\,x)^9$ are equal , then a = ...

ⓐ $\frac{9}{7}$ ⓑ $\frac{7}{9}$ ⓒ $\frac{-9}{7}$ ⓓ $\frac{-7}{9}$

54 In the expansion $\left(a\,x^2+\frac{1}{a\,x}\right)^{11}$ if the two coefficients of x^4 and x^7 are equal , then a = ...

ⓐ 1 ⓑ -1 ⓒ ± 1 ⓓ ± 2

55 If the coefficient of the middle term in the expansion of $(1+m\,x)^4$ equals the coefficient of the middle term in the expansion of $(1-m\,x)^6$, then m = ...

ⓐ $\frac{-3}{10}$ ⓑ $\frac{3}{10}$ ⓒ $\frac{-3}{5}$ ⓓ $\frac{3}{5}$

56 In the expansion of $\left(x^2+\frac{a}{x^3}\right)^{15}$ according to the descending powers of x , if the coefficient of x^{10} = twice the coefficient of x^{15} , then a = ...

ⓐ 1 ⓑ $\frac{2}{3}$ ⓒ $\frac{3}{2}$ ⓓ 2

57 In the expansion of $\left(a x + \frac{1}{b x}\right)^{10}$ according to the descending powers of x , if the term free of x equals the coefficient of the seventh term , then a b = ...

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{36}{25}$ (d) $\frac{25}{36}$

58 In the expansion of $(x + y)^{10}$ according to the descending powers of x , the ninth term : the eighth term equals ....

(a) $\frac{3 y}{8 x}$ (b) $\frac{3 x}{8 y}$ (c) $\frac{8 y}{3 x}$ (d) $\frac{8 x}{3 y}$

59 In the expansion of $(1 - x)^{12}$ according to the ascending powers of x , the coefficient of sixth term : the coefficient of fifth term equals ....

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$ (c) $\frac{-8}{5}$ (d) $\frac{-5}{8}$

60 In the expansion of : $(x + y)^8$ according to the descending powers of x , the ratio $T_6 : T_4$ equals ....

(a) $25 y^2 : 16 x^2$ (b) $25 x^2 : 16 y^2$ (c) 1 (d) $y^2 : x^2$

61 In the expansion of $(1 + x)^{17}$ according to the ascending powers of x , if the coefficient of T_{r+4} is equal to the coefficient of T_{2r+3} , then r = ...

(a) 3 (b) 4 (c) 17 (d) 7

62 In the expansion of $(1 + x)^{n+1}$ according to the ascending powers of x , if $\frac{T_2}{T_1} \times \frac{T_{n+1}}{T_{n+2}} = 64$, then $\underline{|n - 3}$ = ...

(a) 120 (b) 24 (c) 720 (d) 1

63 In the expansion of $\left(x^2 + \frac{1}{x}\right)^{15}$, the ratio between the term free of x and the sum of the coefficients of the two middle terms = ...

(a) $\frac{7}{15}$ (b) $\frac{7}{30}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

64 In the expansion of : $(3 + a x)^9$ according to the ascending power of x , if the coefficient of T_3 = the coefficient of T_4 , then a = ...

(a) $\frac{9}{5}$ (b) $\frac{9}{4}$ (c) $\frac{9}{7}$ (d) $\frac{9}{2}$

65 In the expansion of $(1 + X)^{27}$ according to the ascending powers of X , if the ratio between the first middle term : the second middle term = 3 : 1 , then X = ...

(a) 4 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

---

66 In the expansion of $(3a - 2b)^{11}$ according to the descending powers of a , if the ratio between the two middle terms respectively equals $\frac{-3}{2}$, then a : b = ...

(a) 9 : 4 (b) 4 : 9 (c) 1 (d) − 1

---

67 If $\frac{T_2}{T_3}$ in the expansion $(a + b)^n$ according to descending power of a equals $\frac{T_3}{T_4}$ in the expansion of $(a + b)^{n+3}$, then n = ...

(a) 3 (b) 4 (c) 5 (d) 6

---

68 The two consecutive terms in the expansion of $(3 + 2X)^{74}$ according to the ascending powers of X that have equal coefficients are ....

(a) T_{29} , T_{30} (b) T_{30} , T_{31} (c) T_{31} , T_{32} (d) T_{28} , T_{29}

---

69 In the expansion of $(1 + X)^{2n}$ according to the ascending powers of X , if the coefficient of T_3 is the arithmetic mean between the coefficients of T_2 , T_4 , then $2n^2 - 9n$ = ...

(a) − 8 (b) − 6 (c) − 7 (d) − 18

---

70 If the ratio between the coefficients of two consecutive terms in the expansion of $(1 + X)^{24}$ according to the ascending power of X is 4 : 1 then the two terms are ....

(a) T_4 , T_5 (b) T_{20} , T_{21} (c) T_3 , T_4 (d) T_{21} , T_{22}

---

71 If the ratio among the fifth , the sixth and the seventh terms in the expansion of $\left(\frac{3X}{2} + \frac{2}{3X}\right)^n$ is 40 : 24 : 11 according to the descending power of X , then X = ...

(a) $\pm \frac{4}{3}$ (b) $\pm \frac{1}{2}$ (c) $\pm \frac{2}{3}$ (d) $\pm \frac{3}{8}$

---

72 In the expansion of $(X + y)^n$ according to the descending powers of X , if $(9T_4)^2 = 9T_2 \times 44T_6$, then n = ...

(a) 11 (b) 12 (c) 13 (d) 14

---

73 In the expansion of $(2 + X)^9$ according to descending power of X if the sixth term added to $\frac{1}{4}$ of the sevenths term equals 7 times the eighth term then X = ...

(a) 3 or $\frac{8}{3}$ (b) − 3 or $\frac{-8}{3}$ (c) 3 or $\frac{-8}{3}$ (d) − 3 or $\frac{8}{3}$

74 In the expansion $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ if the ratio between the seventh term from the beginning to the seventh term from the end $1:6$, then n equals ....

ⓐ 7 ⓑ 8 ⓒ 9 ⓓ 10

75 In the expansion of $\left(\sqrt{x}+\frac{1}{x}\right)^8$ according to descending power of x , if T_4 , T_5 , $25\,T_7$, T_6 are in proportion , then the value of $x =$ ....

ⓐ $\frac{5}{8}$ ⓑ $\frac{3}{5}$ ⓒ $\frac{8}{5}$ ⓓ $\frac{5}{2}$

76 Coefficient of x^7 in the expansion of $(1-x^4)(1+x)^9$ is ....

ⓐ 27 ⓑ -24 ⓒ 48 ⓓ -48

77 The coefficient of x^6 in the expansion of $(1+x^2-x^3)^8$ equals ....

ⓐ 80 ⓑ 84 ⓒ 88 ⓓ 92

78 The coefficient of x^9 in the expansion of $(1+3x+3x^2+x^3)^{15}$ equals ....

ⓐ ${}^{45}C_9$ ⓑ ${}^{45}C_8$ ⓒ ${}^{15}C_9$ ⓓ ${}^{15}C_8$

79 In the expansion of $\left(\frac{1}{x^4}+x^2\log_2 x\right)^6$, if $T_5 = 15$, then $x =$ ....

ⓐ 2 or 8 ⓑ $\frac{1}{2}$ of 4 ⓒ 2 or $\frac{1}{2}$ ⓓ 2 or 4

80 In the expansion of $(x+2y)^n = \dots + k\,x^2 y^3 + \dots$ where $n \in \mathbb{Z}^+$, then $\frac{k}{n} =$ ....

ⓐ 20 ⓑ 16 ⓒ 15 ⓓ 10

81 In the expansion of $(1-m\,x)^n$ according to the ascending powers of x if $T_2 = -\frac{1}{4}x$, $T_3 = \frac{3}{100}x^2$, then $n =$ ....

ⓐ 3 ⓑ 5 ⓒ 15 ⓓ 25

82 If $(m+x)^n = 3a + 6a\,x + 5a\,x^2 + \dots + x^n$, where $n \in \mathbb{Z}^+$, then $m+n+a =$ ....

ⓐ 3 ⓑ 6 ⓒ 243 ⓓ 252

83 In the expansion of $(1+x)^n = 1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots + a_n x^n$ and $\frac{a_2+a_3}{a_2} = 3$, then $n =$ ....

ⓐ 4 ⓑ 6 ⓒ 8 ⓓ 9

84 If $1 + \frac{5}{2} x + \frac{5 \times 4}{4 \lfloor 2} x^2 + \frac{5 \times 4 \times 3}{8 \lfloor 3} x^3 + \cdots + \frac{1}{32} x^5 = 1024$, then $x =$ ....

(a) 5 (b) 4 (c) 6 (d) 8

---

85 The set of solution of the equation : $1 - 6x + \frac{6 \times 5}{2 \times 1} x^2 - \frac{6 \times 5 \times 4}{3 \times 2 \times 1} x^3 + \cdots + x^6 = 64$ in $\mathbb{R}$ is ....

(a) $\{-3 , 1\}$ (b) $\{1 , 3\}$ (c) $\{-1 , 3\}$ (d) $\{-1 , -3\}$

---

86 The solution set of the equation :

$1 + 20x + 190x^2 + 1140x^3 + \cdots + x^{20} = x^{18} + 18x^{17} + 153x^{16} + \cdots + 1$ is ....

(a) {Zero} (b) $\{0 , -1\}$ (c) $\{0 , -1 , 1\}$ (d) $\{0 , -1 , -2\}$

---

87 Number of integer terms in the expansion $(1 + \sqrt{3})^6$ is ....

(a) 3 (b) 4 (c) 5 (d) 6

---

88 Number of integer terms in the expansion $\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)^7$ is ....

(a) zero (b) 2 (c) 3 (d) 4

---

89 Number of terms of the expansion $(x + y)^{2000} + (x - y)^{2000}$ in simplest form is ....

(a) 1000 (b) 2000 (c) 2001 (d) 1001

---

90 Number of terms in the expansion $(x + y)^{1000} - (x - y)^{1000}$ in simplest form is ....

(a) 1000 (b) 500 (c) 501 (d) 1001

---

91 Number of terms in the expansion $(x + y)^{16} + (x - y)^{14}$ in simplest form is ....

(a) 15 (b) 17 (c) 30 (d) 32

---

92 The coefficient of the term contains $x^3 y^4 z^5$ in the expansion of $(x + y + z)^{12}$ is ....

(a) ${}^{12}C_9$ (b) ${}^{12}C_8$ (c) $\frac{\lfloor 12}{\lfloor 7 \lfloor 5}$ (d) $\frac{\lfloor 12}{\lfloor 3 \lfloor 4 \lfloor 5}$

---

93 $\sum_{r=0}^{13} {}^{27}C_r =$ ....

(a) 2^{13} (b) 2^{14} (c) 2^{26} (d) 2^{27}

94. $\sum_{r=0}^{20} {}^{20}C_r = \text{..............}$

(a) 2^{20} (b) 2^{19}
(c) $2^{19} + \frac{1}{2}\ {}^{20}C_{10}$ (d) $2^{19} - \frac{1}{2}\ {}^{20}C_{10}$

95. ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \cdots (-1)^n\ {}^nC_n = \text{..............}$

(a) 2^n (b) $(-1)^n$ (c) zero (d) $(-2)^n$

96. If $(1 + a\,x)^n = 1 + 8\,x + 24\,x^2 + \cdots + a^n x^n$ then $\frac{a-n}{a+n} = \text{..............}$

(a) 3 (b) -3 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

97. ${}^nC_0 + {}^nC_1\left(\frac{1}{3}\right) + {}^nC_2\left(\frac{1}{3}\right)^2 + {}^nC_3\left(\frac{1}{3}\right)^3 + \cdots + {}^nC_n\left(\frac{1}{3}\right)^n = \text{..............}$

(a) $\left(\frac{1}{3}\right)^n$ (b) $\left(\frac{2}{3}\right)^n$ (c) $\left(\frac{4}{3}\right)^n$ (d) $\left(\frac{5}{3}\right)^n$

98. ${}^nC_0 + 2 \times {}^nC_1 + 2^2 \times {}^nC_2 + \cdots + 2^r \times {}^nC_r + \cdots + 2^n \times {}^nC_n = \text{..............}$

(a) 5^n (b) 6^n (c) 4^n (d) 3^n

99. The value of the term free of x in the expansion of $\left(x + \frac{1}{x}\right)^{2n} + \left(x - \frac{1}{x}\right)^{2n}$ equals .... (where n is odd)

(a) zero (b) $-2\ {}^{2n}C_r$ (c) $2\ {}^{2n}C_n$ (d) ${}^{2n}C_n$

100. The term which has greatest coefficient in the expansion of $(1 + x)^{10}$ according to the ascending power of x is ....

(a) T_{11} (b) T_5 (c) T_6 (d) T_{10}

101. The term which has greatest coefficient in the expansion of $(3 + 2\,x)^6$ according to ascending power of x is ....

(a) T_1 (b) T_3 (c) T_4 (d) T_7

102. The term who has the smallest coefficient in the expansion of $(2\,x + 7\,y)^3$ according to the descending powers of x is ....

(a) T_4 (b) T_3 (c) T_2 (d) T_1

103 In the expansion of $(x + y)^n$ according to the descending powers of x if the seventh term is the term of greatest coefficient , then n = ...

(a) 12 (b) 13 (c) 14 (d) 15

104 The greatest coefficient in the expansion of $(2x + y)^8$ equals ....

(a) 1120 (b) 448 (c) 1792 (d) 1028

105 If the sum of the coefficients of all terms of the expansion $(1 + 2x)^n$ according to the ascending power of x equals 6561 , then the greatest coefficient in the expansion equals ....

(a) 896 (b) 3594 (c) 1792 (d) 1972

106 The greatest term in the expansion of $(1 + 4x)^8$ according to the ascending power of x at $x = \frac{1}{3}$ equals ....

(a) $56 \times \left(\frac{4}{3}\right)^5$ (b) $56\left(\frac{3}{4}\right)^5$ (c) $56\left(\frac{3}{4}\right)^4$ (d) $56\left(\frac{2}{5}\right)^4$

107 If the number of terms in the expansion of : $(a + b)^{2n} + (a - b)^{2n}$ equals 11 , then n = ...

(a) 8 (b) 9 (c) 10 (d) 11

108 If the number of terms in the expansion of $(a + b)^{2n+1} - (a - b)^{2n+1}$ equals 7 , then n = ...

(a) 5 (b) 6 (c) 7 (d) 8

109 If the number of terms of the expansion $(x + y)^n + (x - y)^n$ after reducing is 16 , then n = ...

(a) 32 only. (b) 31 only. (c) 30 or 31 (d) 31 or 32

110 The number of terms in the expansion of $(1 + x)^{101} (1 + x^2 - x)^{100}$ equals ....

(a) 302 (b) 301 (c) 202 (d) 101

111 The degree of the expression $\left[\left(x + (x^3 - 1)^{\frac{1}{2}}\right)^5 + \left(x - (x^3 - 1)^{\frac{1}{2}}\right)^5\right]$ equals ....

(a) 5 (b) 6 (c) 7 (d) 8

112 In the expansion of $(a + b)^n$ if ℓ is the sum of the odd-order terms , m is the sum of the even-order terms , **then :**

First : $(a^2 - b^2)^n = ...$

(a) $\ell^2 + m^2$ (b) $\ell^2 - m^2$ (c) $\ell \times m$ (d) $\ell - m$

Second : $(a + b)^{2n} - (a - b)^{2n} = ...$

(a) $\ell + m$ (b) ℓm (c) $2 \ell m$ (d) $4 \ell m$

---

113 If $i^2 = -1$, $x + y i = \left(\sqrt{3} + \frac{i}{2}\right)^5 + \left(\sqrt{3} - \frac{i}{2}\right)^5$, then ....

(a) $y = 0$ (b) $x = 0$ (c) $x > 0 , y > 0$ (d) $x > 0 , y < 0$

---

114 The coefficient of x^{20} in the expansion of : $(1 + x^2)^{40} \left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$ equals ....

(a) 1 (b) ${}^{30}C_{10}$ (c) ${}^{30}C_5$ (d) 30

---

115 If $x + y = 6$, then $\sum_{r=0}^{n} {}^nC_r \, x^{n-r} y^r = ...$

(a) 2^n (b) 3^n (c) 4^n (d) 6^n

---

116 The coefficient of x^{13} in the expansion of $\sum_{r=0}^{50} {}^{50}C_r \, (x - 1)^{50-r} \times 2^r$ equals ....

(a) ${}^{50}C_{13}$ (b) ${}^{50}C_{12}$ (c) 1 (d) zero

---

117 In the expansion of $(1 + x)^{59}$ according to the ascending power of x , sum of coefficients of the last 30 terms = ...

(a) 2^{29} (b) 2^{28} (c) 2^{58} (d) 2^{59}

---

118 $1 + {}^{20}C_1 \, i + {}^{20}C_2 \, i^2 + \cdots + {}^{20}C_{20} \, i^{20} = ...$

(a) 20 (b) $\lfloor 20$ (c) -1024 (d) 1024

---

119 The number of integers terms in the expansion of $\left(\sqrt{3} + \sqrt[8]{5}\right)^{256}$ is ....

(a) 32 (b) 33 (c) 34 (d) 35

---

120 If $(1 + x - 2x^2)^6 = 1 + a_1 x + a_2 x^2 + \cdots + a_{12} x^{12}$, then $a_2 + a_4 + a_6 + \cdots + a_{12} = ...$

(a) 31 (b) 32 (c) 64 (d) 63

121 The coefficients of x^6 in the expansion of $[(1+x)^6+(1+x)^7+(1+x)^8+\cdots+(1+x)^{15}]$ equals ....

(a) ${}^{16}C_9$ (b) ${}^{16}C_5-{}^{5}C_4$ (c) ${}^{16}C_7-1$ (d) ${}^{15}C_6$

122 If $0\le k\le n$, then the coefficient of x^k in the expansion :
$1+(1+x)+(1+x)^2+(1+x)^3+\cdots+(1+x)^n$ is ....

(a) ${}^{n+1}C_{k+1}$ (b) ${}^{n}C_k$ (c) ${}^{n+1}C_{n+k}$ (d) ${}^{n}C_{n-k+1}$

123 ${}^{10}C_0+\frac{1}{2}\,{}^{10}C_1+\frac{1}{3}\,{}^{10}C_2+\frac{1}{4}\,{}^{10}C_3+\cdots+\frac{1}{11}\,{}^{10}C_{10}=$ ....

(a) 2^9 (b) 2^{55} (c) $\frac{1}{11}(2)^{11}$ (d) $\frac{33}{110}(2)^{10}$

124 In the expansion of $(4-x)^8$ according to the ascending powers of x , if T_5 is the greatest numerical value then $x\in$ ....

(a) $[-5,5]$ (b) $]\frac{-16}{5},\frac{16}{5}[$

(c) $[-5,5]-[-\frac{16}{5},\frac{16}{5}]$ (d) $[-5,\frac{-16}{5}]\cup[\frac{16}{5},5]$

Fourth Questions on complex numbers

1 Choose the correct answer from the given ones :

1 $i+i^2+i^3+\cdots+i^{100}=$ ....

(a) zero (b) 1 (c) 2 (d) 100

2 $(1+i)^{12}=$ ....

(a) -8 (b) 32 (c) 64 (d) -64

3 If $z=1+i$, then $\frac{z}{\bar{z}}=$ ....

(a) 2 (b) 1 (c) $-i$ (d) i

4 If $a+b\,i=i\,(1-5\,i)^2$, then $b=$ ....

(a) 10 (b) -10 (c) 24 (d) -24

5 If $\frac{a^2+b^2}{a+b\,i}=2+3\,i$, then $a\times b=$ .... when $a , b\in\mathbb{R}^*$

(a) -6 (b) -5 (c) 5 (d) 6

6. $\left(\frac{2i}{1+i}\right)^5 = \dots\dots$

(a) $1+i$ (b) $1-i$ (c) $-4-4i$ (d) $4-4i$

7. $i^n \times i^{n+1} \times i^{n+2} \times i^{n+3} = \dots\dots$

(a) zero (b) 1 (c) -1 (d) i

8. $i^n + i^{n+1} + i^{n+2} + i^{n+3} = \dots\dots$

(a) zero (b) 1 (c) -1 (d) i

9. If $(1+i)^{2n} = (1-i)^{2n}$, then the smallest value of the number n which satisfy that is ……

(a) 4 (b) 8 (c) 2 (d) 12

10. The smallest value of the number n which makes $\left(\frac{1+i}{1-i}\right)^n$ is a real number is ……

(a) 2 (b) 4 (c) 8 (d) 1

11. If a , b , c , d are four positive consecutive integers : $i^a + i^b + i^c + i^d = \dots\dots$

(a) zero (b) -1 (c) 1 (d) i

12. If z_1 , z_2 are conjugate numbers, then $\frac{1}{z_1} + \frac{1}{z_2}$ could be equal to ……

(a) 0.2 (b) $2+3i$ (c) $5i$ (d) $1+i$

13. Which of the following is true ?

(a) $2+3i < 3+4i$ (b) $3-4i < 2-3i$

(c) $1+i > -1-i$ (d) Nothing of the previous.

14. If L , M are the roots of a quadratic equations : $x^2 + 1 = 0$, then $L^{2022} + M^{2022} = \dots\dots$

(a) $-2i$ (b) $2i$ (c) -2 (d) 2018

15. If L , M are the two roots of the equation : $2i x^2 - 4i x + 3i^3 = 0$

, then $L^2 + M^2 = \dots\dots$

(a) 2 (b) 4 (c) 5 (d) 7

16 If 3 , 2 – i are two roots for the 3rd degree equation whose coefficients are real , then the 3rd root for this equation is ....

(a) -3 (b) $2+i$ (c) $-2+i$ (d) $2-i$

17 $i^{\underline{|0}} + i^{\underline{|1}} + i^{\underline{|2}} + i^{\underline{|3}} + \cdots + i^{\underline{|25}} =$ ....

(a) zero (b) $2i$ (c) $2i+25$ (d) $2i+20$

2 Choose the correct answer from the given ones :

1 If $z = 3 + 4i$, then $|z| =$ ....

(a) 3 (b) 4 (c) 5 (d) 6

2 If $z = 6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, then $|\bar{z}| =$ ....

(a) 6 (b) -6 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$

3 If $z = -1 + \sqrt{3}\,i$ then $|\bar{z}| =$ ....

(a) $-1-\sqrt{3}\,i$ (b) $\sqrt{2}$ (c) 2 (d) -2

4 The principle amplitude of the complex $z = -3$ equals ....

(a) zero (b) 90° (c) 180° (d) –90°

5 The complex number $z = -2i$ in trigonometric form equals ....

(a) 2 (cos 90° + i sin 90°) (b) 2 (cos – 90° + i sin – 90°)

(c) 2 (cos 0° + i sin 0°) (d) 2 (cos 180° + i sin 180°)

6 $3i =$ .... (in the exponential form).

(a) $e^{-\frac{\pi}{2}i}$ (b) $3e^{\frac{\pi}{2}i}$ (c) $3e^{\frac{-\pi}{2}i}$ (d) $3e^{\pi i}$

7 The principle amplitude of the complex number $z = 1 - i$ is ....

(a) $\frac{\pi}{4}$ (b) $\frac{-\pi}{4}$ (c) $\frac{-7\pi}{4}$ (d) $\frac{7\pi}{4}$

8 Which of the following represents the algebraic form of $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$?

(a) $-\sqrt{3}+i$ (b) $-1+\sqrt{3}\,i$ (c) $-\sqrt{2}+\sqrt{2}\,i$ (d) $-\sqrt{3}-\sqrt{3}\,i$

9 The algebraic form of the complex number $z=\sqrt{2}\,e^{\frac{\pi}{4}i}$ is ....

(a) $1+i$ (b) $1-i$ (c) $-1+i$ (d) $-1-i$

10 If $z=\sqrt{5}\,e^{\theta i}$ where θ is an acute angle , $\tan\theta=\frac{1}{2}$, then $z=$ .... (is algebraic form)

(a) $1+2i$ (b) $3+i$ (c) $2+i$ (d) $3+4i$

11 If $z=\sqrt{2}\,(\sin 30°+i\cos 30°)$, then the principle amplitude of z is ....

(a) $30°$ (b) $60°$ (c) $90°$ (d) $120°$

12 If $z=-3\,(\cos 45°+i\sin 45°)$, then the amplitude of $z=$ ....

(a) -135 (b) $135°$ (c) $45°$ (d) $-45°$

13 If $z=2+2\sqrt{3}\,i$, then the exponential form for z equals ....

(a) $4\,e^{\frac{-\pi}{3}i}$ (b) $4\,e^{\frac{\pi}{3}i}$ (c) $4\,e^{\frac{-\pi}{6}i}$ (d) $4\,e^{\frac{\pi}{6}i}$

14 $\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}=$ ....

(a) $e^{\frac{\pi}{6}i}$ (b) $e^{\frac{-\pi}{6}i}$ (c) e^{6i} (d) $\frac{1}{6}\,e^{\pi i}$

15 If $z_1=-1-i$, then the exponential form of the number $z=$ ....

(a) $e^{\frac{3\pi}{4}i}$ (b) $e^{\frac{5\pi}{4}i}$ (c) $\sqrt{2}\,e^{\frac{-3\pi}{4}i}$ (d) $\sqrt{3}\,e^{225\,i}$

16 *(2nd session 2021)* If the point $A\left(\sqrt{k}\,,-\sqrt{k}\right)$ represents the complex number z on the Argand's plane , where $k>1$, then the exponential form of the number z is ....

(a) $\sqrt{2k}\,e^{-\frac{\pi}{4}i}$ (b) $\sqrt{2}\,k\,e^{-\frac{\pi}{4}i}$

(c) $\sqrt{2k}\,e^{\frac{\pi}{4}i}$ (d) $\sqrt{2}\,k\,e^{\frac{\pi}{4}i}$

17 The real part of the complex number whose modulus $\sqrt{2}$ and argument $\frac{7\pi}{6}$ is ....

(a) $-\sqrt{3}$ (b) $-\frac{\sqrt{6}}{2}$ (c) $-\frac{\sqrt{2}}{6}$ (d) $-\frac{\sqrt{6}}{3}$

18 If $z_1 = 2i$, $z_2 = -1 + 3i$ where $i^2 = -1$, then arg $(z_1 - z_2)$ equals ....

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{4}$ (d) $-\frac{3\pi}{4}$

19 If $z_1 = 3 + 3\sqrt{3}\,i$, $z_2 = -4 - 4\sqrt{3}\,i$, then the amplitude of $(z_1 + z_2) =$ ....

(a) $60°$ (b) $-120°$ (c) $180°$ (d) $-60°$

20 The trigonometric form for the number $(i^{25})^3$ where $i^2 = -1$ is ....

(a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (b) $\cos\pi + i\sin\pi$

(c) $\cos\pi - i\sin\pi$ (d) $\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)$

21 If $z = i^{10} + i^{8k-5}$, where $k \in \mathbb{Z}$, then the principle amplitude of $z =$ ....

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{-\pi}{4}$ (d) $\frac{-3\pi}{4}$

22 $\left(\frac{\sqrt{2}\,i^5}{i^4 + i^5}\right)^4 =$ ....

(a) $e^{\frac{\pi}{2}i}$ (b) $e^{\frac{-\pi}{2}i}$ (c) $e^{\pi i}$ (d) $e^{\frac{\pi}{4}i}$

23 If z is a complex number , its principle amplitude is θ , then ....

First : Amplitude $(\bar{z}) =$ ....

(a) θ (b) $-\theta$ (c) $\frac{\pi}{2} - \theta$ (d) $\pi - \theta$

Second : Amplitude $(2z) =$ ....

(a) θ (b) $-\theta$ (c) 2θ (d) -2θ

Third : Amplitude $\left(\frac{1}{z}\right) =$ ....

(a) θ (b) $-\theta$ (c) $\pi - \theta$ (d) $-\pi + \theta$

24 If $z = \frac{1}{\bar{z}}$, then $|z| =$ ....

(a) zero (b) 1 (c) -1 (d) ± 1

25 If $|z| = 10$, then $z\bar{z} =$ ....

(a) 10 (b) 100 (c) 1 (d) -100

26 If $|z| = 6$, then $|-\bar{z}| =$ ....

(a) 6 (b) -6 (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

27 If $|z| + |\bar{z}| = 12$, then $|iz| =$ ....

(a) 12 (b) $12i$ (c) 6 (d) -6

28 If $\bar{z} = \frac{36}{z}$, then $|z|^2 - 4|\bar{z}| - 10 =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

29 If z is a non-real complex number , which of the following numbers have same amplitude z ?

(a) $-z$ (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) $\frac{1}{\bar{z}}$

30 If z is a non-zero complex number , then the principle amplitude of (z) + the principle amplitude of $(\bar{z}) =$ ....

(a) zero (b) 90° (c) 180° (d) $-90°$

31 If z is a complex number. Which of the following statements is false ?

(a) $z\bar{z} = |z|^2$ (b) $z\bar{z} = |\bar{z}|^2$

(c) Amplitude (z) = Amplitude $(\bar{z})$ (d) $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$

32 If the number z is pure imaginary and has a principle amplitude is $(4\theta + 10°)$, then $\theta =$ ....

(a) 20° (b) 20 or $-25°$ (c) 90° or $-90°$ (d) $-90°$

33 If z_1 , z_2 are two conjugate numbers and the amplitude of $z_1 = (3\theta - 110°)$ and the amplitude of $z_2 = (2\theta + 50°)$, then $\theta =$ ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{15}$

34 If the modulus of a complex number z is 2 and its amplitude is $\frac{\pi}{2}$, then ....

First : Modulus of the complex number $(z + 2) =$ ....

(a) 2 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) $2\sqrt{2}$

Second : Principle amplitude of the complex number $(z + 2) =$ ....

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) π

---

35 If $z + 2 = i(z - 2)$, then the complex number $z =$ .... (in the trigonometric form)

(a) $2(\cos 0° + i \sin 0°)$ (b) $2(\cos 90° + i \sin 90°)$

(c) $2(\cos 180° + i \sin 180°)$ (d) $2(\cos -90° + i \sin -90°)$

---

36 If $z_1 = 2\sqrt{3} - i$, $z_2 = 2(\cos 30° + i \sin 30°)$, then $\overline{z_1} - \overline{z_2} =$ ....

(a) $2\sqrt{3} + i$ (b) $\sqrt{3} - i$ (c) $\sqrt{3} + 2i$ (d) $3\sqrt{3} + 2i$

---

37 Multiplicative inverse of the complex number z where $|z| \neq 0$ is ....

(a) $\bar{z}$ (b) $\frac{\bar{z}}{|z|^2}$ (c) $\frac{\bar{z}}{|z|}$ (d) $\frac{z}{|z|^2}$

---

38 $e^{\theta i} + e^{-\theta i} =$ ....

(a) $e^{2\theta i}$ (b) $2\cos\theta$ (c) $2\sin\theta$ (d) $e^{-2\theta i}$

---

39 The numerical value of the expression $e^{\pi i} - e^{-\pi i} =$ ....

(a) -2 (b) 0 (c) 1 (d) 2

---

40 $e^{2\pi i} =$ .... (in the algebraic form).

(a) $1 + i$ (b) $1 - i$ (c) 1 (d) -1

---

41 $e^{2 + \pi i} - e^{-\pi i} =$ ....

(a) 1 (b) e^2 (c) $1 - e^2$ (d) $e^2 - 1$

---

42 If argument of z_1 is θ_1 , argument of z_2 is θ_2 , then argument of $z_1 z_2$ is ....

(a) $\theta_1 + \theta_2$ (b) $\theta_1 \times \theta_2$ (c) $\theta_1 - \theta_2$ (d) $\theta_1 \div \theta_2$

---

43 If the principle amplitude of the number $z_1 = \frac{4\pi}{5}$ and the principle amplitude of $z_2 = \frac{2\pi}{3}$, then the principle amplitude of $\left(\frac{z_1}{z_2}\right) =$ ....

(a) $\frac{2\pi}{15}$ (b) $\frac{\pi}{15}$ (c) $\frac{22\pi}{15}$ (d) $\frac{3\pi}{10}$

44 $3(\cos 30° + i \sin 30°) \times 6(\cos 70° + i \sin 70°) = \dots\dots$

(a) $18(\cos 210° + i \sin 100°)$
(b) $9(\cos 100° + i \sin 100°)$
(c) $18(\cos 100° + i \sin 100°)$
(d) $9(\cos 40° + i \sin 40°)$

45 $6(\cos 210° + i \sin 210°) \div 3(\cos 70° + i \sin 70°) = \dots\dots$

(a) $2(\cos 3° + i \sin 3°)$
(b) $3(\cos 30° + i \sin 30°)$
(c) $3(\cos 140° + i \sin 140°)$
(d) $2(\cos 140° + i \sin 140°)$

46 $[5(\cos 10° + i \sin 10°)]^2 = \dots\dots$

(a) $25(\cos 100° + i \sin 100°)$
(b) $10(\cos 100° + i \sin 100°)$
(c) $25(\cos 20° + i \sin 20°)$
(d) $10(\cos 20° + i \sin 20°)$

47 $\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)^4 = e^{\dots}$

(a) $\frac{2\pi}{3} i$
(b) $-\frac{1}{3}\pi i$
(c) $\frac{1}{3}\pi i$
(d) $\frac{-5}{3}\pi i$

48 If $z = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$, then $\frac{1}{z} = \dots\dots$

(a) $\frac{1}{2} e^{\frac{2\pi}{3} i}$
(b) $\frac{1}{2} e^{\frac{-\pi}{3} i}$
(c) $2 e^{\frac{2\pi}{3} i}$
(d) $2 e^{\frac{-\pi}{3} i}$

49 If $z_1 = 6(\cos 240° + i \sin 240°)$, $z_2 = 2(\cos 300° + i \sin 300°)$, then $\frac{z_2}{z_1} = \dots\dots$

(a) $3 e^{\frac{-\pi}{2} i}$
(b) $3 e^{\frac{-\pi}{3} i}$
(c) $\frac{1}{3} e^{\frac{\pi}{3} i}$
(d) $\frac{1}{3} e^{\frac{\pi}{2} i}$

50 If $z_1 = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$, $z_2 = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$, then the amplitude of $\left(\frac{z_1}{z_2}\right)^7$ is $\dots\dots$

(a) $\frac{\pi}{6}$
(b) π
(c) $\frac{-1}{2}\pi$
(d) 2π

51 If $x = \cos 17° + i \sin 17°$, $y = \cos 11° + i \sin 11°$, then complex number $z = x^3 y^9$ in the trigonometric form is $\dots\dots$

(a) $\cos 30° + i \sin 30°$
(b) $\cos 150° + i \sin 150°$
(c) $\cos -120° + i \sin -120°$
(d) $\cos -60° + i \sin -60°$

52 $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5} = \dots\dots$

(a) $\sin 9\theta - i\cos 9\theta$ (b) $\cos 9\theta - i\sin 9\theta$

(c) $\cos\theta - i\sin\theta$ (d) $\sin\theta - i\cos\theta$

53 If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ and if $\theta_1 + \theta_2 = \pi$, then $z_1 z_2 = \dots\dots$

(a) $r_1 r_2$ (b) $-r_1 r_2$ (c) $r_1 r_2 i$ (d) $-r_1 r_2 i$

54 ***(1st Session 2021)*** If $z_1 = \cos\theta + i\sin\theta$, $z_2 = \cos 2\theta + i\sin 2\theta$, then the principle amplitude of the complex number $3 z_1 z_2 = \dots\dots$, where $\theta \in]0 , \frac{\pi}{6}[$

(a) 9θ (b) θ (c) 5θ (d) 3θ

55 ***(1st Session 2021)*** If $Z = k\left(\sin\frac{4}{3}\pi - i\cos\frac{4}{3}\pi\right)$, then $z^6 = \dots\dots$, where $k >$ zero

(a) k^6 (b) $6k$ (c) $-k^6$ (d) $-6k$

56 If $z_1 = 10(\cos\theta + i\sin\theta)$, $z_2 = 5(\cos\alpha + i\sin\alpha)$ and if $\theta - \alpha = \frac{3\pi}{4}$, then $\frac{z_1}{z_2} = \dots\dots$

(a) $-\sqrt{2} + \sqrt{2}\,i$ (b) $\sqrt{2} - \sqrt{2}\,i$ (c) $1 - \sqrt{3}\,i$ (d) $-1 + \sqrt{3}\,i$

57 If $\frac{\cos\theta - i\sin\theta}{\cos 3\theta - i\sin 3\theta} = i$, then $\theta = \dots\dots$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{12}$

58 If $z = \frac{i}{\cos 75^\circ + i\sin 75^\circ}$, then the principle amplitude for the number (z^6) equals $\dots\dots$

(a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

59 If $z = \left(1 + \sqrt{3}\,i\right)^n$ and $|z| = 8$, then the principle amplitude of z is $\dots\dots$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

60 If $\left|\frac{3+2i}{2+ai}\right| = 1$, then $a = \dots\dots$

(a) 3 only. (b) -3 only. (c) ± 3 (d) $\pm\sqrt{3}$

61 If $z = 1 + \cos\theta + i\sin\theta$ and $|z| = \sqrt{3}$, then $\theta =$ .... where θ is an acute angle.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

62 If $z = 1 + e^{\frac{\pi}{2}i}$, then $|3 + \bar{z}| =$ ....

(a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) $\sqrt{17}$

63 If $(a + b\,i)(x + y\,i) = 1 + i$ for all $a, b, x, y \in \mathbb{R}^+$, then $\tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{y}{x}\right) =$ ....

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

64 If $z_1 = r_1 e^{\theta i}$, $z_2 = r_2 e^{\theta i}$, then $|z_1 + z_2|$ ....

(a) less than $(r_1 + r_2)$ (b) greater than $(r_1 + r_2)$

(c) equals $(r_1 + r_2)$ (d) equals $(r_1 - r_2)$

65 If $z = \sqrt{2}\, e^{\theta i}$ and $z^6 = 8\ e^{\frac{3\pi}{2}i}$, then $\theta =$ ....

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

66 If $z = x + y\,i$, $z^4 = 4\,e^{\pi i}$, $z^5 = 4\sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$, then $x + y =$ ....

(a) -1 (b) zero (c) 1 (d) 2

67 If $a \neq 0$ and $z_1 = 1 + \sqrt{2}\,i$, $z_2 = 2\left(\cos\frac{\pi}{a} + i\sin\frac{\pi}{a}\right)$, then $\left|\frac{z_1}{z_2}\right| =$ ....

(a) 2 (b) $\sqrt{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{6}$

68 If $z = 1 - i$, then the principle amplitude for the number $\left(\frac{\bar{z}}{i}\right) =$ ....

(a) $\frac{-3\pi}{4}$ (b) $\frac{-\pi}{2}$ (c) $\frac{-\pi}{3}$ (d) $\frac{-\pi}{4}$

69 If the amplitude for the number $z = 28°$, then the amplitude for the number $\left(\frac{i}{\bar{z}}\right) =$ ....

(a) 28° (b) 62° (c) 118° (d) $-28°$

70 If z is a real number, then for sure $\sqrt[5]{z} \in$ ....

(a) $\mathbb{Z}$ (b) $\mathbb{N}$ (c) $\mathbb{R}$ (d) $\mathbb{C}$

71 If $z = x + iy$, then the real part of the number e^z is ....

(a) $e^{x} \cos y$ (b) $e^{x} \sin y$ (c) e^{x} (d) $\cos y$

72 The real part of the complex number $(e^{3+\frac{\pi}{2}i})$ equals ....

(a) zero (b) e^{3} (c) e^{-3} (d) $-e^{3}$

73 If $z = \frac{2-i}{2+i}$, then $|z| =$ ....

(a) 3 (b) 4 (c) 1 (d) 5

74 If $a \in \mathbb{R}$, $b \in \mathbb{R}$, $z = \frac{(a+b)+(a-b)i}{(a-b)-(a+b)i}$, then the amplitude of (z) = ...

(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) π

75 If $z_1 = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}$ and $|z_2| = 4$, amp. $(z_1 z_2) = \frac{-2\pi}{3}$, then $z_2 =$ ....

(a) $4e^{\frac{2\pi}{7}i}$ (b) $4e^{\frac{\pi}{3}i}$ (c) $4e^{\frac{\pi}{6}i}$ (d) $4e^{\frac{\pi}{12}i}$

76 If the argument of the complex number $z \in]-\pi, 0[$, then $\arg(-z) - \arg(z) =$ ....

(a) π (b) $-\pi$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

77 If z_1 , z_2 are two non-zero complex numbers and $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then $z_1 =$ ....

(a) z_2 (b) $\overline{z_2}$ (c) $-z_2$ (d) $-\overline{z_2}$

78 If z_1 , z_2 are two complex numbers , $\arg(z_1 z_2) = \frac{5\pi}{18}$, $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{9}$, then $\arg(z_1) =$ ....

(a) $\frac{5\pi}{36}$ (b) $\frac{7\pi}{36}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

79 If $\arg(z_1 z_2) = \frac{\pi}{6}$, $\arg(z_1 z_3) = \frac{2\pi}{9}$, $\arg(z_2 z_3) = \frac{5\pi}{18}$, then $\arg(z_1 z_2 z_3) =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{6}$

80 If z is a complex number , and $\arg(z-2) = \frac{\pi}{2}$, $\arg(z-4) = \frac{3\pi}{4}$, then $\arg(z) =$ ....

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

81 If $|z_1| = |z_2|$ and amplitude of (z_1) + amplitude of $(z_2) = 0$, then ....

(a) $\overline{z_1} = z_2$ (b) $z_1 = z_2$ (c) $z_1 + z_2 = 0$ (d) $\overline{z_1} = \overline{z_2}$

82 If z is a complex number whose argument θ , $|z| = 1$, then the argument of $\left(\frac{1+z}{1+\bar{z}}\right)$ equals ....

(a) $-\theta$ (b) $\frac{\pi}{2} - \theta$ (c) θ (d) $\pi - \theta$

83 If $|z| = |z-2|$, then the real part of the number z equals ....

(a) 1 (b) -1 (c) -2 (d) 2

84 If z is a complex number and $|z-3| = |z|$ and the principle amplitude for the number (z) equals $\frac{-\pi}{4}$ then $|z| =$ ....

(a) $3\sqrt{2}$ (b) $\frac{3\sqrt{2}}{2}$ (c) $2\sqrt{2}$ (d) 2

85 If $|z-i| = |z-1|$ where z is a complex number , then the amplitude of the complex number (z) could be equals ....

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

86 *(2nd session 2021)* If z_1 , z_2 are two complex numbers , $z_1 = e^{5 + k\pi i}$, $z_2 = e^{(5 + k i)\pi}$, where $-\frac{1}{2} < k < \frac{1}{2}$, then the principle amplitude of the complex number $z_1 + z_2$ could be equal ....

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{2}$ (c) π (d) $-\frac{\pi}{6}$

87 If the principle amplitude for the number $(\bar{z}) = \frac{-\pi}{3}$, then the principle amplitude for the number $(z + i z)$ equals .... where z is a complex number.

(a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{3\pi}{4}$

88 If the principle amplitude for the complex number (z^4) more than three times the principle amplitude for the number $(5z)$ by $\frac{\pi}{3}$, then the principle amplitude for the number $(\bar{z})$ equals ....

(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{3}$ (c) $\frac{-\pi}{33}$ (d) $\frac{2\pi}{3}$

89 If z is a complex number and $z = \sin 20° (1 + i \cot 20°)$, then

First : $|z| =$ ....

(a) 1 (b) $\sin 20°$ (c) $\cos 20°$ (d) $\tan 20°$

Second : The principle amplitude for the number (z) equals ....

(a) 20°   (b) 40°   (c) 50°   (d) 70°

---

90 If $z_1 = \cos\theta_1 + i\sin\theta_1$ where $0 < \theta_1 < \frac{\pi}{2}$, $z_2 = \cos\theta_2 + i\sin\theta_2$, where $0 < \theta_2 < \frac{\pi}{2}$ and if $z_1^{-4} z_2^3 = 1$, then $\frac{\theta_1}{\theta_2} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{7}{8}$ (d) $\frac{2}{3}$

---

91 If the amplitude for the complex number $\left(\frac{4}{i}\right)$ + the amplitude for the complex number $\left(-1+\sqrt{3}\,i\right) = \frac{k\pi}{6}$, then k = ...

(a) -2 (b) -1 (c) 1   (d) 2

---

92 If the amplitude for the complex number $(z - 1 + i) = \pi$ and if $|z| = 2$, then z = ...

(a) $1-\sqrt{3}\,i$ (b) $-1+\sqrt{3}\,i$ (c) $-\sqrt{3}-i$ (d) $3-i$

---

93 The modulus of the complex number $z = (1 + i\tan 15°)$ equals ....

(a) sin 15°   (b) cos 15°   (c) tan 15°   (d) sec 15°

---

94 If z is a complex number where $z + |z| = 8 + 12\,i$ then $|z|^2 =$ ....

(a) 121   (b) 144   (c) 169   (d) 228

---

95 If $x + y\,i = \frac{a+b\,i}{a-b\,i}$, then $x^2 + y^2 =$ ....

(a) a^2+b^2 (b) a^2-b^2 (c) $-a\,b$ (d) 1

---

96 If a is a real number , then the conjugate of the number $\frac{a^3+i}{a^2+a\,i-1}$ is ....

(a) $a-i$ (b) $a+i$ (c) $\frac{a^3-i}{a^2+a\,i-1}$ (d) $\frac{a^3+i}{a^2-a\,i-1}$

---

97 If $z = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right)^n$ where n is a positive integer , z = 1 , then the least value of n is = ...

(a) 9   (b) 6   (c) 3   (d) 1

98 Amplitude of the complex number $[(1 - \cos\theta) + i \sin\theta]$ equals ...., where $0 < \theta < \pi$

(a) $\frac{\pi}{4} - \frac{\theta}{2}$ (b) $\frac{\pi}{2} - \frac{\theta}{2}$ (c) $\frac{\pi}{2} - \theta$ (d) $\frac{\pi}{2} - \frac{\theta}{4}$

99 If $z = 1 + \cos 2\theta + i \sin 2\theta$, then $|z| =$ .... where θ is an acute angle.

(a) 1 (b) 2 (c) $2 \cos\theta$ (d) $2 \sin 2\theta$

100 If $z = 1 + \cos 40° + i \sin 40°$, then $\arg(z) =$ ....

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{9}$ (c) $\frac{\pi}{18}$ (d) $\frac{2\pi}{9}$

101 If $z = \cos\frac{\pi}{3} + i \sin\frac{\pi}{3}$, then $z + 1 =$ .... (in the exponential form)

(a) $e^{\frac{\pi}{6} i}$ (b) $\sqrt{3}\, e^{\frac{\pi}{6} i}$ (c) $\sqrt{3}\, e^{\frac{\pi}{3} i}$ (d) $(\sqrt{2} + 1) e^{\frac{\pi}{3} i}$

102 If $z = \cos\theta - i \sin\theta$, then $\frac{z^2 - 1}{z\, i} =$ ....

(a) $-2 \sin\theta$ (b) $2 \sin\theta$ (c) $\sin\theta$ (d) $\sin 2\theta$

103 If $a = \cos\theta\, i + \sin\theta$, then $\frac{1 + a}{1 - a} =$ ....

(a) $\cot\frac{\theta}{2}$ (b) $\cot\theta$ (c) $i \cot\frac{\theta}{2}$ (d) $i \tan\frac{\theta}{2}$

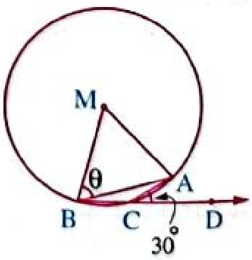
104 The amplitude of the complex number $z = \frac{1 + i \tan 18°}{1 - i \tan 18°}$ is ....

(a) $\frac{1}{2}\pi$ (b) $\frac{1}{4}\pi$ (c) $\frac{1}{5}\pi$ (d) $\frac{1}{12}\pi$

105 If $z = r\, e^{\theta i}$, then $r\, e^{\left(\frac{\pi}{2} + \theta\right) i}$ represents by the complex number ....

(a) $\frac{\pi}{2} z$ (b) $z + \frac{\pi}{2}$ (c) $\frac{z}{i}$ (d) $z\, i$

106 **In the opposite figure :**



Circle with center (M)

in which $m(\angle ACD) = 30°$

, $m(\angle MBA) = \theta$ and $z = \sin 3\theta + i \cos 3\theta$

, then $z =$ .... in the exponential form

(a) $e^{\frac{3\pi}{4} i}$ (b) $e^{\frac{-3\pi}{4} i}$ (c) $e^{\frac{\pi}{2} i}$ (d) $e^{\frac{-\pi}{2} i}$

107 *(1st Session 2021)* If z is a complex number and $z + \bar{z} = 2\,e^{\pi i}$, then z may be equal ....

(a) $e^{\pi i}$ (b) $2\,e^{\frac{\pi}{2} i}$ (c) $e^{\frac{-\pi}{2} i}$ (d) $2\,e^{\pi i}$

---

108 If $z_1 = \cos 70° + i \sin 70°$, $z_2 = \cos 20° + i \sin 20°$, then

First : $\arg (z_1 + z_2) =$ ....

(a) 90° (b) 50° (c) 45° (d) 140°

Second : The modulus of the number $(z_1 + z_2) =$ ....

(a) $\sqrt{2}$ (b) $\cos 20°$

(c) $\sin 20°$ (d) $\sqrt{2}\,(\cos 20° + \sin 20°)$

---

109 If $A = \cos \theta + i \sin \theta$; $B = \cos \propto + i \sin \propto$, then the value of $\frac{1}{2}\left(AB + \frac{1}{AB}\right)$ equals ....

(a) $\sin (\theta + \propto)$ (b) $\sin (\theta - \propto)$ (c) $\cos (\theta + \propto)$ (d) $\cos (\theta - \propto)$

---

110 If z_1 , z_2 are two non-zero complex numbers and $|z_1 + z_2| = |z_1| + |z_2|$; then amplitude (z_1) – amplitude $(z_2) =$ ....

(a) π (b) $\frac{\pi}{2}$ (c) $-\pi$ (d) zero

---

111 The equation $z^2 = \bar{z}$ has .... in $\mathbb{C}$

(a) unique solution (b) two solutions

(c) four solutions (d) has no solution

---

112 The product of the roots of the equation : $x^4 - 1 = 0$ equals ....

(a) zero (b) 1 (c) – 1 (d) i

---

113 If $\sqrt{7 + 24\,i} = x + iy$, then $(x + y)^2 =$ ....

(a) 7 (b) 24 (c) 49 (d) 576

---

114 $\sqrt{5 + 12\,i} =$ ....

(a) $\pm (2 + 3\,i)$ (b) $\pm (3 + 2\,i)$ (c) $\pm (2 - 3\,i)$ (d) $\pm (3 - 2\,i)$

115 If $z_1 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $z = -i\,\overline{z_1}$, then one of the square roots for the number (z) is ....

(a) $\sqrt{2}\,e^{\frac{-5\pi}{12}i}$ (b) $2\,e^{\frac{5\pi}{12}i}$ (c) $\sqrt{2}\,e^{\frac{-7\pi}{12}i}$ (d) $2\,e^{\frac{7\pi}{12}i}$

116 If $z_1 = 4\,e^{\frac{5\pi}{6}i}$, $z_2 = i$, then $\sqrt{z_1 z_2} =$ ....

(a) $\pm(1+\sqrt{3}\,i)$ (b) $\pm(1-\sqrt{3}\,i)$ (c) $\pm\sqrt{3}\,i$ (d) $2\pm\sqrt{3}\,i$

117 The square roots of the number i are ....

(a) $e^{\frac{\pi}{4}i}, e^{\frac{-3}{4}\pi i}$ (b) $e^{\frac{-\pi}{4}i}, e^{\frac{-3}{4}\pi i}$

(c) $e^{\frac{-\pi}{4}i}, e^{\frac{\pi}{4}i}$ (d) $e^{\frac{-3}{4}\pi i}, e^{\frac{3}{4}\pi i}$

118 The sum of the square roots of the complex number (3 – 4 i) equals ....

(a) zero (b) $\pm(\sqrt{3}-2i)$ (c) $\pm(2-i)$ (d) 6

119 If z_1, z_2, z_3 are the cubic roots of the number $z = 6+8i$, then $\frac{25\,z_1^3 \times 4\,z_2^6}{z_3^{12}} =$ ....

(a) $6+8i$ (b) $6-8i$ (c) $3+4i$ (d) $3-4i$

120 If $z_1 = 2\,e^{\frac{2\pi}{3}i}$, $z_1 z_2 = 8\,e^{\frac{11\pi}{3}i}$, then the product of the square roots of the number $z_2 =$ ....

(a) $4\,e^{\pi i}$ (b) 4 (c) $4\,e^{\frac{\pi}{2}i}$ (d) 16

121 If $x \in \mathbb{C}$, then the solution set of the equation $x^2 - (2+i)\,x - (1+5i) = 0$ is ....

(a) $\{7+24i, 7-24i\}$ (b) $\{3+2i, -1-i\}$

(c) $\{4+3i, -4-3i\}$ (d) $\{3-2i, 1+i\}$

122 If $x, y \in \mathbb{R}$ and $x + iy = i + \sqrt{i}$, then $x + y =$ ....

(a) $1+\sqrt{2}$ only. (b) $1-\sqrt{2}$ only. (c) $1\pm\frac{1}{\sqrt{2}}$ (d) $1\pm\sqrt{2}$

123 If $x, y \in \mathbb{R}$ and $(1-i)\,x + (1+i)\,y + 2i = 0$, then the expression $\sqrt{3x+4yi} =$ ....

(a) $\pm(5-i)$ (b) $\pm(2-i)$ (c) $3+i, 1-i$ (d) $6, -5$

124 $i^i =$ ....

(a) $e^{\frac{\pi}{2}}$ (b) $e^{-\frac{\pi}{2}}$ (c) -1 (d) 1

125 If $a e^{2\theta i} + b e^{-2\theta i} = 4 \cos 2\theta - 6 i \sin 2\theta$, then $a \times b =$ ....
where a , b are real numbers.

(a) -1 (b) 5 (c) -5 (d) 4

126 If $x + y i = e^{\theta i}$, then the least value for the number $x y$ is ....

(a) $-\frac{1}{2}$ (b) -1 (c) -2 (d) 1

127 If $x - \frac{1}{x} = i$, then $x^{32} + x^{-32} =$ ....

(a) -1 (b) zero (c) 1 (d) 2

128 If $z + \frac{1}{z} = 2 \cos \theta$, then $z^n - \frac{1}{z^n} =$ ....

(a) $\pm 2 i \sin n\theta$ (b) $\pm 2 \cos n\theta$ (c) $\pm 2 \tan n\theta$ (d) $\pm 2 \csc n\theta$

129 If z_1 , z_2 , z_3 are three complex numbers such that :
$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ and $|z_1 + z_2 + z_3| = a$, then ....

(a) $a = 1$ (b) $a < 1$ (c) $a > 3$ (d) $a = 3$

130 If a , b $\in \mathbb{R}$ and $\left(2\sqrt{2} + i\right)^{50} = 3^{49} (a + b i)$, then $a^2 + b^2 =$ ....

(a) 9 (b) 25 (c) 49 (d) 50

131 If $z_1 = 4\sqrt{3} (\cos 140° + i \sin 140°)$, $z_2 = 4 (\cos 230° + i \sin 230°)$
, then $|z_1 - z_2| =$ ....

(a) 8 (b) $8\sqrt{2}$ (c) 12 (d) $12\sqrt{2}$

132 If z is a complex number and $|z - 4| < |z - 2|$, then the real part of the complex number could be equal to ....

(a) 1 (b) 2 (c) 3 (d) 4

133 If ABCD is a quadrilateral and if $z_1 = 3 (\cos A + i \sin A)$, $z_2 = 2 (\cos B + i \sin B)$
, $z_3 = 4 (\cos C + i \sin C)$, $z_4 = \cos D + i \sin D$, then $z_1 z_2 z_3 z_4 =$ ....

(a) 48 (b) 24 (c) $24 i$ (d) $48 i$

134 If $z_n = \cos \frac{\pi}{2^n} + i \sin \frac{\pi}{2^n}$, then $(z_1 \times z_2 \times z_3 \times$ .... to $\infty)$ equals ....

(a) -1 (b) zero (c) 2 (d) ∞

135 If $z = 1 + {}^8C_1 (\sqrt{3}\, i) + {}^8C_2 (\sqrt{3}\, i)^2 + ... + (\sqrt{3}\, i)^8$, then the amplitude of z is ....

ⓐ $\frac{2\pi}{3}$ ⓑ $\frac{\pi}{3}$ ⓒ $\frac{\pi}{6}$ ⓓ $\frac{-\pi}{6}$

3 Choose the correct answer from the given ones :

1 The number $z = 3 - 4\, i$ is represented on Argand's diagram by the point A where A = ...

ⓐ (3 , 4) ⓑ (3 , – 4) ⓒ (– 3 , 4) ⓓ (– 3 , – 4)

2 If the point $A\left(\sqrt{3}, -1\right)$ represents a complex number on Argand's diagram , then the modulus and the principle amplitude for the number z are ....

ⓐ $\left(2, \frac{\pi}{6}\right)$ ⓑ $\left(2, \frac{5\pi}{6}\right)$ ⓒ $\left(2, \frac{-5\pi}{6}\right)$ ⓓ $\left(2, \frac{-\pi}{6}\right)$

3 ABCD is a square drawn on Argand's diagram its centre is the origin ; the point A represents the complex number $\left(1 + \sqrt{3}\, i\right)$, then the point C represents the complex number ....

ⓐ $-1 - \sqrt{3}\, i$ ⓑ $1 - \sqrt{3}\, i$ ⓒ $-1 + \sqrt{3}\, i$ ⓓ $2(1 + \sqrt{3}\, i)$

4 If the point A represents the complex number z on Argand's plane and point B represents the number $\bar{z}$ on Argand's plane , then B is the image of A by reflection in ....

ⓐ the origin point. ⓑ the x-axis.

ⓒ the y-axis. ⓓ the straight line $y = x$

5 If $z = x + y\, i$ is a complex number and $\left|\frac{z - 3\, i}{z + 3\, i}\right| = 1$, then z lies .... on Argand's plane

ⓐ on the x-axis ⓑ on the y-axis

ⓒ in the first quadrant ⓓ in the second quadrant

6 The complex number $\left(\frac{1 + 2\, i}{1 - i}\right)$ lies in Argand's plane in the .... quadrant.

ⓐ 1^{st} ⓑ 2^{nd} ⓒ 3^{rd} ⓓ 4^{th}

7 If z_1 , z_2 , z_3 , z_4 are roots of the equation $z^4 = a$, then polygon whose vertices are the points represent z_1 , z_2 , z_3 , z_4 on Argand's plane represents ....

ⓐ rectangle. ⓑ square. ⓒ parallelogram. ⓓ trapezium.

8 The fifth roots of unity represented on Argand diagram vertices of ....

(a) an equilateral triangle. (b) a square.

(c) a regular pentagon. (d) a regular hexagon.

9 If z_0 , z_1 ... , z_5 represent the sixth roots of unity on Argand diagram , then $m(\angle z_r o z_{r+1})$ = ... where $0 \le r \le 5$, $r \in z$

(a) 30° (b) 60° (c) 90° (d) 120°

10 If the point (A) represents on Argand's plane the complex number (z) and the point (B) represents the complex number (i z) on the same plane , then $m(\angle AOB)$ = ... , where (O) is origin point.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

11 If z_1 , z_2 , z_3 are vertices of a triangle , the medians of the triangle are intersected at $-3+4i$, then $|z_1+z_2+z_3|$ = ...

(a) 5 (b) $10\sqrt{3}$ (c) 15 (d) $15\sqrt{3}$

12 If z_1 , z_2 , z_3 are the roots of the equation $z^3 = a$ where $a \neq$ zero and $\arg(z_1) = \theta$, then argument of z_2 , z_3 are .... respectively.

(a) θ , θ

(b) $\theta + \frac{\pi}{3}$, $\theta + \frac{2\pi}{3}$

(c) $\theta + \frac{2\pi}{3}$, $\theta + \frac{4\pi}{3}$

(d) $\frac{2\pi}{3}$, $\frac{4\pi}{3}$

13 If z , z^2 , z^3 are complex numbers on Argand diagram and they are on one circle whose centre at the origin. Which of the following statements is true ?

(a) $\arg(z) = \arg(z^2) = \arg(z^3)$

(b) $|z| = 1$

(c) $|z| = |z^2| = |z^3| \neq 1$

(d) The triangle whose vertices are z , z^2 , z^3 is a right-angled triangle.

14 If z is a complex number , $z = r(\cos\pi + i\sin\pi)$, then the two roots of z are ....

(a) real and have same sign. (b) real and have different signs.

(c) imaginary and have same sign. (d) imaginary and have different signs.

15 If the points A , B , C represent on an Argand's plane the complex numbers z , $-z$, $\bar{z}$ respectively , where $z = 5(\cos\theta + i\sin\theta)$, θ is an acute angle , where $\sin\theta = \frac{3}{5}$, then the area of the triangle ABC = ... area unit.

(a) 5 (b) 10 (c) 24 (d) 25

16 The points represents the complex numbers z , $z\,i$, $z + i\,z$ in the Argand's plane are vertices of Δ ABC , then the area of Δ ABC = ...

(a) $|z|$ (b) $|z|^2$ (c) $2|z|^2$ (d) $\frac{1}{2}|z|^2$

17 The modulus of a complex number z is 16 and z_1 , z_2 are the square root for the number z , then the length of the line segment whose end points z_1 and z_2 equals ....

(a) 32 (b) 16 (c) 8 (d) 4

18 If z_1 , z_2 , z_3 are the roots of the equation $z^3 = 4 + 4\sqrt{3}\,i$, then the area of the polygon whose vertices are the points z_1 , z_2 , z_3 on Argand's diagram equals .... square unit.

(a) $3\sqrt{3}$ (b) $6\sqrt{3}$ (c) $9\sqrt{3}$ (d) $12\sqrt{3}$

19 If $z_1 = 1 - \sqrt{3}\,i$, $|z_1 z_2| = 1$, $\arg(z_2) - \arg(z_1) = \frac{\pi}{2}$, then area of the triangle whose vertices z_1 , z_2 , O (the origin) equals ....

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

20 If z is a complex number , then the equation : $|z| = 3$ represents on Argand diagram by ....

(a) a circle , its centre at the origin and its raduis is 3
(b) a circle , its centre is the point (0 , 3) and its raduis is 3
(c) a circle , its centre is the point (3 , 0) and its raduis is 3
(d) a circle , its centre is the origin and its raduis is 9

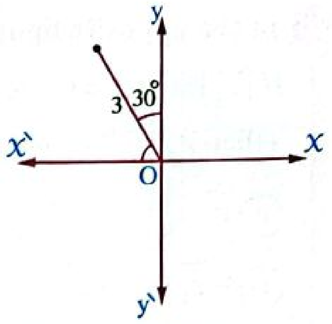
21 If z is a complex number , then the equation $|z|^2 + 8\left(z + \bar{z}\right) + z\,\bar{z} = 8$ represents a circle , its surface area = ... area units.

(a) 9π (b) 16π (c) 20π (d) 25π

22 If z is a complex number , then the number of values of the number $z^{\frac{4}{3}}$ is ....

(a) 1 value. (b) 2 values. (c) 3 values. (d) 4 values.

23 The opposite figure represents the complex number z = ...



(a) 3 (cos 30° + i sin 30°)

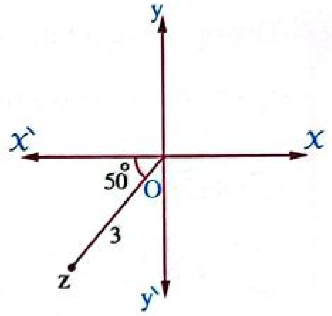
(b) 3 (cos 60° + i sin 60°)

(c) 3 (cos 120° + i sin 120°)

(d) 3 (cos 150° + i sin 150°)

---

24 **In the opposite figure :**



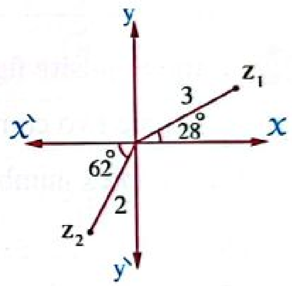
The number z in the exponential form = ...

(a) $e^{\frac{13}{18}\pi i}$ (b) $e^{-\frac{13}{18}\pi i}$

(c) $3e^{\frac{13}{18}\pi i}$ (d) $3e^{-\frac{13}{18}\pi i}$

---

25 The opposite figure represents two complex numbers z_1 , z_2 , then the amplitude of $(z_1 z_2)$ = ...

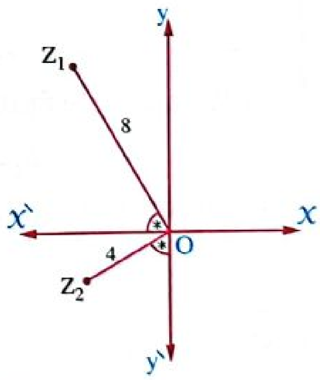


(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$

(c) π (d) $\frac{-\pi}{8}$

---

26 **In the opposite figure :**



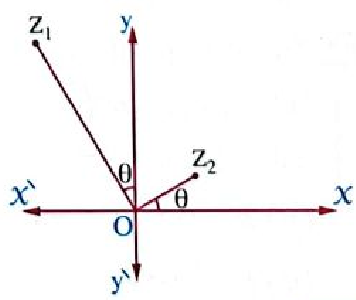
z_1 , z_2 are two complex numbers , then $\frac{z_1}{z_2}$ = ...

(a) −2 (b) 2

(c) −2 i (d) 2 i

---

27 **In the opposite figure :**



z_1 , z_2 are two complex numbers in Argand's plane , $|z_1| = 3|z_2|$, then $\frac{z_1}{z_2}$ = ...

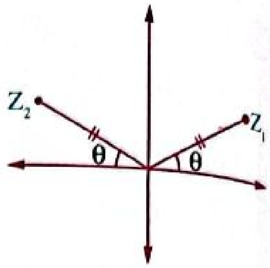
(a) $3e^{-\frac{\pi}{2}i}$ (b) $3e^{\frac{\pi}{2}i}$

(c) $\frac{1}{3}e^{\frac{\pi}{2}i}$ (d) $\frac{1}{3}e^{-\frac{\pi}{2}i}$

28 **In the opposite figure :**

If $|z_1| = |z_2| = r$

, then $z_2 = \ldots\ldots\ldots\ldots$

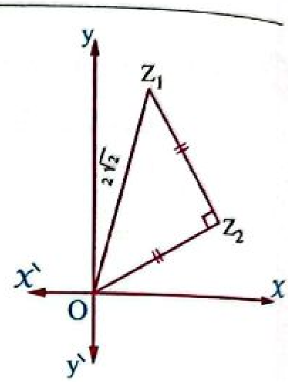


(a) $\overline{z_1}$ (b) $-z_1$

(c) $-\overline{z_1}$ (d) $z_1\, i$

---

29 The opposite figure represents two complex numbers z_1 , z_2 on Argand's diagram , then $\left(\frac{z_1}{z_2}\right)^6 = \ldots\ldots\ldots\ldots$



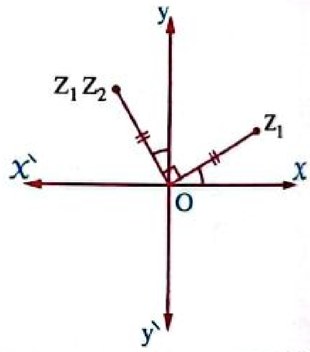
(a) $8\, e^{\frac{-\pi}{6} i}$ (b) $8\, e^{\frac{-\pi}{4} i}$

(c) $8\, e^{\frac{-\pi}{3} i}$ (d) $8\, e^{\frac{-\pi}{2} i}$

---

30 **In the opposite figure :**

z_1 , z_2 are two complex numbers and $(z_1\, z_2)$ is a complex number , then $z_2 = \ldots\ldots\ldots\ldots$



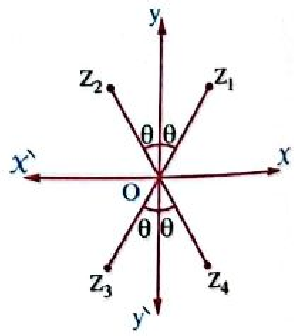
(a) $-z\, i$ (b) $-i$

(c) i (d) $z\, i$

---

31 **In the opposite figure :**

If z_1 , z_2 , z_3 and z_4 are complex numbers , then $\arg(z_1\, z_2\, z_3\, z_4) = \ldots\ldots\ldots\ldots$



(a) $-90°$ (b) zero

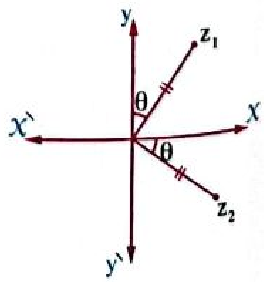
(c) $90°$ (d) $180°$

---

32 **In the opposite figure :**

If $|z_1| = |z_2| = \sqrt{3}$

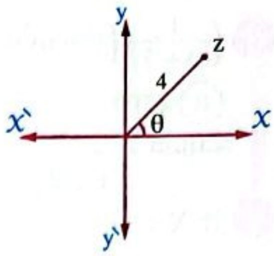
, then $|z_1 + z_2| = \ldots\ldots\ldots\ldots$

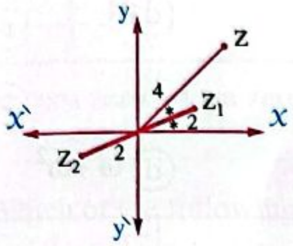


(a) 2 (b) $2\sqrt{3}$

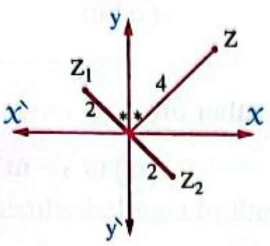
(c) $\sqrt{6}$ (d) 3

33 If the opposite figure represents the complex number z, which of the following figures represent the two square roots of z

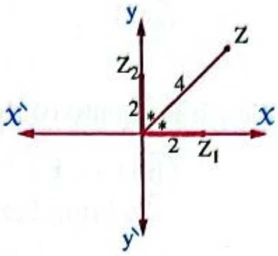




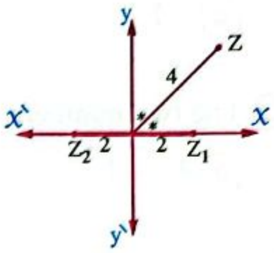
(a)



(b)



(c)



(d)

4 Choose the correct answer from the given ones :

1 $\omega^{22} + \omega^{32} = ...$

(a) ω (b) 1 (c) -1 (d) zero

2 $\omega^{-13} + \frac{1}{\omega} = ...$

(a) ω^2 (b) 1 (c) $2\omega^2$ (d) 2ω

3 $\omega^{2020} + \omega^{2021} + \omega^{2022} = ...$

(a) ω (b) ω^2 (c) zero (d) 1

4 $\left(\frac{1}{\omega} - \frac{1}{\omega^2}\right)^2 = ...$

(a) $\pm\sqrt{3}\,i$ (b) ± 3 (c) 3 (d) -3

5 $\left(\omega^2 + \frac{1}{\omega}\right)\left(1 + \frac{1}{\omega^2}\right)^2 = ...$

(a) 2 (b) zero (c) -3 (d) -5

6 $\left(\omega + \frac{1}{\omega}\right)^2 \left(\omega^2 + \frac{1}{\omega^2}\right)^2 = ...$

(a) zero (b) 1 (c) -1 (d) ω^2

7 $(1 + \omega)^4 + (1 + \omega^2)^4 + (\omega + \omega^2)^4 = ...$

(a) zero (b) 1 (c) -1 (d) ω

8. $\left(\frac{1}{\omega+1}\right)\left(1+\omega-\frac{3}{\omega}\right) = \dots\dots$

(a) zero (b) 1 (c) 2 (d) 4

9. If $x = \frac{-1+\sqrt{3}\,i}{2}$ where $i^2 = -1$, then $x^5 + x^4 + 5 = \dots\dots$

(a) -1 (b) 1 (c) ω (d) 4

10. The two numbers that each is square of the other are $\dots\dots$

(a) $1 , -1$ (b) $i , -i$ (c) $\omega , -\omega$ (d) ω , ω^2

11. If a , b , c are three consecutive integers then $\omega^a + \omega^b + \omega^c = \dots\dots$

(a) zero (b) 1 (c) ω (d) ω^2

12. The conjugate of the number ω equals $\dots\dots$

(a) ω (b) ω^2 (c) 1 (d) $-\omega$

13. The conjugate of the number $1+\omega$ is $\dots\dots$

(a) $1-\omega$ (b) $-\omega^2$ (c) $1+\omega^2$ (d) $1-\omega^2$

14. The conjugate of the number $i-\omega^2$ is $\dots\dots$

(a) $i+\omega^2$ (b) $-i+\omega^2$ (c) $-i-\omega$ (d) $-i+\omega$

15. The conjugate of the complex number $(i+\omega+2021)$ is the complex number $\dots\dots$

(a) $-i+\omega+2021$ (b) $-i+\omega^2+2021$

(c) $-i-\omega+2021$ (d) $-i-\omega^2-2021$

16. The conjugate of the number $2\,\omega+3\,\omega^2$ is $\dots\dots$

(a) $2\,\omega-3\,\omega^2$ (b) $2\,\omega^2+3\,\omega$ (c) $3\,\omega-2\,\omega^2$ (d) $3\,\omega+3\,\omega^2$

17. $\frac{3}{2+\omega} = \dots\dots$

(a) $2-\omega$ (b) $2+\omega^2$ (c) $3\,(1+\omega)^2$ (d) $3\,\omega^2$

18. If $z = 2+3\,\omega^2$, then : $z\,\bar{z} = \dots\dots$

(a) 4 (b) 6 (c) 7 (d) 9

19 If ω , ω^2 are the non real cubic roots of unity , then $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=$ ....

(a) -1 (b) zero (c) 1 (d) 2

20 Which of the following is not a cubic root of unity ?

(a) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (b) $e^{\frac{2\pi}{3}i}$

(c) cos zero + i sin zero (d) $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$

21 Which of the following results belongs to the real numbers ?

(a) $\frac{1+\omega}{1+\omega^2}$ (b) $(1+\omega)(2+\omega^2)$ (c) $(3+\omega)(3+\omega^2)$ (d) $(\omega^2-\omega)^3$

22 If z_1 , z_2 are conjugate numbers and $z_1=(c+d)\,\omega^2+(c-d)\,\omega$, $z_2=7\,\omega^2+5\,\omega$, then $c\times d=$ ....

(a) 7 (b) -5 (c) -4 (d) -6

23 If $z\in\mathbb{C}$, ω is one of the cubic roots of unity and $|\,\omega z^2\,|-5\,|\,z\,|=14$, then $\left|\frac{2\,\bar{z}+49}{z+2}\right|=$ ....

(a) 7 (b) 14 (c) 21 (d) 28

24 If ω , ω^2 are the non real cubic roots of unity , then the solution set of $x^3=8$ in C is ....

(a) $\{2\}$ (b) $\{2, 2\,\omega, 4\,\omega^2\}$

(c) $\{2, 2\,\omega, 2\,\omega^2\}$ (d) $\{8, 8+\omega, 8+\omega^2\}$

25 If $x=a$, $y=b\,\omega$, $z=c\,\omega^2$, then $\frac{x}{a}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=$ ....

(a) zero (b) 1 (c) 2 (d) ω

26 $\cos\left[(\omega^{10}+\omega^{23})\,\pi+\frac{\pi}{4}\right]=$ ....

(a) $\frac{-\sqrt{3}}{2}$ (b) $\frac{-\sqrt{2}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{\sqrt{3}}{2}$

27 $1+\omega+\omega^2+\omega^3+\cdots+\omega^{100}=$ ....

(a) zero (b) 1 (c) ω (d) $-\omega^2$

28 $\left(2+\frac{3}{\omega}\right)\left(2+\frac{3}{\omega^2}\right)\left(3-\frac{2}{\omega}\right)\left(3-\frac{2}{\omega^2}\right)=$ ....

(a) 7 (b) 9 (c) 26 (d) 133

29 *(Trial 2021)* $\left(\frac{a}{\omega}-\frac{a}{\omega^2}+\frac{3a}{\omega^4}-\frac{3a}{\omega^5}\right)^2=$ ....

(a) $-48a^2$ (b) $48a^2$ (c) $16a^2$ (d) $-16a^2$

30 *(1st Session 2021)* If 1 , ω , ω^2 are the cubic roots of unity , and $X=\frac{1}{1+\omega i}$, $y=\frac{\omega+i}{1+\omega^2 i}$, then $X-y=$ ....

(a) $i+1$ (b) $1-i$ (c) 1 (d) i

31 *(Trial 2021)* $\frac{14+6\omega+21\omega^2}{8\omega^2-7}=$ ....

(a) $-\omega$ (b) ω (c) ω^2 (d) $-\omega^2$

32 $\frac{1}{2}$, $\frac{1}{2}\omega$, $\frac{1}{2}\omega^2$ are the cubic roots of the number ....

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{2}i$ (d) $\frac{1}{8}$

33 2 , $\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ are the cubic roots of the equation ....

(a) $z^3=1$ (b) $z^3-8=0$ (c) $(z-1)^3=1$ (d) $(z+1)^3=1$

34 $(\omega+i)^{100}(\omega^2+i)^{100}=$ ....

(a) -1 (b) 1 (c) ω (d) $101i$

35 $\left(1-\frac{1}{\omega}\right)\left(1-\frac{1}{\omega^2}\right)\left(1-\frac{1}{\omega^4}\right)\left(1-\frac{1}{\omega^8}\right)$ ... to 10 factors = ...

(a) 243 (b) 15 (c) 5 (d) -5

36 $\left(\frac{3+5\omega}{5+3\omega^2}-\frac{5+3\omega^2}{3+5\omega}\right)^8=$ ....

(a) 1 (b) 27 (c) 81 (d) 243

37 If $a=2\omega-3\omega^2$, $b=3+5\omega^2$, then $a^2+b^2=$ ....

(a) -37 (b) -19 (c) 1 (d) 38

38. If $x^2 - x + 1 = 0$, then $x^{3n} =$ .... where n is an even number.

(a) ± 1 (b) 1 (c) -1 (d) zero

39. $(a + b\omega + a\omega^2)(a + b\omega^2 + a\omega^4) =$ ....

(a) 1 (b) $a - b$ (c) $(a - b)^2$ (d) $b^2 - a^2$

40. $\left(1 + 2\omega^5 + \frac{1}{\omega^2}\right)\left(1 + 2\omega + \frac{1}{\omega^4}\right) =$ ....

(a) 0 (b) 1 (c) -1 (d) 2

41. $\frac{a - d\omega}{a\omega^2 - d} - \omega^2 =$ ....

(a) $3i$ (b) $\pm\sqrt{3}\,i$ (c) -3 (d) 3

42. If $z = \omega^x$, where x is a positive integer , then $|z| =$ ....

(a) 1 (b) ω (c) x (d) ω^2

43. $\sum_{r=1}^{5} \omega^r =$ ....

(a) zero (b) -1 (c) 1 (d) ω^2

44. $\sum_{r=1}^{6} (1 + \omega^r) =$ ....

(a) zero (b) 6 (c) 1 (d) $1 + \omega$

45. $1 + \sum_{r=1}^{6} \omega^r =$ ....

(a) zero (b) 1 (c) ω (d) $2 - \omega^2$

46. $\sum_{r=0}^{10} (1 + \omega^r + \omega^{2r}) =$ ....

(a) 6 (b) 8 (c) 10 (d) 12

47. If $i^{12} = \omega^{12}$, then ....

(a) $i = \omega$ (b) $i = \pm\omega$

(c) i , ω each is a root of the equation $z^{12} = 1$ (d) i , ω have no relation.

48 If $(1 + \omega)^7 = a + b\,\omega$ where a and b are real numbers , then $(a , b) =$ ....

(a) $(0 , -1)$ (b) $(1 , 1)$ (c) $(0 , 1)$ (d) $(1 , -1)$

---

49 If $(1 + \omega^2)^n = (1 + \omega)^n$, then the least value of the positive integer n is ....

(a) 2 (b) 3 (c) 5 (d) 6

---

50 If a , b are the non real cubic roots of unity then $a\,b + a^5 + b^5 =$ ....

(a) 1 (b) zero (c) -1 (d) 3

---

51 If 3 , $2 + \omega^2$ are two roots of cubic equation that has real coefficient then the third root equals ....

(a) $2 + \omega$ (b) $2 - \omega$ (c) $\omega^2 - 2$ (d) $2 - \omega^2$

---

52 The value of the number $(-1 + \sqrt{3}\,i)^{48}$ where $i^2 = -1$ equals ....

(a) 1 (b) 2 (c) 2^{24} (d) 2^{48}

---

53 $\left(\frac{-1-\sqrt{3}\,i}{2}\right)^{3n} + \left(\frac{-1+\sqrt{3}\,i}{2}\right)^{3n} =$ .... where $n \in \mathbb{Z}$

(a) $\frac{3}{2}$ (b) 3 (c) zero (d) 2

---

54 If 1 , ω , ω^2 are the cubic roots of unity , then $\left(\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + ...\right) + \omega + \omega^2 =$ ....

(a) -1 (b) 1 (c) $-i$ (d) i

---

55 The product of the cubic roots of the real number x equals ....

(a) x (b) $\omega\,x$ (c) $\omega^2\,x^2$ (d) x^3

---

56 $\left(\frac{-1-\sqrt{-3}}{2}\right)^{22} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{17} =$ ....

(a) zero (b) -1 (c) 1 (d) ω

---

57 If z_1 , z_2 , z_3 are complex numbers represented by vertices of an equilateral triangle whose centroid is the origin , then $z_1 + z_2 + z_3 =$ ....

(a) zero (b) 1 (c) ω (d) ω^2

58 Set of solutions of the equation : $x = 1 + \omega^{x} + \omega^{2x}$ is ....

(a) $\{3\}$ (b) $\{0, 3\}$ (c) $\{-3, 3\}$ (d) $\{0, -3, 3\}$

59 If $L = a + b$, $M = a\omega + b\omega^2$, $N = a\omega^2 + b\omega$, then $\frac{LMN}{a^3 + b^3} =$ ....

(a) zero (b) -1 (c) 1 (d) $\frac{a^3 - b^3}{a^3 + b^3}$

60 If $\frac{1 + 10\omega + 10\omega^2}{1 - 3\omega - 3\omega^2} = (k\,i)^2$, then $k =$ ....

(a) $\pm \frac{1}{2}$ (b) ± 1 (c) $\pm \frac{3}{2}$ (d) $\pm \frac{3}{4}$

61 If z is a complex number, where $z^2 = 1 + \omega^2$, then $z =$ ....

(a) $\pm (1 + \omega)$ (b) $\pm \omega i$ (c) $\pm \omega^2 i$ (d) $\pm \omega$

62 If $L = A\omega + B\omega^2$, $m = A\omega^2 + B\omega$ are complex numbers, then the false statement in each of following .... where A and B $\in \mathbb{R}$

(a) L, m each is a multiplicative inverse of the other.

(b) L, m are conjugate numbers

(c) $L^2 - m^2 = \pm\sqrt{3}\, i\,(A^2 - B^2)$

(d) $\omega L + \omega^2 m \in \mathbb{R}$

63 ***(Trial 2021)*** On Argand diagram, area of the circle passes through the points represent the cubic roots of unity equals .... square units.

(a) π (b) $2\sqrt{3}\,\pi$ (c) $\sqrt{3}\,\pi$ (d) 2π

64 ***(Trial 2021)*** On Argand diagram, area of the triangle whose vertices are the points represent the cubic roots of one equals .... square units.

(a) $\frac{3\sqrt{3}}{4}$ (b) $\frac{3\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$

65 If A, B and C are the cubic roots of the complex $z = 8i$, then the area of the triangle ABC = ... square units.

(a) $2\sqrt{3}$ (b) $3\sqrt{3}$ (c) $4\sqrt{3}$ (d) $6\sqrt{3}$

66 If $x + \frac{1}{x} = -1$, then $x^{2018} + x^{-2018} =$ ....

(a) -1 (b) zero (c) 1 (d) 2

67 Sum of roots of the equation $(z-2)^3 = 1$ equals ....

(a) zero (b) 2 (c) 1 (d) 6

68 $\sqrt{2(\omega + i)(\omega^2 + i)} =$ ....

(a) $1 + i$ (b) $1 - i$ (c) $\pm(1 + i)$ (d) $\pm(1 - i)$

69 If $\lfloor x + 2\omega + 2\omega^2 = x + 2\omega + 2\omega^2$, then $x =$ ....

(a) -3 or -4 (b) 3 or 4 (c) ± 3 (d) ± 4

70 If $z = \frac{\sqrt{3} + i}{2}$, then $z^{69} =$ ....

(a) $-i$ (b) i (c) 1 (d) -1

71 If $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \cdots + a_{2n} x^{2n}$

, then $a_0 + a_1 \omega + a_2 \omega^2 + \cdots + a_{2n} \omega^{2n} =$ ....

(a) zero (b) ω (c) ω^2 (d) 1

72 If $x^2 + x + 1 = 0$

, then $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \cdots + \left(x^6 + \frac{1}{x^6}\right)^2 =$ ....

(a) 6 (b) 12 (c) 18 (d) 54

73 $\sum_{n=1}^{12} \left(\cos \frac{2n\pi}{3} + i \sin \frac{2n\pi}{3}\right) =$ ....

(a) zero (b) 1 (c) 12 (d) 78

74 $1 + {}^{100}C_1\, \omega + {}^{100}C_2\, \omega^2 + \ldots + {}^{100}C_{100}\, \omega^{100} =$ ....

(a) zero (b) 1 (c) ω (d) ω^2

75 If $1, \omega, \omega^2$ are the cubic roots of one , then $\sum_{r=0}^{100} {}^{100}C_r\, (2 + \omega^2)^{100 - r}\, \omega^r =$ ....

(a) -1 (b) zero (c) 1 (d) 2

Fifth Questions on determinants

Choose the correct answer from the given ones :

1 $\begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} =$ ....

(a) zero (b) 1 (c) -1 (d) $\cos 2x$

2. $\begin{vmatrix} \omega & i \\ i & \omega \end{vmatrix} = \dots\dots\dots\dots$

(a) 1 (b) -1 (c) ω (d) $-\omega$

3. If $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} + \begin{vmatrix} 5 & 7 \\ -2 & 3 \end{vmatrix} = 32$, then $x = \dots\dots\dots\dots$

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

4. $\begin{vmatrix} 3 & 5 & 3 \\ 4 & 8 & 4 \\ 7 & 2 & 7 \end{vmatrix} = \dots\dots\dots\dots$

(a) 0 (b) -1 (c) 1 (d) 3×48

5. $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \dots\dots\dots\dots$

(a) ω (b) ω^2 (c) i (d) zero

6. Value of the determinant : $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$ equals $\dots\dots\dots\dots$

(a) $3\sqrt{3}\,i$ (b) $\pm 3\sqrt{3}\,i$ (c) $-\sqrt{3}\,i$ (d) $\sqrt{3}\,i$

7. $\begin{vmatrix} 3 & 1 & 2 \\ 4 & 0 & 5 \\ 5 & 3 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 5 \\ 1 & 3 & 7 \end{vmatrix} + \dots\dots\dots\dots$

(a) $\begin{vmatrix} 2 & 1 & 2 \\ 3 & 0 & 5 \\ 4 & 3 & 7 \end{vmatrix}$ (b) $\begin{vmatrix} 3 & 1 & 2 \\ 4 & 0 & 5 \\ 5 & 3 & 7 \end{vmatrix}$ (c) $\begin{vmatrix} 2 & 1 & 2 \\ 4 & 0 & 5 \\ 2 & 3 & 7 \end{vmatrix}$ (d) $\begin{vmatrix} 2 & 1 & 2 \\ 2 & 0 & 5 \\ 3 & 3 & 7 \end{vmatrix}$

8. $\begin{vmatrix} 4 & 6 & 8 \\ 8 & 6 & 2 \\ 12 & 20 & 10 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 3 & 1 \\ 3 & 10 & 5 \end{vmatrix} \times \dots\dots\dots\dots$

(a) 2 (b) 4 (c) 8 (d) 16

9 Which of the following determinants not equal zero ?

(a) $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 5 & 5 \end{vmatrix}$ (b) $\begin{vmatrix} 0 & 0 & 0 \\ 3 & 5 & 4 \\ 2 & 1 & 3 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 0 & 0 \\ 3 & 2 & -1 \\ 5 & -4 & 2 \end{vmatrix}$ (d) $\begin{vmatrix} 1 & 0 & 0 \\ 3 & 2 & -1 \\ -4 & -6 & 5 \end{vmatrix}$

10 $\begin{vmatrix} 24 & 25 & 26 \\ 27 & 28 & 29 \\ 30 & 31 & 32 \end{vmatrix} = \text{.............}$

(a) 0 (b) 12 (c) 24 (d) 56

11 $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & t \end{vmatrix} = (-1) \times \text{.............}$

(a) $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & t \end{vmatrix}$ (b) $\begin{vmatrix} a+d & d & g \\ b+e & e & h \\ c+f & f & t \end{vmatrix}$ (c) $\begin{vmatrix} d & a & g \\ e & b & h \\ f & c & t \end{vmatrix}$ (d) $\begin{vmatrix} 2a & d & g \\ 2b & e & h \\ 2c & f & t \end{vmatrix}$

12 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 12$, then $\begin{vmatrix} a & d & x \\ b & e & y \\ c & f & z \end{vmatrix} = \text{.............}$

(a) -12 (b) 12 (c) zero (d) 24

13 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 15$, then $\begin{vmatrix} a & b & c \\ x & y & z \\ d & e & f \end{vmatrix} = \text{.............}$

(a) -30 (b) -15 (c) zero (d) 15

14 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 12$, then $\begin{vmatrix} z & x & y \\ f & d & e \\ c & a & b \end{vmatrix} = \text{.............}$

(a) -12 (b) -6 (c) 6 (d) 12

15 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 20$, then $\begin{vmatrix} x & y & z \\ a & b & c \\ d & e & f \end{vmatrix} = \text{.............}$

(a) -20 (b) 10 (c) 20 (d) 40

16 $\begin{vmatrix} a & a & a \\ a & b & c \\ c & c & c \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) zero (b) a c (c) b c (d) a b c

17 $\begin{vmatrix} ab & a & \frac{1}{c} \\ ac & c & \frac{1}{b} \\ bc & b & \frac{1}{a} \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) zero (b) abc (c) 1 (d) 2

18 If $\begin{vmatrix} a & b & c \\ x & y & z \\ d & e & f \end{vmatrix} = 40$, then $\begin{vmatrix} a + 100\,x & b + 100\,y & c + 100\,z \\ x & y & z \\ d & e & f \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) 40 (b) 140 (c) 60 (d) 4000

19 If $\begin{vmatrix} a & b & 1 \\ x & y & 1 \\ d & e & 1 \end{vmatrix} = 30$, then $\begin{vmatrix} a+4 & b & 1 \\ x+4 & y & 1 \\ d+4 & e & 1 \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) 7.5 (b) 30 (c) 34 (d) 120

20 $\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) a + b + c (b) zero (c) 1 (d) a b c

21 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 2$, then $\begin{vmatrix} 5\,a & b & c \\ 5\,d & e & f \\ 35\,x & 7\,y & 7\,z \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) 700 (b) 10 (c) 35 (d) 70

22 $\begin{vmatrix} a+b & 5 & c \\ b+c & 5 & a \\ a+c & 5 & b \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) 5 (b) 4 (c) 3 (d) zero

23 $\begin{vmatrix} x+y & z+y & x+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = \ldots\ldots$

(a) $x+y+z$ (b) -1 (c) 0 (d) xyz

---

24 *(Trial 2021)* $\begin{vmatrix} x & y & y \\ y & x & y \\ y & y & x \end{vmatrix} = (x+2y) \times \ldots\ldots$

(a) $\begin{vmatrix} 1 & y & y \\ 0 & x-y & 0 \\ 0 & 0 & x-y \end{vmatrix}$ (b) $\begin{vmatrix} 1 & y & y \\ 0 & x+y & 0 \\ 0 & 0 & x-y \end{vmatrix}$

(c) $\begin{vmatrix} 1 & y & y \\ 0 & x+y & 0 \\ 0 & 0 & x+y \end{vmatrix}$ (d) $\begin{vmatrix} 1 & y & y \\ 0 & x-y & 2y \\ 0 & 0 & x+y \end{vmatrix}$

---

25 If $\begin{vmatrix} -2 & 3 & 0 \\ a & 1 & -2 \\ 4 & b & 1 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 0 \\ -3 & 1 & -2 \\ 5 & 2 & 1 \end{vmatrix} + \begin{vmatrix} -4 & 3 & 0 \\ 2 & 1 & -2 \\ -1 & 2 & 1 \end{vmatrix}$, where a , b are integers

, then $a \times b$ could be equal ……

(a) -4 (b) -2 (c) zero (d) 2

---

26 If $\begin{vmatrix} -1 & a & b \\ 4 & 2 & c \\ 0 & 5 & 3 \end{vmatrix} = -6$, then which of the following is true ?

(I) $a = b = c = 0$ **(II)** $12a = 5c + 20b$ **(III)** $5c = 12a + 20b$

(a) **(I)** only (b) **(II)** only (c) both **(I)** , **(II)** (d) both **(II)** , **(III)**

---

27 The solution set of the equation : $\begin{vmatrix} x & 1 & 2 \\ 0 & x & 3 \\ 0 & 0 & x \end{vmatrix} - 8 = 0$ in $\mathbb{R}$ is ……

(a) $\{-2\}$ (b) $\{2\}$ (c) $\{2, -2\}$ (d) $\{8\}$

---

28 The solution set of the equation : $\begin{vmatrix} a+1 & 3 & 2 \\ 0 & a-1 & 5 \\ 0 & 0 & 7 \end{vmatrix} = 21$ in $\mathbb{R}$ is ……

(a) $\{2, -2\}$ (b) $\{-3, 7\}$ (c) $\{2, 3\}$ (d) $\{-2, 7\}$

29 The solution set of the equation : $\begin{vmatrix} x+3 & 5 & -1 \\ 0 & x+2 & x \\ 0 & 0 & x \end{vmatrix} = 0$ in $\mathbb{R}$ is ....

(a) $\{0 , -2\}$ (b) $\{-2 , -3\}$ (c) $\{0 , 2 , 3\}$ (d) $\{0 , -2 , -3\}$

30 The solution set of the equation : $\begin{vmatrix} 2 & 5 & 0 \\ 4 & x & 0 \\ x & 7 & 5 \end{vmatrix} = 0$ is ....

(a) $\{2\}$ (b) $\{5\}$ (c) $\{7\}$ (d) $\{10\}$

31 If $\begin{vmatrix} x & 1 & 2 \\ 0 & x^2 & 3 \\ 0 & 0 & x \end{vmatrix} = 16$ where $x \in \mathbb{R}$; then $x^6 =$ ....

(a) 16 (b) ± 16 (c) ± 64 (d) 64

32 Solution set of the equation : $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ in $\mathbb{R}$ is ....

(a) $\{1 , 2\}$ (b) $\{-1 , 2\}$ (c) $\{1 , -2\}$ (d) $\{-1 , -2\}$

33 If $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = 0$, then $x =$ ....

(a) 1 or $\frac{1}{2}$ (b) 1 or $-\frac{1}{2}$ (c) -1 or $\frac{1}{2}$ (d) -1 or $-\frac{1}{2}$

34 If $\begin{vmatrix} 2x & 1 & 1 \\ 1 & 2x & 1 \\ 1 & 1 & 2x \end{vmatrix} = 0$, then $x =$ ....

(a) 1 or $\frac{1}{2}$ (b) 1 or $-\frac{1}{2}$ (c) -1 or $\frac{1}{2}$ (d) -1 or $-\frac{1}{2}$

35 If x one of the factor of the determinant : $\begin{vmatrix} 3 & 2 & k \\ x+3 & k-2 & x \\ 2 & x+1 & x+k \end{vmatrix}$, then k = ...

(a) zero or 1 (b) zero or -5 (c) zero or 5 (d) -5 or 5

36 If $(x-2)$ is a factor of the determinant $\begin{vmatrix} x-1 & x+3 & 2 \\ -3 & x+5 & -6 \\ x+3 & 2 & x+k \end{vmatrix}$, then k = ...

(a) 2 (b) 4 (c) 6 (d) 8

37 The solution set of the equation $\begin{vmatrix} |x-1| & 2 \\ 4 & 3 \end{vmatrix} = 7$ is ....

(a) $\{6, 4\}$ (b) $\{4, -6\}$ (c) $\{6, -4\}$ (d) $\varnothing$

38 The solution set of the equation $\begin{vmatrix} 2\sin x & 1 \\ 1 & \cos x \end{vmatrix} = 0$ is .... where $x \in [0, \pi]$

(a) $\{0\}$ (b) $\{\frac{\pi}{2}\}$ (c) $\{0, \frac{\pi}{2}\}$ (d) $\{\frac{\pi}{4}\}$

39 If $\begin{vmatrix} 1 & -x & 0 \\ x & 1 & x \\ 1 & -1 & x+1 \end{vmatrix} = \begin{vmatrix} x^2 & 1 \\ -x & x \end{vmatrix}$, then x = ...

(a) zero (b) 1 (c) -1 (d) 2

40 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 15$, then $\begin{vmatrix} a-5d & b-5e & c-5f \\ d & e & f \\ x+2d & y+2e & z+2f \end{vmatrix}$ = ...

(a) zero (b) 15 (c) 30 (d) -150

41 If $\begin{vmatrix} x & y & z \\ a & b & c \\ 2 & -1 & 8 \end{vmatrix} = 10$, then $\begin{vmatrix} 2a & 2b & 2c \\ x & y & z \\ 4 & -2 & 16 \end{vmatrix}$ = ...

(a) -40 (b) -20 (c) -10 (d) 40

42 $\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & y+1 \end{vmatrix}$ = ...

(a) 0 (b) $x+y$ (c) xy (d) $-xy$

43 If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a & b & c \end{vmatrix} = k$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a+1 & b+1 & c+1 \end{vmatrix} = \dots\dots\dots\dots$

(a) k (b) $k-1$ (c) $k-6$ (d) $k-3$

---

44 $\begin{vmatrix} 1 & \sin x & \sin x \\ 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \end{vmatrix} = \dots\dots\dots\dots$

(a) $\sin 2x$ (b) $1-\sin 2x$ (c) 1 (d) 0

---

45 $\begin{vmatrix} 2 & 1 & 3 \\ 0 & \sin 3x & 5 \\ 0 & 0 & \cos 3x \end{vmatrix} = \sin \dots\dots\dots\dots$

(a) x (b) $3x$ (c) $6x$ (d) $12x$

---

46 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 3$, then $\begin{vmatrix} 4a+3b+2c & b & c \\ 4d+3e+2f & e & f \\ 4x+3y+2z & y & z \end{vmatrix} = \dots\dots\dots\dots$

(a) 3 (b) 6 (c) 12 (d) 72

---

47 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 10$, then $\begin{vmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2x & 2y & 2z \end{vmatrix} = \dots\dots\dots\dots$

(a) 10 (b) 20 (c) 40 (d) 80

---

48 If $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then the cofactor of element a_{12} equals $\dots\dots\dots\dots$

(a) -46 (b) -4 (c) 4 (d) 46

---

49 If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 15$, then $a\begin{vmatrix} b & c \\ e & f \end{vmatrix} - b\begin{vmatrix} a & c \\ d & f \end{vmatrix} + c\begin{vmatrix} a & b \\ d & e \end{vmatrix} = \dots\dots\dots\dots$

(a) zero (b) 15 (c) -15 (d) 30

50 If $z = \cos x + i \sin x$, $\bar{z}$ is the conjugate of z, then $\begin{vmatrix} z & 1 \\ 1 & \bar{z} \end{vmatrix} = \ldots\ldots\ldots$

(a) zero (b) 1 (c) -1 (d) 7

---

51 If x is a complex number, then the number of different solutions for the equation : $\begin{vmatrix} x^3+1 & x-1 \\ x+1 & x^3-1 \end{vmatrix} = 0$ equals $\ldots\ldots\ldots$

(a) 6 (b) 5 (c) 4 (d) 3

---

52 If $\begin{vmatrix} \log_2 3 & 3 & 9 \\ 0 & \log_3 5 & 7 \\ 0 & 0 & \log_5 x \end{vmatrix} = 4$, then $x = \ldots\ldots\ldots$

(a) 16 (b) 32 (c) 64 (d) 128

---

53 If L, m are two roots of the equation : $x^2 - 2x - 4 = 0$, then $\begin{vmatrix} 2L & -m \\ 4m & 2L \end{vmatrix} = \ldots\ldots\ldots$

(a) 84 (b) 48 (c) 36 (d) 63

---

54 If $i^2 = -1$, then $\begin{vmatrix} 1 & i & i+1 \\ 0 & 1 & i-1 \\ 0 & i & i \end{vmatrix} = \ldots\ldots\ldots$

(a) $2i-1$ (b) $2i+1$ (c) i (d) 1

---

55 *(2nd session 2021)* The value of the determinant $\begin{vmatrix} 7^{2n} & 7^{3n} & 7^{4n} \\ 7^{3n} & 7^{4n} & 7^{5n} \\ 7^{4n} & 7^{5n} & 7^{6n} \end{vmatrix} = \ldots\ldots\ldots$ where $n \in \mathbb{Z}^+$

(a) zero (b) 7^{2n} (c) 7^n (d) 7^{3n}

---

56 $\begin{vmatrix} \log a & \log b & \log c \\ \log 2a & \log 2b & \log 2c \\ \log 3a & \log 3b & \log 3c \end{vmatrix} = \ldots\ldots\ldots$

(a) $\log 3(abc)$ (b) $\log a + \log b + \log c$

(c) zero (d) 1

57 If T_1 , T_2 , T_3 , ... , T_9 are terms of an arithmetic sequence with common difference (d)

, then $\begin{vmatrix} T_1 & T_2 & T_3 \\ T_4 & T_5 & T_6 \\ T_7 & T_8 & T_9 \end{vmatrix} = \text{..............}$

(a) zero
(b) the common difference (d)
(c) 1
(d) $(T_5)^2$

---

58 The solution set of the equation : $\begin{vmatrix} x & 2x & 3x \\ 3x & 2x & x \\ x & -x & 0 \end{vmatrix} = 96$ in $\mathbb{R}$ is ....

(a) $\{4\}$
(b) $\{3\}$
(c) $\{2\}$
(d) $\{-2\}$

---

59 In Δ ABC , $\begin{vmatrix} a & b & c \\ 5 & 7 & 8 \\ \sin A & \sin B & \sin C \end{vmatrix} = \text{............}$

(a) 5 a
(b) 7 b
(c) 8 c
(d) 0

---

60 If ABC is a triangle , then $\begin{vmatrix} b^2+c^2 & b\,c\cos A & a^2 \\ c^2+a^2 & c\,a\cos B & b^2 \\ a^2+b^2 & a\,b\cos C & c^2 \end{vmatrix} = \text{.............}$

(a) zero
(b) $a^2+b^2+c^2$
(c) $\cos A + \cos B + \cos C$
(d) 6

---

61 *(2nd Session 2021)* In the triangle ABC , if $\begin{vmatrix} a^2 & c^2 & -b^2 \\ 1 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix} = ac$,

where a , b and c are the side lengths of triangle ABC , then m ($\angle$ B) = ...°

(a) 45
(b) 90
(c) 60
(d) 120

---

62 *(2nd Session 2021)* In the triangle ABC , if $\begin{vmatrix} a+2 & 3 & \sin C \\ 1 & b & 0 \\ 2 & 3 & \sin C \end{vmatrix} = 12$,

where a , b and c are the side lengths of the triangle ABC , then the surface area of the triangle ABC = ... unit area.

(a) 12
(b) 6
(c) 24
(d) 8

63) $\begin{vmatrix} 1 & x & y \\ x & 1+x^2 & xy \\ y & xy & 1+y^2 \end{vmatrix} = \dots\dots\dots\dots$

(a) zero (b) 1 (c) -1 (d) $3xy$

---

64) $\begin{vmatrix} 1 & 1 & 1 \\ 1+y & 1 & 1 \\ 1 & 1+y & 1 \end{vmatrix} = \dots\dots\dots\dots$

(a) zero (b) y (c) y^2 (d) $3y$

---

65) If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$, then $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ equals $\dots\dots\dots\dots$

(a) zero (b) abc (c) $-abc$ (d) $a+b+c$

---

66) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \dots\dots\dots\dots$

(a) $4abc$ (b) $-4abc$ (c) $4a^2b^2c^2$ (d) $-4a^2b^2c^2$

---

67) $\begin{vmatrix} a & a-c & 2a \\ b & a-c & 2b \\ c & a & b+c \end{vmatrix} = \dots\dots\dots\dots$

(a) abc (b) $abc-1$

(c) $a+b+c$ (d) $(a-b)(a-c)(b-c)$

---

68) $\begin{vmatrix} x-y & y-z & z-x \\ y-z & z-x & x-y \\ z-x & x-y & y-z \end{vmatrix} = \dots\dots\dots\dots$

(a) 0 (b) $x-y$ (c) $y-z$ (d) $z-x$

---

69) If $N=\begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & 5 \\ 1 & 4 & 2 \end{vmatrix}$, $M=\begin{vmatrix} 3 & 0 & 9 \\ 4 & 6 & 10 \\ 5 & 20 & 10 \end{vmatrix}$, then $M = \dots\dots\dots\dots$

(a) N (b) 10 N (c) 20 N (d) 30 N

70 $\begin{vmatrix} 1 & i & \omega \\ \omega i & -\omega & 0 \\ \omega^2 i & -\omega^2 & \omega \end{vmatrix} = \dots\dots\dots\dots$

(a) zero (b) $i + \omega$ (c) $\omega^2 + 1$ (d) 3

71 If $\begin{vmatrix} x-1 & 0 & 0 \\ 0 & x^2+x+1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 9$, then $x^6 = \dots\dots\dots\dots$

(a) 1 (b) 10 (c) 27 (d) 100

72 The solution set of the equation : $\begin{vmatrix} 1+x & 1-x & 1-x \\ 1-x & 1+x & 1-x \\ 1-x & 1-x & 1+x \end{vmatrix} = 0$ in $\mathbb{R}$ is $\dots\dots\dots\dots$

(a) $\{1 , 0\}$ (b) $\{0 , -1 , 1\}$ (c) $\{1 , -1 , 3\}$ (d) $\{0 , 3\}$

73 If $\begin{vmatrix} a & b & c \\ x & y & z \\ d & e & f \end{vmatrix} = 10$ and $\begin{vmatrix} a x & b x & c x \\ x^2 & y x & z x \\ d x & e x & f x \end{vmatrix} = 5120$, then $\sqrt[3]{x} = \dots\dots\dots\dots$

(a) 2 (b) 8 (c) 512 (d) $\sqrt[3]{2}$

74 If all elements of the determinant of the 3rd degree whose value = m are multiplied by 2 , then the value of the result equals $\dots\dots\dots\dots$

(a) m (b) 2 m (c) 4 m (d) 8 m

75 If $\Delta = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix}$, then $2\Delta \neq \dots\dots\dots\dots$

(a) $\begin{vmatrix} 2 & 6 \\ 10 & 14 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 3 \\ 10 & 7 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 6 \\ 5 & 14 \end{vmatrix}$ (d) $\begin{vmatrix} 1 & 3 \\ 10 & 14 \end{vmatrix}$

76 If $\Delta = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ 0 & 4 & 2 \end{vmatrix}$, then $\begin{vmatrix} 4 & 12 & 4 \\ 8 & -4 & 4 \\ 0 & 16 & 8 \end{vmatrix} = \dots\dots\dots\dots$

(a) 12Δ (b) 64Δ (c) 4Δ (d) 16Δ

77 If $\begin{vmatrix} \omega & i & \omega^2 \\ i & \omega^2 & \omega \\ \omega^2 & \omega & i \end{vmatrix} = x + i\,y$, then $x + y =$ ....

(a) zero (b) 1 (c) 2 (d) -2

---

78 If 1 , ω , ω^2 are the cubic roots of unity , $n \in \mathbb{Z}^+$, then $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} =$ ....

(a) 1 (b) ω (c) ω^2 (d) 0

---

79 *(2nd Session 2021)* If 1 , ω , ω^2 are the cubic roots of unity ,

then the value of the determinant $\begin{vmatrix} 1 & \omega & \omega - 1 \\ 1 & -1 & \omega + 1 \\ 1 & \omega & \omega \end{vmatrix} =$ ....

(a) $\omega - 1$ (b) ω^2 (c) ω (d) $\omega^2 + 1$

---

80 $\begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} + \cdots + \begin{vmatrix} 10 & 1 \\ 1 & 1 \end{vmatrix} =$ ....

(a) 32 (b) 30 (c) 29 (d) 27

---

81 $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} + \cdots + \begin{vmatrix} 2019 & 2020 \\ 2021 & 2022 \end{vmatrix} =$ ....

(a) 2019 (b) -2019 (c) -4038 (d) 4038

---

82 If c , b are the two roots of the equation : $x^2 - 11x + 27 = 0$, then $\begin{vmatrix} \log_3 c & \log_3 b \\ -1 & 1 \end{vmatrix} =$ ....

(a) 3 (b) 1 (c) -1 (d) -3

---

83 If the roots of the equation $a\,x^2 + b\,x + c = 0$ are the same roots of $k\,x^2 + h\,x + r = 0$,

then $\begin{vmatrix} a & b & c \\ k & h & r \\ 6 & 7 & 8 \end{vmatrix} =$ ....

(a) zero (b) 1 (c) 13 (d) $7a - 6b$

84 If $90° \le x \le 180°$ and $\begin{vmatrix} 1 & 0 & 0 \\ 6 & 2\sin x - 1 & 0 \\ 8 & m & 1+\cos 2x \end{vmatrix} = 0$, then m ($\angle x$) = ...

(a) 30° or 150°
(b) 90° or 150°
(c) 120° or 150°
(d) 90° or 120°

---

85 ABC is a triangle in which $\begin{vmatrix} BC & 0 & 0 \\ 0 & AC & 0 \\ 0 & 0 & AB \end{vmatrix} = 32 \text{ cm}^3$, if the area of Δ ABC = 4 cm^2 , then the length of the radius of the circumcircle of the triangle = ... cm.

(a) 4
(b) 3
(c) $2\frac{1}{2}$
(d) 2

---

86 $\begin{vmatrix} \sin A & \cos A & \sin(A+\theta) \\ \sin B & \cos B & \sin(B+\theta) \\ \sin C & \cos C & \sin(C+\theta) \end{vmatrix} = $ ....

(a) zero
(b) $\sin^2 A + \cos^2 B + \sin^2 C$
(c) $\cos^2 A + \cos^2 B + \cos^2 C$
(d) $\sin^2 (A+B+C+\theta)$

---

87 If $x = -9$ is one of the roots of the equation : $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$, then the other two roots are ....

(a) 2 , 6
(b) 3 , 6
(c) 2 , 7
(d) 3 , 7

---

88 If $f(x) = \begin{vmatrix} \sin x & 0 & 0 \\ 0 & \sin 2x & 0 \\ 0 & 0 & \cos 3x \end{vmatrix}$, then $\lim\limits_{x \to 0} \frac{f(x)}{x^2} = $ ....

(a) 1
(b) 2
(c) 3
(d) 6

---

89 If $f(x) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & x & 5 \\ 1 & 2 & x+3 \end{vmatrix}$, then $\sum\limits_{r=1}^{10} f(r) = $ ....

(a) 55
(b) 25
(c) 165
(d) 385

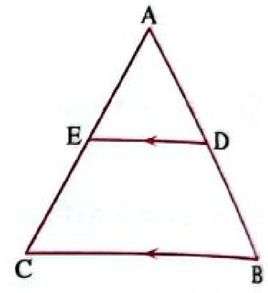
90 $\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+1}C_1 & {}^{n+2}C_1 \\ {}^{n}C_2 & {}^{n+1}C_2 & {}^{n+2}C_2 \end{vmatrix} = \cdots\cdots\cdots$

(a) 1 (b) -1 (c) zero (d) n

---

91 **In the opposite figure :**

$\overline{DE} \parallel \overline{BC}$, then $\begin{vmatrix} 5 & 6 & 7 \\ ED & AD & AE \\ BC & AB & AC \end{vmatrix} = \cdots\cdots\cdots$

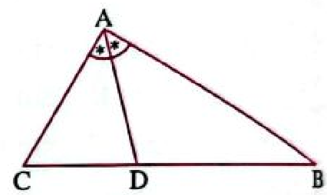


(a) 7 (b) 6
(c) 5 (d) zero

---

92 **In the opposite figure :**

If $\overrightarrow{AD}$ bisects $\angle A$, then $\begin{vmatrix} 2 & 3 & 7 \\ AB & AC & AB + AC \\ BD & DC & BC \end{vmatrix} = \cdots\cdots\cdots$



(a) zero (b) 6
(c) 21 (d) 2BC

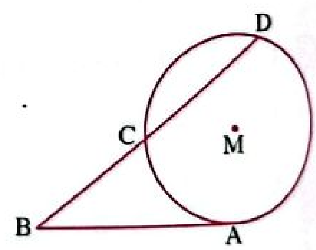
---

93 ***(1st Session 2021)*** **In the opposite figure :**

$\overleftrightarrow{AB}$ is a tangent to circle M at A

$\overline{DC}$ is a chord in the circle

where $\overleftrightarrow{DC} \cap \overleftrightarrow{AB} = \{B\}$



If $\begin{vmatrix} 1 & 0 & CD \\ -1 & AB & BC \\ 0 & -BC & AB \end{vmatrix} = 32$, then AB = ………… length units.

(a) 8 (b) 4 (c) 16 (d) 6

Questions on matrices and solving system of linear equations

Choose the correct answer from the given ones :

1 The singular matrix from the following matrices is …………

(a) $\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 4 \\ -3 & 6 \end{pmatrix}$

2. All the following matrices has multiplicative inverse except ....

ⓐ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ⓑ $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

ⓒ $\begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$ ⓓ $\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$

3. All the following matrices has no multiplicative inverse except ....

ⓐ $\begin{pmatrix} 3 & -6 \\ -2 & 4 \end{pmatrix}$ ⓑ $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$

ⓒ $\begin{pmatrix} 4 & -8 \\ 2 & -4 \end{pmatrix}$ ⓓ $\begin{pmatrix} 4 & -2 \\ 10 & 5 \end{pmatrix}$

4. Value of x which makes the matrix $\begin{pmatrix} x & 2 \\ -3 & 3 \end{pmatrix}$ is singular is ....

ⓐ 2 ⓑ -2 ⓒ $\frac{1}{2}$ ⓓ -3

5. Value of a which makes the matrix $\begin{pmatrix} 2 & a \\ a & 8 \end{pmatrix}$ has no multiplicative inverse = ...

ⓐ -4 ⓑ 4 ⓒ ± 4 ⓓ 16

6. Value of x which makes the matrix $\begin{pmatrix} x-1 & 2 \\ 4 & x+1 \end{pmatrix}$ singular matrix is ....

ⓐ -30 ⓑ 3 ⓒ ± 3 ⓓ 9

7. If the matrix $A = \begin{pmatrix} \log x & 1 \\ 1 & \log x \end{pmatrix}$ is a singular matrix, then $x =$ ....

ⓐ -1 or 1 ⓑ 10 or $\frac{-1}{10}$ ⓒ $\frac{1}{10}$ or -10 ⓓ 10 or $\frac{1}{10}$

8. If $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$, then Adj (A) = ...

ⓐ $\begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix}$ ⓑ $\begin{pmatrix} 3 & -5 \\ 1 & 2 \end{pmatrix}$ ⓒ $\begin{pmatrix} 2 & 1 \\ -5 & 3 \end{pmatrix}$ ⓓ $\begin{pmatrix} -5 & 3 \\ 2 & 1 \end{pmatrix}$

9. The multiplicative inverse of the matrix $A = \begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$ is ....

ⓐ $\begin{pmatrix} 1 & -2 \\ -3 & 5 \end{pmatrix}$ ⓑ $\begin{pmatrix} 1 & -3 \\ -2 & 5 \end{pmatrix}$ ⓒ $\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix}$ ⓓ $\begin{pmatrix} -1 & 3 \\ 2 & -5 \end{pmatrix}$

10 If $A = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then $A^{-1} =$ ....

(a) $\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

---

11 *(Trial 2021)* If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, then $(A^2)^{-1} =$ ....

(a) $\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} -\cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$

---

12 The sum of the matrix $A = \begin{pmatrix} 2 & -3 \\ 5 & -7 \end{pmatrix}$ and its multiplicative inverse equals ....

(a) $\begin{pmatrix} 4 & -6 \\ 10 & -14 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$

---

13 If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then ....

(a) $A^t = A$ (b) $A^t = \text{Adj}(A)$ (c) $A^t = A^{-1}$ (d) All the previous.

---

14 If $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, then $A^{-2} =$ ....

(a) I (b) $-I$ (c) $-2I$ (d) A

---

15 If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, then $A \times \text{Adj}(A) =$ ....

(a) $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 10 \\ 10 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 10 & 1 \\ 1 & 10 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 10 \\ 10 & 1 \end{pmatrix}$

16 For any square matrix A if $A^2 - A + I = O$, then $A^{-1} =$ ....

(a) A^{-2} (b) $A + I$ (c) $I - A$ (d) $A - I$

---

17 The value of a which makes the matrix $\begin{pmatrix} 2 & 3 & 5 \\ 1 & a & 2 \\ 0 & 1 & -1 \end{pmatrix}$ singular is ....

(a) $\frac{-5}{3}$ (b) $\frac{5}{3}$ (c) 2 (d) 3

---

18 If $A = \begin{pmatrix} 7 & 2 & 5 \\ 0 & k & 1 \\ 1 & k & 4 \end{pmatrix}$ and the cofactor of a_{11} is 6 , then the cofactor of $a_{13} =$ ....

(a) -2 (b) 2 (c) 6 (d) -6

---

19 If A and B are two non singular matrices , then $(AB)^{-1}$ equals ....

(a) AB (b) $A^{-1} B^{-1}$ (c) $B^{-1} A^{-1}$ (d) $(BA)^{-1}$

---

20 If A and B are non singular matrices then all of the following are true except ....

(a) $(A + B)^{-1} = A^{-1} + B^{-1}$ (b) $(A \times B)^t = B^t \times A^t$

(c) $(A \times B)^{-1} = B^{-1} \times A^{-1}$ (d) $(A + B)^t = A^t + B^t$

---

21 If A is a non-singular matrix of order 3 × 3 and $k \in \mathbb{R}^*$, then which of the following is always true ?

(a) $|kA| = k\ |A|$ (b) $|A^{-1}| = |A|^3$ (c) $|A| = |A^t|$ (d) $|A| = |A^{-1}|$

---

22 If $X = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ and $a X^{-1} + X + b I = \boxed{}$ where a and b are constants ,

then (a , b) = ...

(a) (1 , 1) (b) (1 , − 1) (c) (1 , − 4) (d) (1 , 2)

---

23 If A is square matrix , $|A| = 4$, then $A \times \text{Adj}(A) =$ ....

(a) 4 (b) 4 I (c) I (d) $\frac{1}{4}$ I

---

24 If $A = \begin{pmatrix} 2 & 5 \\ 5 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 3 \\ -46 & 17 \end{pmatrix}$, $A \times C = B$, then C = ...

(a) $\begin{pmatrix} -8 & 1 \\ 3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -11 & 4 \\ 3 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} -17 & 4 \\ 6 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} -9 & 6 \\ 3 & -1 \end{pmatrix}$

25 If A is a matrix of order 2×2 and $|A| = 5$, then $|3A| =$ ....

(a) 5 (b) 15 (c) 45 (d) 10

26 If A , B are two matrices of order 3×3 and $A = 2B$, $|B| = 5$, then $|A| =$ ....

(a) 8 (b) 16 (c) 32 (d) 40

27 If I is the unit matrix of order 2×2 , then $|4I| =$ ....

(a) 4 (b) 6 (c) 8 (d) 16

28 If A is a matrix of order 2×2 and $|A| = 5$, then $|A \times \text{Adj}(A)| =$ ....

(a) 5 (b) 25 (c) 125 (d) 1

29 If A is a matrix of order 3×3 and $|A| = -3$, then $|A \times \text{adj}(A)| =$ ....

(a) 3 (b) -27 (c) 27 (d) -9

30 If A is a square matrix of order 3×3 and $|A| = 5$, then $|\text{Adj}(A)| =$ ....

(a) 5 (b) 25 (c) 125 (d) $\frac{1}{5}$

31 If A is a non singular matrix , then Adj (A) = ...

(a) $|A|A^{-1}$ (b) $\left(\text{Adj}(A)\right)^{-1}$ (c) $\frac{1}{|A|}A^{-1}$ (d) $|\text{Adj } A|$

32 If A is a non singular matrix , then the false statement in each of following ....

(a) A has a multiplicative inverse. (b) $RK(A) = RK(A^{-1})$

(c) $|A| = |A^t|$ (d) $A + A^{-1} = \square$

33 If $A = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$, then $|A|\,|\text{Adj}(A)| =$ ....

(a) a^3 (b) a^6 (c) a^9 (d) a^{27}

34 If A , B , C are three matrices of order $n \times n$ and $ABC = I$, then $B^{-1} =$ ....

(a) $A^{-1}C^{-1}$ (b) $(AC)^{-1}$ (c) $C^{-1} + A^{-1}$ (d) CA

35 If A is a square matrix of order 3×3 and $|A| = 5$, then $|2\,\text{Adj}(A)| =$ ....

(a) 250 (b) 200 (c) 50 (d) 25

36 If A is a matrix of order 3×3 , then Adj (2 A) = ...

(a) 2 Adj (A) (b) $\frac{1}{2}$ Adj (A) (c) 4 Adj (A) (d) 8 Adj (A)

37 If A is a matrix of order 3×3 and c_{jk} denoted the cofactor of the element a_{jk} in the matrix A , then $a_{11}\, c_{31} + a_{21}\, c_{32} + a_{31}\, c_{33} =$ ....

(a) zero (b) 5 (c) 15 (d) 25

38 If A is a matrix of order 3×3 and $|A| = 5$ and c_{jk} denoted the cofactor of the element a_{jk} in the matrix , then $a_{11}\, c_{11} + a_{21}\, c_{21} + a_{31}\, c_{31} =$ ....

(a) zero (b) 5 (c) 15 (d) 25

39 The rank of the identity matrix I_3 is ....

(a) 3 (b) 2 (c) 1 (d) 0

40 The rank of the matrix $\square$ of the order 3×3 is ....

(a) 0 (b) 1 (c) 2 (d) 3

41 From the following linear systems , the homogeneous equations are ....

(a) $2x + y = 1$, $x + 2y = 4$
(b) $x - y = 0$, $x + 2y = 5$
(c) $3x + y = 3$, $2x + y = 0$
(d) $x - 2y = 0$, $x + y = 0$

42 If m is the number of linear equations and n is the number of variables , then the augmented matrix is of the order ....

(a) $m \times n$
(b) $m \times (n + 1)$
(c) $(m + 1) \times n$
(d) $(m + 1)(n + 1)$

43 If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$, then RK (A) = ...

(a) 0 (b) 1 (c) 2 (d) 3

44 If the matrix $A = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & 6 \\ 3 & -6 & -9 \end{pmatrix}$, then RK (A) = ...

(a) 0 (b) 1 (c) 2 (d) 3

45 If $A = \begin{pmatrix} 0 & 2 & -3 \\ -2 & -4 & 6 \\ 3 & 6 & -9 \end{pmatrix}$, then RK (A) = ...

(a) 0 (b) 1 (c) 2 (d) 3

46 If $A = \begin{pmatrix} 2 & 3 & -1 \\ -2 & -3 & 1 \\ 6 & 9 & -3 \end{pmatrix}$, then RK (A^t) = ...

(a) 0 (b) 1 (c) 2 (d) 3

47 The rank of the augmented matrix for the system : $x - 2y = 3$, $3x - 6y = 9$ is ....

(a) 0 (b) 1 (c) 2 (d) 3

48 If A is a non-zero matrix of order 1×3 , B is a non-zero matrix of order 3×1 , then RK (BA) = ...

(a) zero (b) 1 (c) 2 (d) 3

49 If $A^{-1} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, then y = ...

(a) 5 (b) 6 (c) 7 (d) 8

50 If $\begin{pmatrix} 2 & -1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, then $\begin{pmatrix} x \\ y \end{pmatrix}$ = ...

(a) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

51 The system $\begin{pmatrix} 2 & 4 & 3 \\ 1 & -4 & -3 \\ 4 & -4 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \square$ has ....

(a) the trivial solution only.

(b) infinite number of solutions other than the zero solution.

(c) finite number of solution except the zero solution.

(d) no solution at all.

52 If A is a matrix of order $m \times n$, then ....

(a) $Rk(A) \leq$ the smallest number of m and n

(b) $Rk(A) <$ the smallest number of m and n

(c) $Rk(A) \geq$ the smallest number of m and n

(d) $Rk(A) >$ the smallest number of m and n

---

53 The system of linear equations : $x + y = 2$, $2x + 2y = 3$ ....

(a) has no solutions.

(b) has unique solution.

(c) has infinite number of solutions.

(d) has two solutions.

---

54 The system of equations : $2x + 5y = 0$, $3x - z = 0$, $2y - 3z = 0$ has ....

(a) only the zero solution.

(b) no solution.

(c) finite number of solutions except the zero solution.

(d) infinite number of solutions other than the zero solution.

---

55 The system of equations $3x + y - z = 0$, $5x + 2y - 3z = 2$, $15x + 6y + 9z = 5$ has ....

(a) a unique solution

(b) an infinite number of solutions

(c) three solutions

(d) no solution

---

56 If $A = \begin{pmatrix} 1 & -2 & 3 \\ k & 0 & 1 \\ 3 & 2 & -1 \end{pmatrix}$, $RK(A) = 2$, then $k =$ ....

(a) -2

(b) 0

(c) 2

(d) 6

---

57 *(2nd Session 2021)* If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & m & 6 \\ 5 & 7 & 9 \end{pmatrix}$, $RK(A) = 3$, then $m \in$ ....

(a) $\{4\}$

(b) $R - \{0\}$

(c) $R - \{4\}$

(d) $R - \{6\}$

58 *(Trial 2021)* If A^* is the augmented matrix of the system of equations :

$3x + 2y - z = 4$, $x + y - z = 3$, $x = 2z$, then ....

(a) $2 < RK(A^*) < 4$ (b) $RK(A^*) < 3$

(c) $1 < RK(A^*) \leq 2$ (d) $1 \leq RK(A^*) < 3$

---

59 If $A^* = \left(\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 0 \end{array}\right)$, then $RK(A) + RK(A^*) =$ ....

(a) 6 (b) 5 (c) 4 (d) 2

---

60 If the two equations $2x + y = 1$, $4x + 2y = k$ have infinite number of solutions , then $k =$ ....

(a) 0 (b) 1 (c) 2 (d) 3

---

61 If the equations $3x - 2y + z = 0$, $6x - 5y + 2z = 0$, $9x - 6y + kz = 0$ have solutions other than the zero solution then $k =$ ....

(a) zero (b) 1 (c) 3 (d) 4

---

62 If the equations : $x + 2y + 3z = 5$, $2x - 3y + kz = 13$, $3x + ky + 2z = 3$ have unique solution then $k \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-1\}$

(c) $\mathbb{R} - \{13\}$ (d) $\mathbb{R} - \{-1, 13\}$

---

63 The solution set of system : $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, $\frac{1}{x} - \frac{1}{y} + \frac{2}{z} = \frac{1}{2}$, $\frac{2}{x} + \frac{3}{y} - \frac{4}{z} = \frac{4}{3}$ is ....

(a) $\{(2, 3, 5)\}$ (b) $\{(2, 3, 6)\}$

(c) $\{(\frac{1}{2}, \frac{1}{3}, \frac{1}{5})\}$ (d) $\{(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})\}$

---

64 If ABC is a triangle and $X = \begin{pmatrix} a & b & c \\ \sin A & \sin B & \sin C \end{pmatrix}$, then $RK(X) =$ ....

(a) zero (b) 1 (c) 2 (d) 3

65. If $A = (a_{yz})$ where $a_{yz} = y \times z$ where $y , z \in \{1 , 2 , 3\}$, then Rk (A) = ...

(a) zero (b) 1 (c) 2 (d) 3

---

66. ***(1st session 2021)*** If the matrix (A_{xy}) of order 3×3 where $a_{xy} = 2x - y$, then the rank of the matrix A is ....

(a) 3 (b) 2 (c) 1 (d) zero

---

67. The rank of the matrix $X = \begin{pmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix}$ equals ....

(a) zero if $a = 6$ (b) 1 if $a = -1$ (c) 3 if $a = 2$ (d) 1 if $a = -6$

---

68. If the matrix $C = \begin{pmatrix} -2 & 1 & 5 \\ a^2 - b & 0 & -a \\ b & 0 & b^2 + a \end{pmatrix}$ and $a \times b = -3$ and the rank of the matrix C equals 2 , then $a^6 + b^6 =$ ....

(a) 9 (b) 18 (c) 27 (d) 36

---

69. In the non-homogeneous equations in 3 variables on the form $AX = B$ where A is the coefficients matrix if $R(A) = R(A^*)$, then the equations have ....

(a) A unique solution. (b) infinite number of solutions.

(c) no solution. (d) (a) or (b)

---

70. If the non-homogeneous equations in 3 variables has no solutions , then the planes represent these equations in the space ....

(a) the three planes are parallel.

(b) the third plane intersects two parallel planes.

(c) the planes are mutually intersecting and the three planes do not intersect at one point

(d) each of the previous is true.

Second: Multiple choice question bank in Analytic Solid Geometry

First Questions on 3-dimensional orthogonal coordinate system

Choose the correct answer from the given ones :

1. The point $(0 , 0 , -3)$ lies on ....
   (a) y-axis. (b) z-axis. (c) x y-plane. (d) x z-plane.

2. If $A(m , 2m - n , m + n + 3) \in x$-axis , then $m =$ ....
   (a) 0 (b) -2 (c) 3 (d) -1

3. The point $(2 , 0 , -3)$ lies in the coordinates plane whose equation is ....
   (a) $z = 0$ (b) $y = 0$ (c) $x = 0$ (d) $x + y = -1$

4. All points in space in the form $(x , 5 , z)$ lies in the plane whose equation is ....
   (a) $x = 5$ (b) $y = 5$ (c) $z = 0$ (d) $y = 0$

5. The point $A(1 - k , 2k , 3 + k)$ lies on the plane x y , then $A =$ ....
   (a) $(-2 , 6 , 0)$ (b) $(0 , 2 , 4)$ (c) $(1 , 0 , 3)$ (d) $(4 , -6 , 0)$

6. The point $A(n - 1 , n + 4 , 2n)$ lies on the plane $y = 6$, then $n =$ ....
   (a) 7 (b) 2
   (c) 3 (d) any real number $\neq 0$

7. If the point (x , y , z) lies in the x z-plane , then ....
   (a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $x + y = 0$

8. The point $A(2 , -3 , 0)$ lies ....
   (a) on the z-axis. (b) in the y z-plane.
   (c) in the x y-plane. (d) on the x-axis.

9. If the point $(2a , a + 3 , 5)$ lies in the cartesian plane x z , then its distance from y z plane equals .... unit length.
   (a) 3 (b) 5 (c) 6 (d) zero

10 If the point $(a-2, 5, a-4)$ at a distance 5 units from the y z-plane and at a distance 3 units from the x y-plane , then a = ...

(a) 2 (b) 4 (c) 7 (d) 7 or -3

11 The distance between the point (a, b, c) and y-axis equals ....

(a) $\sqrt{a^2+c^2}$ (b) $\sqrt{a^2+b^2}$ (c) $\sqrt{b^2+c^2}$ (d) $\sqrt{a^2+b^2+c^2}$

12 The perpendicular distance from the point $(-5, -3, 4)$ to the x-axis = ... length unit.

(a) 3 (b) 5 (c) 4 (d) 10

13 If the point A $(\ell+5, 2\ell, \ell)$ is at a distance $2\sqrt{5}$ length unit from the x-axis , then A = ...

(a) $(3, 4, 2)$ (b) $(3, -4, -2)$

(c) $(7, 4, 2)$ (d) b , c together.

14 If the point $(3, k, -2)$ equidistant from the two axes y and z , then k = ...

(a) ± 3 (b) ± 2 (c) $\pm\sqrt{13}$ (d) ± 5

15 The smallest distance between the point $(-4, 5, -2)$ and the plane $z = 0$ equals .... length unit.

(a) -2 (b) 5 (c) 4 (d) 2

16 The distance between the point $(-2, -4, 5)$ and the y z-plane equals .... length units.

(a) 2 (b) 4 (c) 5 (d) $\sqrt{41}$

17 The point A $(3, -5, 1)$ in the space , then the sum of its dimensions from the three coordinate planes = ... length units.

(a) -1 (b) 1 (c) 9 (d) 35

18 The equation of z-axis in the space is ....

(a) $x = 0, y = 0$ (b) $x = 0, z = 0$

(c) $y = 0, z = 0$ (d) $x = 0$

19 The two coordinate planes $z = 0$, $x = 0$ are intersecting at ....

(a) origin point (b) x-axis (c) y-axis (d) z-axis

20 The x-axis and the y-axis belong to the plane with equation ....

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $x + y = 0$

21 x y-plane and y z-planes intersect at ....

(a) origin point. (b) x-axis. (c) y-axis. (d) z-axis.

22 The coordinate planes x y , x z , z y intersecting at ....

(a) the origin. (b) x-axis. (c) y-axis. (d) z-axis.

23 The straight lines $\overleftrightarrow{xx}$, $\overleftrightarrow{zz}$ form the coordinate plane whose equation is ....

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $y = 2$

24 The coordinates of the midpoint of the line segment its end points are $(-3, 2, 4)$, $(5, 1, 8)$ is ....

(a) $\left(1, \frac{3}{2}, 6\right)$ (b) $(2, -1, 4)$ (c) $(8, -1, 4)$ (d) $\left(1, \frac{3}{2}, 2\right)$

25 If A (a, b, c) is the midpoint of $(-4, 0, 5)$, $(-2, 4, -13)$, then $a + b + c =$ ....

(a) -5 (b) -6 (c) 3 (d) 4

26 If C $(-1, 6, -5)$ is the midpoint of $\overline{AB}$ where A $(k-2, -1, m+3)$, B $(2, n-7, -2)$, then the value of $k + m - n =$ ....

(a) 33 (b) 23 (c) -27 (d) -33

27 If $(5, 6, -3)$ is the midpoint of $\overline{AB}$ where A $(3, -1, 5)$, then B = ...

(a) $\left(4, \frac{5}{2}, 1\right)$ (b) $(7, 13, -11)$ (c) $(-2, -7, 8)$ (d) $(3, 2, 13)$

28 If the midpoint of $\overline{AB} \in x$-axis where A $(2, 12+k, k)$, B $(4, m, 8-m)$, then $k - 3m =$ ....

(a) 4 (b) -4 (c) -2 (d) -10

29 If the midpoint of $\overline{AB}$ lies in the cartesian plane x z and A $(-3, 12+k, 5)$, B $(1, 3k, -2)$, then $k =$ ....

(a) 5 (b) -3 (c) -2 (d) 1

30 If A (7 , – 1 , 8) , B (11 , 2 , – 4) , then the length of $\overline{AB}$ = ... cm.

(a) 10 (b) 11 (c) 12 (d) 13

31 The distance between the point (– 3 , 4 , 5) from the origin = ... length unit.

(a) 5 (b) $5\sqrt{2}$ (c) 10 (d) 25

32 The image of the point (– 2 , 3 , 4) by reflection in the z-axis is ....

(a) (– 2 , 3 , 4) (b) (2 , 3 , 4) (c) (2 , – 3 , – 4) (d) (2 , – 3 , 4)

33 The projection of the point (3 , 2 , 1) on the x y-plane is ....

(a) (3 , 2 , 0) (b) (0 , 2 , 1) (c) (3 , 2 , 1) (d) (3 , 0 , 1)

34 The point lies on the y-axis and at a distance $\sqrt{10}$ length units from the point (1 , 2 , 3) is ....

(a) (0 , – 2 , 0) (b) (0 , 2 , 0) (c) (0 , 4 , 0) (d) (0 , – 4 , 0)

35 If L , M , N are the projections of the point A (3 , 4 , 5) on the x-axis , y-axis and z-axis respectively , then M = ...

(a) (3 , 0 , 5) (b) (0 , 4 , 0) (c) (0 , 4 , 5) (d) (0 , – 4 , 0)

36 If L , M , N are the projections of the point A (3 , 4 , 5) on the x y-plane , y z-plane and x z-plane respectively , then L = ...

(a) (– 3 , – 4 , 0) (b) (0 , 4 , 5) (c) (3 , 4 , 0) (d) (3 , 0 , 0)

37 If A (– 4 , – 2 , 3) , B (1 , 2 , k) and the length of $\overline{AB} = \sqrt{77}$, then k = ...

(a) – 3 or 6 (b) – 3 or 12 (c) 9 or 6 (d) 9 or – 3

38 If the points A (6 , 0 , 3) , B (7 , 1 , 7) , C (9 , 3 , 15) lie on the same straight line , then A divides $\overline{BC}$ in the ratio ....

(a) 3 : 1 internally. (b) 1 : 3 externally. (c) 2 : 3 internally. (d) 1 : 2 externally.

39 The perimeter of the triangle OAB where O is the origin , A (0 , 3 , 0) and B (4 , 0 , 0) is .... length unit.

(a) 5 (b) 7 (c) 12 (d) 13

40 If M (1 , 2 , 3) , N (4 , 2 , 3) , R (1 , 6 , 3) are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively , then the perimeter of Δ ABC is .... length unit.

(a) 12 (b) 13 (c) 14 (d) 24

41 If A (1 , 2 , 5) , B (2 , 5 , 3) and C (– 1 , 3 , 2) , then ....

(a) A , B and C are on the same straight line.

(b) ABCO is a square where O is the origin point.

(c) ABC is right-angled triangle. (d) ABC is equilateral triangle.

42 The points A (2 , – 1 , 3) , B (– 4 , 4 , 2) and C (– 2 , 5 , 1) represent ....

(a) vertices of right angled triangle. (b) vertices of equeilateral triangle.

(c) vertices of isosceles triangle. (d) three colinear points.

43 The points (2 , – 1 , 5) , (6 , 0 , 6) and .... are collinear.

(a) (14 , – 2 , 8) (b) (14 , 2 , 8) (c) (8 , 2 , 2) (d) $\left(4 , -1 , 5\frac{1}{2}\right)$

44 *(2nd Session 2021)* If ABC is a triangle in which D is the midpoint of $\overline{BC}$, A (3 , 1 , 5) , B (2 , 3 , 7) , C (0 , 3 , 1) , then the length of $\overline{AD}$ = ... length unit.

(a) 9 (b) 2 (c) 7 (d) 3

45 If the point E is equidistant from the points O (0 , 0 , 0) , A (ℓ , 0 , 0) , B (0 , m , 0) and C (0 , 0 , n) , then the point E = ...

(a) (ℓ , m , n) (b) $\left(\frac{-\ell}{2} , \frac{-m}{2} , \frac{-n}{2}\right)$ (c) (– ℓ , – m , – n) (d) $\left(\frac{\ell}{2} , \frac{m}{2} , \frac{n}{2}\right)$

46 If A (6 , – 2 , 4) , B (2 , 4 , – 8) , C (– 2 , 2 , 4) are three consecutive vertices of a parallelogram ABCD , then D = ...

(a) $\left(2 , \frac{4}{3} , 0\right)$ (b) (– 2 , 4 , – 16) (c) (2 , – 4 , 16) (d) (1 , – 2 , 8)

47 *(Trial 2021)* If A (3 , – 4 , 0) , B (15 , 0 , 2) , C (0 , – 8 , 4) are three points in the space and they form triangle ABC , then the distance between its centroid and the x z-plane is ....

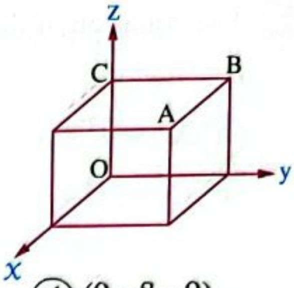
(a) greater than the distance from the x y-plane.

(b) smaller than or equal to the distance from the x y-plane.

(c) greater than the distance from the y z-plane.

(d) greater than or equal to the distance from the y z-plane.

48 The opposite figure is a cuboid : A (5 , 8 , 4) , then :



First : The coordinates of the point B are ....

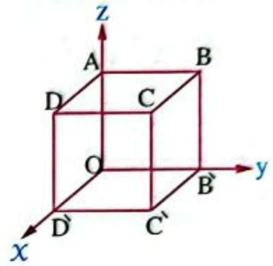
(a) (4 , 8 , 0) (b) (0 , 8 , 4)
(c) (5 , 8 , 0) (d) (5 , 0 , 4)

Second : The coordinates of point C are ....

(a) (0 , 0 , 0) (b) (0 , 0 , 4) (c) (5 , 0 , 4) (d) (0 , 8 , 0)

---

49 $ABCDOB'C'D'$ is a cube of edge length 5 length unit , then :



First : The coordinates of the point C are ....

(a) (5 , 5 , 0) (b) (5 , 5 , 5)
(c) (0 , 0 , 5) (d) (0 , 5 , 0)

Second : The coordinates of D' are ....

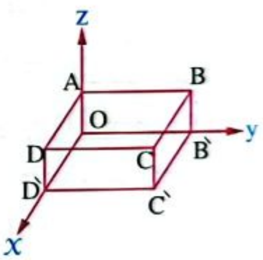
(a) (0 , 0 , 5) (b) (5 , 5 , 0) (c) (0 , 5 , 5) (d) (5 , 0 , 0)

Third : The diagonal length of the cube = ... length units.

(a) $5\sqrt{2}$ (b) $5\sqrt{3}$ (c) 5 (d) $5\sqrt{6}$

---

50 **In the opposite figure :**



A cuboid ; C' (5 , 8 , 0) , D (5 , 0 , 3) , then :

First : The coordinates of C ....

(a) (5 , 8 , 0) (b) (5 , 3 , 8)
(c) (5 , 8 , 3) (d) (5 , 8 , 8)

Second : The volume of the cuboid .... cubic units.

(a) 64 (b) 120 (c) 144 (d) 150

Third : The equation of the plane $OB'C'D'$ is ....

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $z = 3$

Fourth : The equation of the plane $DD'C'C$ is ....

(a) $x = 0$ (b) $y = 0$ (c) $x = 5$ (d) $z = 3$

Second Questions on equation of the sphere

Choose the correct answer from the given ones :

1 The equation of the sphere whose centre (2 , – 3 , 5) and its radius length $2\sqrt{5}$ length unit is ....

(a) $(x + 2)^2 + (y - 3)^2 + (z + 5)^2 = 2\sqrt{5}$ (b) $x^2 + y^2 + z^2 = 20$
(c) $(x - 2)^2 + (y + 3)^2 + (z - 5)^2 = 20$ (d) $(x - 2)^2 + (y + 3)^2 + (z - 5)^2 = 2\sqrt{5}$

2 The equation of the sphere whose centre is the origin and its radius length is 3 units is ....

(a) $x^2 + y^2 + z^2 = 3$ (b) $x^2 + y^2 + z^2 = 9$

(c) $(x-2)^2 + (y-3)^2 + (z-3)^2 = 9$ (d) $x^2 + y^2 + z^2 + 9 = 0$

3 The equation of the sphere whose centre is the origin and cuts 5 units from the positive part of the x-axis is ....

(a) $x^2 + y^2 + z^2 - 25 = 0$ (b) $x^2 + y^2 + z^2 + 25 = 0$

(c) $x^2 + y^2 + z^2 - 100 = 0$ (d) $x^2 + y^2 + z^2 - \sqrt{5} = 0$

4 The equation of the sphere whose centre is the origin and passes through $(3, -1, 2)$ is ....

(a) $x^2 + y^2 + z^2 = 4$ (b) $(x-3)^2 + (y+1)^2 + (z-2)^2 = 14$

(c) $(x-3)^2 + (y+1)^2 + (z-2)^2 = \sqrt{14}$ (d) $x^2 + y^2 + z^2 = 14$

5 If the origin lies on a sphere whose centre $(-1, 2, 2)$, then its equation is ....

(a) $x^2 + y^2 + z^2 + 2x + 4y + 4z = -3$ (b) $x^2 + y^2 + z^2 + 2x - 4y - 4z = 3$

(c) $(x+1)^2 + (y-2)^2 + (z-2)^2 + 9 = 0$ (d) $x^2 + y^2 + z^2 + 2x - 4y - 4z = 0$

6 The equation of the sphere whose centre is the origin and passes through vertices of a cube whose edge length 12 length unit is ....

(a) $x^2 + y^2 + z^2 = 144$ (b) $x^2 + y^2 + z^2 = 108$

(c) $x^2 + y^2 + z^2 = 36$ (d) $x^2 + y^2 + z^2 + 108 = 0$

7 The equation of a sphere with centre $(2, -3, 4)$ and touches xy-plane is ....

(a) $(x-2)^2 + (y+3)^2 + (z-4)^2 = 4$ (b) $(x-2)^2 + (y+3)^2 + (z-4)^2 = 9$

(c) $(x-2)^2 + (y+3)^2 + (z-4)^2 = 16$ (d) $(x+2)^2 + (y-3)^2 + (z+4)^2 = 16$

8 The equation of the sphere whose centre $(-3, -3, 5)$ and touches the two planes xz and yz is ....

(a) $(x+3)^2 + (y+3)^2 + (z-5)^2 = 3$ (b) $(x+3)^2 + (y+3)^2 + (z-5)^2 = 9$

(c) $(x-3)^2 + (y-3)^2 + (z+5)^2 = 34$ (d) $x + y^2 + z^2 + 6x + 6y - 10z = 9$

9 The equation of the sphere with centre $(1, -3, -1)$ and passes through the point $(-2, -1, -1)$ is ....

(a) $(x+2)^2 + (y+1)^2 + (z+1)^2 = 13$

(b) $x^2 + y^2 + z^2 - 2x + 6y + 2z - 13 = 0$

(c) $x^2 + y^2 + z^2 - 2x + 6y + 2z = 2$

(d) $(x-1)^2 + (y+3)^2 + (z+1)^2 = \sqrt{13}$

---

10 The equation of the sphere whose diameter is $\overline{AB}$ where $A(7, 1, -4)$, $B(3, -1, 2)$ is ....

(a) $(x-7)^2 + (y-1)^2 + (z+4)^2 = 28$

(b) $(x-3)^2 + (y+1)^2 + (z-2)^2 = 14$

(c) $x^2 + y^2 + z^2 - 10x + 2z - 14 = 0$

(d) $(x-5)^2 + (z+1)^2 + y^2 = 14$

---

11 The x-axis touches the sphere whose centre $(2, 3, 4)$ then the equation of the sphere is ....

(a) $(x-2)^2 + (y-3)^2 + (z-4)^2 = 15$

(b) $(x-2)^2 + (y-3)^2 + (z-4)^2 = 5$

(c) $x^2 + y^2 + z^2 - 4x - 6y - 8z + 4 = 0$

(d) $x^2 + y^2 + z^2 - 25 = 0$

---

12 ***(2nd Session 2021)*** The equation of the sphere whose centre is $(-1, 0, 5)$ and its volume 36π volume unit is ....

(a) $(x+1)^2 + y^2 + (z-5)^2 = 36$

(b) $(x-1)^2 + y^2 + (z+5)^2 = 6$

(c) $(x+1)^2 + y^2 + (z-5)^2 = 27$

(d) $(x+1)^2 + y^2 + (z-5)^2 = 9$

---

13 Equation of the sphere has centre $(2, -1, 4)$ and its area 100π square units is ....

(a) $(x+2)^2 + (y-1)^2 + (z+4)^2 = 25$

(b) $(x-2)^2 + (y+1)^2 + (z-4)^2 = 25$

(c) $(x-2)^2 + (y+1)^2 + (z-4)^2 = 100$

(d) $(x-2)^2 + (y+1)^2 + (z-4)^2 = 0$

---

14 If the points $(0, 0, 0)$, $(4, 0, 0)$, $(0, 4, 0)$, $(0, 0, 4)$ are 4 vertices of a cube , then the equation of the sphere touches its faces from inside is ....

(a) $(x-4)^2 + (y-4)^2 + (z-4)^2 = 4$

(b) $(x-2)^2 + (y-2)^2 + (z-2)^2 = 4$

(c) $(x-2)^2 + y^2 + z^2 = 4$

(d) $x^2 + (y-2)^2 + (z-2)^2 = 4$

---

15 The centre of the sphere whose equation $x^2 + y^2 + z^2 + 2x - 4y + 6z - 16 = 0$ is ....

(a) $(1, 1, 1)$

(b) $(2, -4, 6)$

(c) $(-1, 2, -3)$

(d) $(-2, 4, -6)$

16 The centre of a sphere has a diameter ends at A $(3, -3, 2)$ and B $(5, -1, 2)$ is ....

(a) $(8, -4, 4)$ (b) $(4, -2, 0)$ (c) $(4, -2, 2)$ (d) $(-2, -2, 0)$

17 If $\overline{AB}$ is a diameter of the sphere which its equation : $x^2 + y^2 + z^2 - x + 2y + 3z - 44 = 0$ and : A $= (2, 4, -6)$, then B = ...

(a) $(1, -2, -3)$ (b) $(-1, -6, 3)$ (c) $(0, 4, 1)$ (d) $(2, 3, -5)$

18 $\overline{AB}$ is a diameter in a sphere whose equation $(x-5)^2 + (y+2)^2 + (z-1)^2 = 11$, and the coordinates of A $(8, -1, 2)$ then the coordinates of B is ....

(a) $(5, -2, 1)$ (b) $(10, -4, 5)$ (c) $(2, -3, 0)$ (d) $(10, 3, 6)$

19 The radius of the sphere $x^2 + y^2 + z^2 - 2x - 6y + 10z - 1 = 0$, equals .... length unit.

(a) 3 (b) 4 (c) 5 (d) 6

20 $x^2 + y^2 + z^2 - 4x - 6y - 8z + 4 = 0$ is an equation of a sphere, whose diameter length = ... cm.

(a) 5 (b) 10 (c) 15 (d) 20

21 $x^2 + y^2 + z^2 - 4kx + 4y - 8z + k = 0$ is the equation of a sphere, the length of its radius 5 length unit, then k = ... or ....

(a) $\frac{-5}{4}, 1$ (b) $\frac{5}{4}, -1$ (c) $\frac{-1}{4}, \frac{5}{4}$ (d) $1, -1$

22 The radius of the sphere whose equation $(x-2)^2 + (y-4)^2 + (z-5)^2 = 64$ equals .... length units.

(a) 64 (b) $3\sqrt{5}$ (c) 8 (d) 5

23 The area of the sphere whose equation $x^2 + y^2 + z^2 - 25 = 0$ equals .... area units.

(a) 20π (b) 40π (c) 25π (d) 100π

24 The volume of a sphere whose equation $x^2 + y^2 + z^2 - 4x - 8y - 10z - 36 = 0$ equals .... volume units.

(a) 324π (b) 36π (c) 782π (d) 972π

25 If point (– 2 , 4 , m) lies on the sphere $(x + 2)^2 + (y - 1)^2 + (z - 3)^2 = 25$, then m = ...

(a) 7 (b) 4 (c) 7 or – 1 (d) 4 or – 1

26 *(1st Session 2021)* If the point (7 , – 2 , 2) lies on the surface of a sphere with equation : $(x - 4)^2 + (y - 1)^2 + (z + 1)^2 = K^2$, then | K | = ...

(a) 3 (b) $3\sqrt{3}$ (c) 27 (d) $\sqrt{3}$

27 A sphere with radius 5 length units , touches the coordinates planes. If the coordinates of its centre are positive , then the distance between the point where it touches the x y-plane and where it touches the y z-plane equals .... length units.

(a) 5 (b) $5\sqrt{2}$ (c) $5\sqrt{3}$ (d) $5\sqrt{6}$

28 The centre of the sphere which touches the positive cartesian planes and its radius length is 5 units is ....

(a) (0 , 0 , 0) (b) (5 , 5 , 5) (c) (5 , 0 , 0) (d) (0 , 5 , 5)

29 The centre of the sphere $x^2 + y^2 + z^2 - 2z = 0$ lies on the ....

(a) x-axis (b) plane z = 0 (c) y-axis (d) z-axis

30 If $x^2 + y^2 + z^2 - 2x + 4y - 4z + k = 0$ is the equation of a sphere , then the value of k could be ....

(a) 9 (b) 18 (c) 5 (d) 10

31 Equation of the sphere whose centre M (4 , – 6 , – 5) and its radius 2 length unit is $x^2 + y^2 + z^2 + ax + by + cz + d = 0$, then a + b + c + d = ...

(a) 18 (b) 73 (c) 87 (d) 304

32 The sphere whose centre lies on y-axis and passes through the two points (1 , 3 , 2) , (– 2 , 4 , 2) its radius = ... length unit.

(a) 6 (b) 8 (c) 3 (d) 9

33 The point A (3 , 2 , 1) lies .... the sphere whose equation $x^2 + (y + 1)^2 + (z - 1)^2 = 4$

(a) on (b) inside (c) outside (d) on the centre of

34 The sphere whose equation : $(x-2)^2+(y+4)^2+(z+3)^2=4$ touches ....

(a) x-axis (b) y z-plane. (c) x y-plane. (d) y-axis.

35 Which of the following represents a sphere , its centre lies on z-axis and touches the x y-plane ?

(a) $x^2+y^2+z^2=25$ (b) $x^2-10x+y^2+z^2=0$

(c) $x^2+y^2+z^2-10z=25$ (d) $x^2+y^2+z^2-10z=0$

36 The sphere lies between the two planes $z=-1$ and $z=5$ and has centre $(2,-1,k)$, then $k=$ ....

(a) zero (b) 1 (c) 2 (d) 3

37 If y-axis intersects the sphere whose centre is $(3,-4,12)$ and its radius length 13 cm. at the two points A , B , then AB = ... length units.

(a) 8 (b) 10 (c) 13 (d) 26

38 If x-axis cuts the sphere $(x-2)^2+(y+3)^2+(z-1)^2=14$ at the two points A and B , then the length of $\overline{AB}$ = ... length units.

(a) 8 (b) 2 (c) 16 (d) 4

39 The two spheres with equations : $x^2+y^2+z^2-2x-2y+2z-1=0$, $(x-5)^2+(y+2)^2+z^2=4$ are ....

(a) touching externally. (b) touching internally.

(c) intersecting. (d) distant.

40 If $(x+3)^2+(y-2)^2+(z-4)^2=1$, $(x+4)^2+(y-4)^2+(z-2)^2=4$ are the equations of two spheres , then the two spheres are ....

(a) intersecting. (b) touching externally.

(c) touching internally. (d) distant.

41 If M and N are two spheres of radii r_1 and r_2 respectively where $r_1>r_2$, if the two spheres are tangential , then MN = ...

(a) r_1+r_2 (b) zero (c) r_1-r_2 (d) a or c

42 If the two spheres $(x-3)^2 + y^2 + (z-3)^2 = 16$, $(x+1)^2 + (y-4)^2 + (z-k)^2 = 25$ are tangentil , then the value of k = ...

(a) ± 10 (b) ± 4 (c) 10 or – 4 (d) 4 or – 10

43 *(1st Session 2021)* If M_1 , M_2 are two spheres touching internally and $M_1\left(-3 , 2 , -6\sqrt{2}\right)$, $r_1 = 8$ length unit , $M_2\left(-2 , 1 , -5\sqrt{2}\right)$, then r_2 = ... length units where $r_1 > r_2$

(a) 5 (b) 2 (c) 7 (d) 6

44 The shortest distance between the point (5 , – 1 , 7) and the surface of the sphere : $(x-2)^2 + (y+5)^2 + (z+5)^2 = 25$ equals ....

(a) 8 (b) 9 (c) 10 (d) 13

45 *(Trial 2021)* If the shortest distance between A (3 , 5 , 1) and the surface of a sphere with centre M (1 , 2 , – 5) equals 2 length units where A lies outside the shpere , then the radius of the sphere = ... length units.

(a) 5 (b) 2 (c) 7 (d) 12

46 A sphere touches the x y-plane , y z-plane and x z-plane and passes through the point (1 , – 4 , 5) , then its radius could be .... length units

(a) 6 (b) 7 (c) 8 (d) 9

47 The equation of the sphere whose centre (3 , m – 1 , 5) and touches the coordinate axes x and y is ....

(a) $x^2 + y^2 + z^2 - 3x - 3y + 5z = 34$

(b) $(x-3)^2 + (y \pm 3)^2 + (z-5)^2 = 34$

(c) $(x+3)^2 + (y \pm 3)^2 + (z+5)^2 = 34$

(d) $(x \pm 3)^2 + (y \pm 3)^2 + (z \pm 5)^2 = 34$

48 The radius of the smallest sphere that the points (0 , 0 , 5) , (5 , 0 , 0) , (0 , 5 , 0) lie on it is .... length units.

(a) 15 (b) $\frac{5\sqrt{6}}{3}$ (c) $5\sqrt{3}$ (d) 5

49 The radius of the smallest sphere passes through (5 , 5 , 0) , (0 , 5 , 5) , (5 , 0 , 5) is .... length units.

(a) 5 (b) 10 (c) $\frac{5\sqrt{6}}{3}$ (d) $5\sqrt{2}$

50 The number of spheres touch the coordinate axes and its diameter length is 16 units is ....

(a) 1 (b) 2 (c) 4 (d) 8

---

51 If $\overrightarrow{AB} \perp \overrightarrow{BC}$ where A (1 , 2 , 3) , B (1 , 8 , 3) , C (9 , 8 , 3) , then the equation of the smallest sphere passes through A , B and C is ....

(a) $(x-5)^2+(y-5)^2+(z-3)^2=25$
(b) $\left(x-\frac{11}{3}\right)^2+(y-6)^2+(z-2)^2=25$
(c) $(x-2)^2+(y+3)^2+(z-4)^2=16$
(d) $(x+2)^2+(y-3)^2+(z+4)^2=16$

---

52 The sphere with the equation : $x^2+y^2+z^2-2a(x+y+z)+2a^2=0$, where $a>0$ is ....

(a) touching the three coordinate planes.
(b) touching the three coordinate axes.
(c) passing through the origin.
(d) otherwise.

---

53 The sphere with the equation : $x^2+y^2+z^2-2kx-2ky-2kz+k^2=0$, where $k>0$ is ....

(a) touching the three coordinate planes.
(b) touching the three coordinate axes.
(c) passing through the origin.
(d) otherwise.

---

54 The sphere equation which touches the cartesian planes and its diameter length 16 units is .... given that its centre coordinates are positive.

(a) $\left(x-4\sqrt{2}\right)^2+\left(y-4\sqrt{2}\right)^2+\left(z-4\sqrt{2}\right)^2=64$
(b) $x^2+y^2+z^2+8\sqrt{2}x+8\sqrt{2}y+8\sqrt{2}z=64$
(c) $x^2+y^2+z^2=64$
(d) $(x-8)^2+(y-8)^2+(z-8)^2=64$

---

55 The sphere equation which touches the positive coordinate axes and its diameter length 16 units is ....

(a) $\left(x-4\sqrt{2}\right)^2+\left(y-4\sqrt{2}\right)^2+\left(z-4\sqrt{2}\right)^2=64$
(b) $x^2+y^2+z^2+8\sqrt{2}x+8\sqrt{2}y+8\sqrt{2}z=64$
(c) $x^2+y^2+z^2=64$
(d) $(x-8)^2+(y-8)^2+(z-8)^2=64$

56 M is the centre of a sphere , placed inside a cube such that it touches all its faces from inside , the side length of the cube is 10 cm. and one of its vertices is the origin and its centre coordinates are positive , then the sphere equation is ....

(a) $x^2 + y^2 + z^2 - 20x - 20y - 20z + 50 =$ zero

(b) $x^2 + y^2 + z^2 + 10xy + 10yz + 50 =$ zero

(c) $x^2 + y^2 + z^2 - 10x - 10y - 10z + 50 =$ zero

(d) $x^2 + y^2 + z^2 + 10x - 10y - 10z + 50 =$ zero

---

57 If sphere $(x-2)^2 + (y+1)^2 + (z-3)^2 = 9$ translated 3 units in $\overrightarrow{Ox}$ direction , then the sphere equation is ....

(a) $(x-2)^2 + (y+1)^2 + (z-3)^2 = 9$

(b) $(x-5)^2 + (y+1)^2 + (z-3)^2 = 9$

(c) $(x-2)^2 + (y-1)^2 + (z-3)^2 = 9$

(d) $(x-2)^2 + (y+1)^2 + (z-6)^2 = 9$

---

58 The equation of the sphere resulted by reflection of the sphere : $x^2 + y^2 + (z+4)^2 = 16$ in the origin is ....

(a) $x^2 + y^2 + z^2 - 8z = 0$

(b) $x^2 + y^2 + z^2 + 8z = 0$

(c) $x^2 + y^2 + z^2 + 8x = 0$

(d) $x^2 + y^2 + z^2 - 8y = 0$

---

59 The equation of the sphere resulted by reflection of the sphere : $(x-3)^2 + (y+2)^2 + (z+4)^2 = 16$ in the x z-plane is ....

(a) $x^2 + y^2 + z^2 - 6x + 4y - 8z + 13 = 0$

(b) $x^2 + y^2 + z^2 + 6x + 4y + 8z + 13 = 0$

(c) $x^2 + y^2 + z^2 - 6x - 4y - 8z + 13 = 0$

(d) $x^2 + y^2 + z^2 - 6x - 4y + 8z + 13 = 0$

---

60 If the points A , B , C are the intersection points between sphere $(x-1)^2 + (y-2)^2 + (z-1)^2 = 6$ and the $\overrightarrow{Ox}$, $\overrightarrow{Oy}$ axes , then the area of triangle ABC equals .... square units.

(a) 2 (b) 3 (c) 4 (d) 6

---

61 Area of largest circle drawn on the surface of a sphere whose centre is (5 , 0 , − 1) and passes through the point (7 , 1 , − 4) equals .... square units.

(a) 25π (b) 49π (c) 16π (d) 14π

---

62 Area of resulting circle from intersecting of the sphere $(x-3)^2 + (y-4)^2 + (z-12)^2 = 169$ with the plane x y = ... square units.

(a) 13π (b) 169π (c) 25π (d) 144π

63 A sphere $x^2 + y^2 + z^2 = 75$ passes through vertices of a cube drawn inside it , then total area of the cube = ... square units.

(a) 125 (b) 600 (c) $25\sqrt{3}$ (d) 300

64 The equation of the sphere with centre at the origin and passes through the vertices of a cube with lateral area of 144 square units is ....

(a) $x^2 + y^2 + z^2 = 27$ (b) $x^2 + y^2 + z^2 = 36$

(c) $x^2 + y^2 + z^2 + 36 = 0$ (d) $x^2 + y^2 + z^2 = 72$

65 If the two spheres :

$(x-3)^2 + y^2 + (z-3)^2 = 16$ and $(x+1)^2 + (y-4)^2 + (z-k)^2 = 25$ are intersecting , then $k \in$ ....

(a) $\mathbb{R} - [-4 , 10]$ (b) $\{4 , -10\}$ (c) $]-4 , 10[$ (d) $\{-4 , 10\}$

66 A point on the 3-dimension space is at the same distance from the 3 cartesian planes which is "a" length unit , then which of the following is correct ?

(a) The point is (a , a , a)

(b) There are 8 points satisfying that and they are vertices of a cube of volume ($8\,a^3$) cube unit.

(c) This point is the centre of the sphere which touches the coordinate axis and its radius a length unit.

(d) All these points lie on the surface of a sphere whose centre is the origin and its radius a length unit.

Third Questions on vectors in the space

Choose the correct answer from the given ones :

1 If $\overrightarrow{A} = (-3 , 7 , 8)$, then $\|\overrightarrow{A}\| =$ ....

(a) 12 (b) $\sqrt{222}$ (c) $\sqrt{122}$ (d) 10

2 If $\overrightarrow{A} = 5\hat{i} - 4\hat{j} + 2\sqrt{2}\,\hat{k}$, then $\|\overrightarrow{A}\| =$ ....

(a) 3 (b) 7 (c) 8 (d) 9

3. The vector which represents a unit vector from the following vectors is ....

(a) $(-3, 2, 2)$ (b) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

(c) $\left(\frac{3}{5}, \frac{-4}{5}, 0\right)$ (d) $\left(\frac{-1}{12}, \frac{-3}{12}, \frac{2}{12}\right)$

4. $\|\overrightarrow{A}\| = 8$, then $\|-5\overrightarrow{A}\| =$ ....

(a) -40 (b) 40 (c) 40 or -40 (d) 20

5. If $\|4\overrightarrow{A}\| = \|3k\overrightarrow{A}\|$, then $k =$ ....

(a) ± 1 (b) $\pm \frac{3}{4}$ (c) $\pm \frac{4}{3}$ (d) $\pm \frac{2}{\sqrt{3}}$

6. If $\|k(3, -3\sqrt{3}, 0)\| = 1$, then $k =$ ....

(a) $\frac{1}{6}$ (b) $\pm \frac{1}{6}$ (c) 6 (d) ± 6

7. If $\overrightarrow{A} = (3, -2, m)$, $\|\overrightarrow{A}\| = \sqrt{22}$, then $m =$ ....

(a) 21 (b) ± 9 (c) ± 3 (d) 17

8. If $\overrightarrow{A} = (-2, k, 1)$ and $\|\overrightarrow{A}\| = 3$ units, then $k =$ ....

(a) 4 (b) -4 (c) ± 2 (d) $\sqrt{14}$

9. If $\overrightarrow{A} = \left(2, \sqrt{2}, \sqrt{3}\right)$, then the component of $\overrightarrow{A}$ in direction of the y-axis is ....

(a) 2 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 4

10. If the vector $\overrightarrow{A}$ lies in the cartesian plan y z, then $\overrightarrow{A}$ can be ....

(a) $(x, 0, y)$ (b) $(x, y, 0)$ (c) $(0, y, z)$ (d) $(x, 0, 0)$

11. If $\overrightarrow{A} = (2, -3, 5)$, $\overrightarrow{B} = (-2, 0, 4)$, then $3\overrightarrow{A} - 2\overrightarrow{B} =$ ....

(a) $(10, -9, 7)$ (b) $(4, -9, 19)$ (c) $(15, 11, 8)$ (d) $(11, 12, 4)$

12. If $\overrightarrow{C} = (2, -3, 1)$, $\overrightarrow{D} = (0, 2, -2)$ and $3\overrightarrow{A} - 4\overrightarrow{D} = \overrightarrow{C}$, then $\overrightarrow{A} =$ ....

(a) $(3, 5, -7)$ (b) $\left(\frac{2}{3}, \frac{-11}{3}, 3\right)$ (c) $\left(\frac{2}{3}, \frac{5}{3}, \frac{-7}{3}\right)$ (d) $(2, -11, 3)$

13. If $\overrightarrow{A} = (-1, 5, -2)$, $\overrightarrow{B} = (3, 1, 1)$ and $\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C} = \hat{i}$, then $\overrightarrow{C} =$ ....

(a) $\hat{i} + 6\hat{j} - \hat{k}$ (b) $-\hat{i} - 6\hat{j} + \hat{k}$ (c) $\hat{i} + 4\hat{j} - 3\hat{k}$ (d) $\hat{i} + 4\hat{j} - \hat{k}$

14 If $(2x+1, 5, k+4) = (-1, y^2-4, x+1)$, then $x+y+k =$ ....

ⓐ -7 or 2 ⓑ 8 or 2 ⓒ 3 or 7 ⓓ -8 or -2

15 If $\ell(-6, m, 2N) = 12\hat{i} + 10\hat{j} - 4\hat{k}$, then $\ell + m + n =$ ....

ⓐ 8 ⓑ 6 ⓒ ± 7 ⓓ -6

16 If $\overrightarrow{A} = 4\hat{i} + 3\hat{j} + 2\hat{k}$, $\overrightarrow{B} = -2\hat{i} + 5\hat{j} - \hat{k}$, $\overrightarrow{C} = 8\hat{i} + 19\hat{j} + 4\hat{k}$, then $\overrightarrow{C} =$ ....

ⓐ $2\overrightarrow{A} - 3\overrightarrow{B}$ ⓑ $3\overrightarrow{A} + 5\overrightarrow{B}$ ⓒ $3\overrightarrow{A} + 2\overrightarrow{B}$ ⓓ $\pm\left(3\overrightarrow{A} + 2\overrightarrow{B}\right)$

17 If $\overrightarrow{A} = (-2, 3, 1)$, $\overrightarrow{B} = (0, 2, -2)$, $\overrightarrow{C} = (1, -3, 5)$, then $\|\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C}\| =$ ....

ⓐ $\sqrt{\sqrt{14} + 2\sqrt{2} + \sqrt{35}}$ ⓑ 21

ⓒ $\sqrt{21}$ ⓓ $\sqrt{57}$

18 If $\overrightarrow{A} + \overrightarrow{B} = (2, 3, -1)$, $\overrightarrow{A} = (4, 1, 0)$, then $\|\overrightarrow{B}\| =$ .... length unit.

ⓐ $\sqrt{3}$ ⓑ 3 ⓒ 9 ⓓ $\sqrt{53}$

19 If $\overrightarrow{A} = (1, -1, 2)$, $\overrightarrow{B} = (0, 2, -3)$, $\overrightarrow{C} = (-2, 1, 0)$, then $\|3\overrightarrow{A} - \overrightarrow{B} + \overrightarrow{C}\| =$ ....

ⓐ $8\sqrt{3}$ ⓑ 11 ⓒ 12 ⓓ $7\sqrt{2}$

20 If $\overrightarrow{A} = (1, 2, -4)$, $\overrightarrow{B} = (1, 1, k-1)$ and $\|\overrightarrow{A} + \overrightarrow{B}\| = 7$ unit of length, then $k =$ ....

ⓐ ± 6 ⓑ 2 or -11 ⓒ 11 or -1 ⓓ ± 7

21 $\|\overrightarrow{A} + \overrightarrow{B}\|$ .... $\|\overrightarrow{A}\| + \|\overrightarrow{B}\|$

ⓐ $\geq$ ⓑ $<$ ⓒ $\leq$ ⓓ $=$

22 If $A = (-2, 0, 3)$, $B = (4, 2, -5)$, then $\overrightarrow{AB} =$ ....

ⓐ $(-6, -2, 8)$ ⓑ $(2, 2, -2)$ ⓒ $(6, 2, -8)$ ⓓ $(1, 1, -1)$

23 If C is the midpoint of $\overrightarrow{AB}$ and $\overrightarrow{AC} = (3, -2, 5)$, then $\overrightarrow{CB} =$ ....

ⓐ $(-3, 2, -5)$ ⓑ $(3, -2, 5)$ ⓒ $\left(\frac{-3}{2}, 1, \frac{-5}{2}\right)$ ⓓ $\left(\frac{3}{2}, -1, \frac{5}{2}\right)$

24 If $\overrightarrow{AB} = (1, 1, 1)$, $\overrightarrow{B} = (0, 1, 3)$, then $\overrightarrow{A} =$ ....

ⓐ $(1, 0, -2)$ ⓑ $(-1, 0, 2)$ ⓒ $(1, 2, 4)$ ⓓ $(-1, -2, -4)$

25 If $\overrightarrow{A} = (2, 3\sqrt{3}, 0)$, $\overrightarrow{B} = (4, 2\sqrt{3}, \sqrt{6})$, then $\|\overrightarrow{AB}\| =$ ....

(a) 12 (b) 13 (c) $\sqrt{13}$ (d) 5

26 If $\overrightarrow{AB} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\overrightarrow{BC} = 3\hat{i} + \hat{k}$, then $\|\overrightarrow{AC}\|^2 =$ ....

(a) 35 (b) 36 (c) 38 (d) 30

27 If $\overrightarrow{A} = (-2, 4, 6)$, $\overrightarrow{B} = (0, k, 3)$ where $k \in \mathbb{Z}^+$ and $\|\overrightarrow{AB}\| = 7$, then the value of $k =$ ....

(a) 10 (b) 8 (c) 6 (d) 4

28 ***(2nd Session 2021)*** If $\overrightarrow{A} + \overrightarrow{BC} = 4\hat{i} + 12\hat{j} + 9\hat{k}$, where $\overrightarrow{A} = (0, -1, 3)$, $B(4, -2, 1)$, then $\overrightarrow{C} =$ ....

(a) $8\hat{i} + 13\hat{j} + 13\hat{k}$ (b) $8\hat{i} + 11\hat{j} + 7\hat{k}$ (c) $8\hat{i} + 9\hat{j} + 7\hat{k}$ (d) $8\hat{i} + 13\hat{j} - 7\hat{k}$

29 If the position vectors of the vertices A , B and C of parallelogram ABCD are $\overrightarrow{r_1}$, $\overrightarrow{r_2}$, $\overrightarrow{r_3}$ respectively , then the position vector of the vertex D is ....

(a) $\overrightarrow{r_1} + \overrightarrow{r_2} - \overrightarrow{r_3}$ (b) $\overrightarrow{r_2} + \overrightarrow{r_3} - \overrightarrow{r_1}$ (c) $\overrightarrow{r_3} + \overrightarrow{r_1} - \overrightarrow{r_2}$ (d) otherwise.

30 If D is the point of intersection of medians of triangle ABC , then $\overrightarrow{DA} + \overrightarrow{DB} + \overrightarrow{DC} =$ ....

(a) $\overrightarrow{O}$ (b) $3\,\overrightarrow{DA}$ (c) $3\,\overrightarrow{DB}$ (d) $3\,\overrightarrow{DC}$

31 If $\overrightarrow{A}$ is a non-zero vector , $\overrightarrow{C} = \dfrac{\overrightarrow{A}}{\|\overrightarrow{A}\|}$, then which of the following statements is always true ?

(a) $\|\overrightarrow{A}\| = \|\overrightarrow{C}\|$ (b) $\|\overrightarrow{A}\| < \|\overrightarrow{C}\|$ (c) $\|\overrightarrow{A}\| > \|\overrightarrow{C}\|$ (d) $\overrightarrow{A} \,//\, \overrightarrow{C}$

32 All of the following represents unit vectors except ....

(a) $\left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{-6}{10}, \frac{2\sqrt{7}}{10}, 0.6\right)$ (c) $(1, 1, 1)$ (d) $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}, 0\right)$

33 If $\overrightarrow{A}$ is a unit vector where $\overrightarrow{A} = \left(k, \frac{2}{3}, \frac{2}{3}\right)$, then $k =$ ....

(a) ± 3 (b) $\pm \frac{1}{3}$ (c) $\pm\sqrt{3}$ (d) $\pm \frac{1}{\sqrt{3}}$

34 If $\overrightarrow{A} = (1, 2, 0)$, $\overrightarrow{B} = (0, 4, -2)$, then the unit vector in direction of the vector $\overrightarrow{AB}$ is ....

(a) $\left(\frac{-1}{2}, 1, -1\right)$ (b) $\left(\frac{-1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$ (c) $\left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$ (d) $(-1, 2, -2)$

35 The unit vector that is parallel to the resultant of the two forces $\overrightarrow{F_1} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\overrightarrow{F_2} = 4\hat{i} - 3\hat{j} + 2\hat{k}$ is ....

(a) $\frac{1}{\sqrt{37}}(6\hat{i} + \hat{k})$ (b) $\frac{1}{\sqrt{37}}(6\hat{i} + \hat{j})$

(c) $\frac{1}{7}(2\hat{i} - 6\hat{j} + 3\hat{k})$ (d) $\frac{1}{7}(-2\hat{i} + 9\hat{j} - 3\hat{k})$

---

36 The unit vector of a vector parallel to the y z-plane could be ....

(a) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ (b) $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ (c) $(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$ (d) $(\frac{1}{\sqrt{2}}, 0, 0)$

---

37 The unit vector that bisects the angle between $\hat{j}$ and $\hat{k}$ is ....

(a) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (b) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{k})$ (c) $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{k} - \hat{j})$

---

38 If $\overrightarrow{A}$ is a non-zero vector , $k \in \mathbb{R}$, where $\|(5 - k)\overrightarrow{A}\| < \|2\overrightarrow{A}\|$, then ....

(a) $0 < k < 3$ (b) $3 < k < 7$ (c) $-7 < k < -3$ (d) $-7 < k < 3$

---

39 If $\|\overrightarrow{A}\| = 3$, $-1 \le k \le 2$, then $\|k\overrightarrow{A}\| \in$ ....

(a) $[-3, 6]$ (b) $[0, 6]$ (c) $[3, 6]$ (d) $[1, 6]$

---

40 If $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are three vectors , then $\left\|\frac{\overrightarrow{A}}{\|\overrightarrow{A}\|}\right\| + 2\left\|\frac{\overrightarrow{B}}{\|\overrightarrow{B}\|}\right\| + 3\left\|\frac{\overrightarrow{C}}{\|\overrightarrow{C}\|}\right\| =$ ....

(a) 4 (b) 5 (c) 6 (d) 8

---

41 The measures of the direction angles of the positive y-axis are ....

(a) $(90°, 90°, 0)$ (b) $(0, 90°, 90°)$

(c) $(0, 90°, 0)$ (d) $(90°, 0, 90°)$

---

42 The vector whose direction angles are $(0, 90°, 90°)$ ....

(a) acts in the positive direction of y-axis. (b) acts in the positive direction of x-axis.

(c) acts in the positive direction of z-axis. (d) lies in the y z-plane.

---

43 The direction cosines of the negative z-axis are ....

(a) $(0, 0, 1)$ (b) $(0, 1, 0)$ (c) $(0, 0, -1)$ (d) $(0, -1, 0)$

44 Which of the following vectors represents a perpendicular unit vector on the cartesian plane x y ?

ⓐ $\hat{i}$ ⓑ $\hat{j}$ ⓒ $\hat{k}$ ⓓ $(0, 0, 2)$

45 The direction angles of a vector are : α , β , γ , then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$ ....

ⓐ 1 ⓑ 2 ⓒ 0 ⓓ -1

46 If the direction angles of a vector are θ , θ , α respectively where $\sin^2\alpha = 3\sin^2\theta$, then $\cos^2\theta =$ ....

ⓐ $\frac{2}{5}$ ⓑ $\frac{1}{5}$ ⓒ $\frac{3}{5}$ ⓓ $\frac{2}{3}$

47 If K , E , F are direction cosines of vector $\overrightarrow{A}$, then ....

ⓐ $K + E + F = 1$ ⓑ $K = E = F$ ⓒ $K^2 + E^2 + F^2 = 1$ ⓓ $K + E + F = \|\overrightarrow{A}\|$

48 If $\overrightarrow{A}$ is a vector , $\frac{\overrightarrow{A}}{\|\overrightarrow{A}\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$, then the direction cosines of $\overrightarrow{A}$ are ....

ⓐ $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ ⓑ $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ ⓒ $\left(\sqrt{3}, \sqrt{3}, -\sqrt{3}\right)$ ⓓ $(1, 1, -1)$

49 The cosine direction of the position vector of the point $(3, 12, 4)$ is ....

ⓐ $3, 12, 4$ ⓑ $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

ⓒ $\frac{3}{\sqrt{13}}, \frac{1}{\sqrt{13}}, \frac{2}{\sqrt{13}}$ ⓓ $\frac{3}{13}, \frac{12}{13}, \frac{4}{13}$

50 The direction cosines of the vector $\overrightarrow{A} = \left(-2, 2, 2\sqrt{2}\right)$ is ....

ⓐ $\left(\frac{-1}{2}, \frac{1}{2}, 1\right)$ ⓑ $\left(\frac{-1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right)$

ⓒ $\left(\frac{1}{2}, \frac{-1}{2}, \frac{\sqrt{2}}{2}\right)$ ⓓ $\left(\frac{-1}{2}, \frac{-1}{2}, \frac{\sqrt{2}}{2}\right)$

51 ***(2nd Session 2021)*** The direction cosines of the vector $\overrightarrow{A} = (-2k, 2k, k)$, where $k \in]0, 1[$ are ....

ⓐ $\left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ ⓑ $\left(\frac{-2k}{3}, \frac{2k}{3}, \frac{-k}{3}\right)$

ⓒ $\left(\frac{-2}{3}, \frac{-2}{3}, \frac{-1}{3}\right)$ ⓓ $\left(\frac{2k}{3}, \frac{2k}{3}, \frac{k}{3}\right)$

52 ABC is a triangle in which $\overrightarrow{AB} = \hat{i} - \hat{j} + 2\hat{k}$, $\overrightarrow{BC} = -2\hat{i} + 2\hat{j} - \hat{k}$, then the direction cosines of the vector $\overrightarrow{AC}$ is ....

(a) $(-1, 1, 1)$ (b) $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (d) $\left(\frac{-1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

---

53 If $\vec{A} = (2\sqrt{2}, 2, -2)$, then the vector which have the same direction angles is ....

(a) $(8, 0, -4\sqrt{3})$ (b) $(4\sqrt{2}, -4, -4)$

(c) $(3\sqrt{2}, 3, -3)$ (d) $(4, 2, \sqrt{3})$

---

54 The norm of a vector is 6 length unit and its direction cosines are $\left(-\frac{1}{2}, 0, \frac{\sqrt{3}}{2}\right)$, then the vector is ....

(a) $(-3, 0, 3\sqrt{3})$ (b) $(-3, 3, 3\sqrt{3})$

(c) $\left(\frac{-3}{2}, 0, 2\sqrt{3}\right)$ (d) $(-1, 0, \sqrt{3})$

---

55 If the direction cosines of a vector are $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$, then its direction angles are ....

(a) $(60°, 120°, 135°)$ (b) $(60°, 120°, 45°)$

(c) $(30°, 150°, 45°)$ (d) $(120°, 60°, 45°)$

---

56 If $(60°, \theta_y, 45°)$ are the measures of direction angles of a vector , then $\theta_y =$ ....

(a) 30° or 60° (b) 60° or 150° (c) 60° or 120° (d) 30° or 150°

---

57 If the direction angles of a vector are $(45°, 45°, \theta)$, then $\theta =$ ....

(a) 45° (b) 90° (c) 0° (d) 60°

---

58 If $(30°, 70°, \theta)$ are the direction angles of a vector , then one of the values of $\theta \simeq$ ....

(a) 100° (b) 80° (c) 260° (d) 68.61°

---

59 If the measures of direction angles of vector $\vec{A}$ are $(45°, 135°, 90°)$, then the unit vector in direction of $\vec{A} =$ ....

(a) $(\sqrt{2}, -\sqrt{2}, 0)$ (b) $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 1\right)$ (c) $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$ (d) $(1, -1, 0)$

60 If the measures of direction angles of vector $\overrightarrow{A}$ are $(45°, 120°, 60°)$, $\|\overrightarrow{A}\| = 12\sqrt{2}$, then $\overrightarrow{A} =$ ....

(a) $(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{1}{2})$
(b) $(12, -6\sqrt{2}, 6\sqrt{2})$
(c) $(12, 6\sqrt{2}, -6\sqrt{2})$
(d) $(\frac{1}{\sqrt{2}}, \frac{-1}{2}, \frac{-1}{2})$

61 If $\overrightarrow{A} = \hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{B} = 4\hat{i} - 2\hat{j} + 3\hat{k}$, $\overrightarrow{C} = \hat{i} - 2\hat{j} + \hat{k}$, then the vector with magnitude 6 units in direction of $2\overrightarrow{A} - \overrightarrow{B} + 3\overrightarrow{C}$ is ....

(a) $2\hat{i} - 4\hat{j} + 4\hat{k}$
(b) $2\hat{i} + 4\hat{j} + 4\hat{k}$
(c) $2\hat{i} - 4\hat{j} - 4\hat{k}$
(d) $2\hat{i} + 4\hat{j} - 4\hat{k}$

62 If $\|\overrightarrow{A}\| = 8$ and makes with the positive direction of the x-axis an angle of measure 45° and with the positive direction of the y-axis an angle of measure 60°, then $\overrightarrow{A} =$ ....

(a) $8(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$
(b) $4(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$
(c) $\frac{1}{4}(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$
(d) $\frac{1}{8}(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$

63 Which of the following could be direction angles of vector in space ?

(a) $(60°, 30°, 30°)$
(b) $(90°, 90°, 60°)$
(c) $(120°, 150°, 90°)$
(d) $(60°, 30°, 0°)$

64 If $(30°, 60°, 90°)$ are the direction angles of vector $\overrightarrow{A}$, then the direction angles of vector $2\overrightarrow{A}$ is ....

(a) $(30°, 60°, 90°)$
(b) $(60°, 120°, 180°)$
(c) $(-30°, -60°, -90°)$
(d) $(60°, 120°, 90°)$

65 If $(60°, 135°, 60°)$ are the direction angles of vector $\overrightarrow{A}$, then the direction angles of vector $-2\overrightarrow{A}$ is ....

(a) $(60°, 135°, 60°)$
(b) $(120°, 270°, 120°)$
(c) $(30°, 45°, 30°)$
(d) $(120°, 45°, 120°)$

66 If the measures of direction angles of position vector $\overrightarrow{A}$ are $(30°, 120°, 90°)$, then $\overrightarrow{A}$ lies in the ....

(a) x z-plane.
(b) y z-plane.
(c) x y-plane.
(d) $\overrightarrow{OX}$ direction.

67 If $(\theta_x , \theta_y , \theta_z)$ are the direction angles of vector $\overrightarrow{A}$, then ....

(I) $90^\circ \leq \theta_x + \theta_y + \theta_z \leq 180^\circ$ (II) $\theta_x + \theta_y + \theta_z = 270^\circ$

(III) $\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$

(a) both (I) and (III) (b) both (II) and (III) (c) only (III) (d) only (I)

---

68 If $(\theta_x , \theta_y , \theta_z)$ are the direction angles of a vector where $\theta_x + \theta_y = 90^\circ$, then which of the following not correct ?

(a) $\theta_z = 90^\circ$

(b) The vector lies on a plane x y

(c) $\cos^2 \theta_x + \cos^2 \theta_y = 1$

(d) The vector makes equal angles with the coordinate axes.

---

69 The vector which lies in x z-plane and makes angle 30° with the direction of z-axis , then the cosine directions is ....

(a) $(\frac{1}{2} , \frac{\sqrt{3}}{2} , 0)$ (b) $(\frac{1}{2} , 0 , \pm\frac{\sqrt{3}}{2})$ (c) $(-\frac{1}{2} , \frac{\sqrt{3}}{4} , 0)$ (d) $(\pm\frac{1}{2} , 0 , \frac{\sqrt{3}}{2})$

---

70 The position vector that lies on the x y-plane and makes an angle of measure 60° with the positive direction of y-axis , then its direction cosines are ....

(a) $(0 , \frac{1}{2} , \frac{\sqrt{3}}{2})$ (b) $(\frac{1}{2} , \pm\frac{\sqrt{3}}{2} , 0)$ (c) $(\pm\frac{\sqrt{3}}{2} , \frac{1}{2} , 0)$ (d) $(\frac{\sqrt{3}}{2} , \pm\frac{1}{2} , 0)$

---

71 The vector $\overrightarrow{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ makes an angle of measure .... (to the nearest minute) with the positive direction of the z-axis.

(a) $143^\circ\ 18'$ (b) $74^\circ\ 30'$ (c) $36^\circ\ 42'$ (d) $85^\circ\ 54'$

---

72 If the vector $\overrightarrow{n} = (a , 4 , c)$ is parallel to the cartesian plane y z and $\|\overrightarrow{n}\| = 5$, then $c^2 =$ ....

(a) 3 (b) 9 (c) 12 (d) 20

---

73 If the measure of the angle which $\overrightarrow{C} = (2 , 4 , k)$ makes with the positive direction of y-axis equals 45° , then k = ...

(a) ± 5 (b) ± 2 (c) $\pm 2\sqrt{3}$ (d) $\pm\sqrt{3}$

74 The vector which makes equal angles with the positive direction of the coordinate axes from the following is ....

ⓐ $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ ⓑ $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

ⓒ $\left(-\sqrt{3}, -\sqrt{3}, \sqrt{3}\right)$ ⓓ $(-1, 1, -1)$

---

75 If $\theta_x, \theta_y, \theta_z$ are the direction angles of a vector with the positive coordinate axes, then $\sin^2 \theta_x + \sin^2 \theta_y + \sin^2 \theta_z =$ ....

ⓐ 1 ⓑ 0 ⓒ 3 ⓓ 2

---

76 If $\theta_x, \theta_y, \theta_z$ are the direction angles of vector, then $\cos 2\,\theta_x + \cos 2\,\theta_y + \cos 2\,\theta_z =$ ....

ⓐ 0 ⓑ -1 ⓒ 2 ⓓ 1

---

77 If $\theta_x, \theta_y, \theta_z$ are the direction cosines of a vector in the space and $\theta_x = \theta_y = \theta_z$, then $\cos^4 \theta_x + \cos^4 \theta_y + \cos^4 \theta_z =$ ....

ⓐ $\frac{1}{81}$ ⓑ $\frac{1}{27}$ ⓒ $\frac{1}{9}$ ⓓ $\frac{1}{3}$

---

78 The vector $\vec{A}$ whose magnitude $= 21\sqrt{3}$ and makes equal angles in measure with the positive directions of the cartesian axis is ....

ⓐ $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ ⓑ $(7, 7, 7)$

ⓒ $(21, 21, 21)$ ⓓ $\pm(21, 21, 21)$

---

79 If $(\theta_x, \theta_y, \theta_z)$ are direction angles of the vector $\vec{A}$ in 3-dimension space, then which of the following is incorrect ?

(1) $\theta_x + \theta_y \geq 90°$

(2) $\sin^2\left(\frac{\pi}{2} - \theta_x\right) + \sin^2\left(\frac{\pi}{2} - \theta_y\right) + \sin^2\left(\frac{\pi}{2} - \theta_z\right) = 1$

(3) $(-\theta_x, -\theta_y, -\theta_z)$ are direction angles of the vector $-\vec{A}$

(4) $\theta_x = \pm \cos^{-1}\left(\frac{a_x}{\|\vec{A}\|}\right)$

ⓐ (1) only. ⓑ (1), (2) only. ⓒ (2), (3) only. ⓓ (3), (4) only.

80 The vector $\overrightarrow{B} = \hat{i} + 2\hat{j}$ makes an angle of measure .... with the positive direction of z-axis.

(a) zero (b) 90° (c) 180° (d) 270°

81 The vector $\overrightarrow{A} = 3\hat{i} - \hat{k}$ makes an angle of measure 90° with the .... axis.

(I) x (II) y (III) z

(a) only I (b) only II (c) only III (d) both I and III

82 If $\overrightarrow{A} = (4 , 0 , 3)$, then tangent of the angle this vector makes with the x-axis = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{4}{\sqrt{7}}$ (d) $\frac{4}{7}$

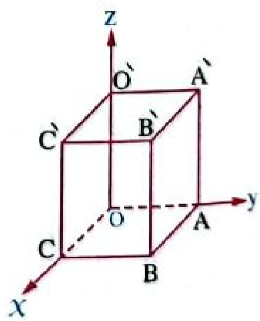
83 ***(1st Session 2021)*** If cosine of the angle between the vector $\overrightarrow{A} = (k , 12 , 4)$ and the x-axis equals $\frac{3}{13}$, then k = ... where $k \in \mathbb{R}$

(a) 4 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) 3

84 If $\overrightarrow{M} = (k - 1 , k , 3)$, then the least value for $\|\overrightarrow{M}\|$ is ....

(a) $\frac{\sqrt{34}}{2}$ (b) $\frac{\sqrt{38}}{2}$ (c) $\sqrt{10}$ (d) $\sqrt{19}$

85 **In the opposite figure :**



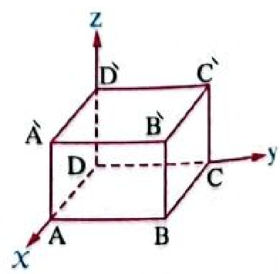
If the vector $\overrightarrow{OB} = (3 , 4 , 5)$

, then the vector $\overrightarrow{BC}$ is ....

(a) (3 , 4 , 0) (b) (0 , 4 , 5)

(c) (0 , – 4 , – 5) (d) (– 3 , 0 , 0)

86 **In the opposite figure :**

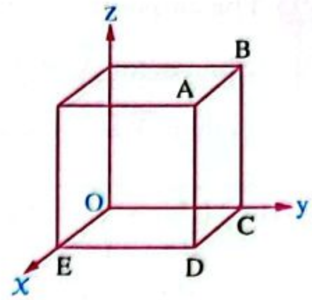


ABCD A`B`C`D` is a cuboid , A (4 , 0 , 0) , C (0 , 9 , 0) , D` (0 , 0 , 7) , then $\|\overrightarrow{AC`}\|$ = ... length unit.

(a) $\sqrt{137}$ (b) $\sqrt{141}$

(c) $\sqrt{146}$ (d) $\sqrt{147}$

87 The opposite figure represents a cuboid , A (6 , 8 , 24) , then the direction angles of the vector $\overrightarrow{OD}$ are ....



(a) $(60^\circ , 120^\circ , 45^\circ)$

(b) $(53^\circ 8' , 36^\circ 52' , 90^\circ)$

(c) $(53^\circ 8' , 90^\circ , 36^\circ 52')$

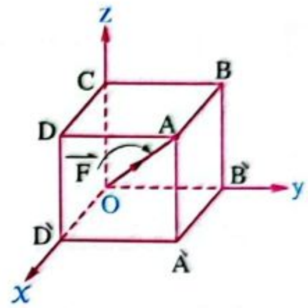
(d) $(36^\circ 52' , 45^\circ , 90^\circ)$

---

88 **In the opposite figure :**

A cube of edge length 5 length unit.

The magnitude of the force $\vec{F}$ is 25 newton , then $\vec{F}$ = ...



(a) (25 , 25 , 25)

(b) ± (5 , 5 , 5)

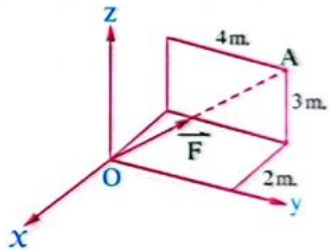
(c) $\frac{1}{\sqrt{3}}$ (25 , 25 , 25)

(d) $\left(\frac{5}{\sqrt{3}} , \frac{5}{\sqrt{3}} , \frac{5}{\sqrt{3}}\right)$

---

89 **In the opposite figure :**

The components of the force $\vec{F}$ whose magnitude is $12\sqrt{29}$ newtons = ...



(a) (12 , 12 , 12)

(b) (– 24 , 48 , 36)

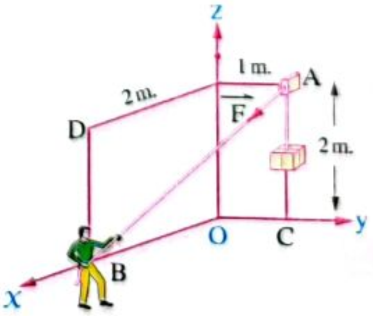
(c) $\left(-2\sqrt{29} , 4\sqrt{29} , 3\sqrt{29}\right)$

(d) $\left(12\sqrt{29} , 12\sqrt{29} , 12\sqrt{29}\right)$

---

90 **In the opposite figure :**

If the tension force in a string equals 21 newtons , then the algebraic components of the forces $\vec{F}$ in the directions of the cartesian axes = ...



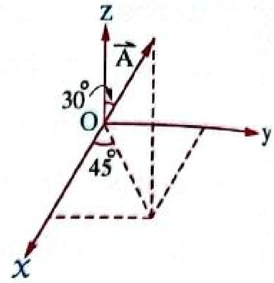
(a) (42 , – 21 , – 42)

(b) (14 , – 7 , – 14)

(c) ± (42 , – 21 , – 42)

(d) ± (14 , – 7 , – 14)

91 The opposite figure represents the vector $\vec{A}$ of norm 10 units , then $\vec{A} =$ ....



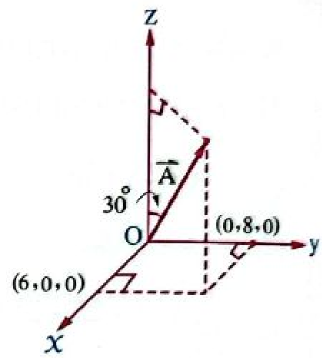
ⓐ $\left(\frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}, 5\sqrt{3}\right)$　ⓑ $\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}, 5\sqrt{3}\right)$

ⓒ $\left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{2}\right)$　ⓓ $\left(5\sqrt{2}, 5\sqrt{2}, 5\sqrt{3}\right)$

---

92 **In the opposite figure :**

$\|\vec{A}\| =$ ....



ⓐ 8

ⓑ $10\sqrt{3}$

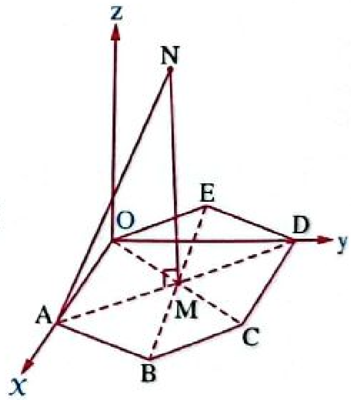
ⓒ 20

ⓓ 10

---

93 **In the opposite figure :**

ABCDEO is a regular hexagon of side length 2 cm. is drawn in the plane x y , N $\notin$ the plane x y such that the projection of N on the plane is M (the centre of the hexagon) , the length of AN = 3 cm. , then $\overrightarrow{AN} =$ ....



ⓐ $\hat{i} + 3\hat{j} + \sqrt{5}\,\hat{k}$　ⓑ $\hat{i} - \sqrt{3}\,\hat{j} + 5\,\hat{k}$

ⓒ $-\hat{i} + \sqrt{3}\,\hat{j} + \sqrt{5}\,\hat{k}$　ⓓ $\hat{i} + \sqrt{3}\,\hat{j} + \sqrt{5}\,\hat{k}$

Fourth　Questions on scalar product

Choose the correct answer from the given ones :

1 If $\vec{A} = -3\hat{i} + 2\hat{j} - 5\hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$, then $\vec{A} \cdot \vec{B} =$ ....

ⓐ zero　ⓑ -4　ⓒ -5　ⓓ -10

---

2 If $\vec{A} = 2\hat{i} + 3\,j - \hat{k}$, $\vec{B} = 4\hat{i} - \hat{j}$, then $\vec{A} \cdot \vec{B} =$ ....

ⓐ 5　ⓑ 4　ⓒ 3　ⓓ 2

3. If $\overrightarrow{A}$, $\overrightarrow{B}$ are two vectors in space , then ....

(a) $\overrightarrow{A} \cdot \overrightarrow{B} = k$ where $k \in \mathbb{R}$
(b) $\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{C}$ where $\overrightarrow{C}$ is a non-zero vector.
(c) $\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{0}$ if $\overrightarrow{A} \parallel \overrightarrow{B}$
(d) all the previous are true.

---

4. If $(3\overrightarrow{A} \cdot 2\overrightarrow{B}) = 12$, then $(-\overrightarrow{B} \cdot 5\overrightarrow{A}) =$ ....

(a) -10 (b) -5 (c) 5 (d) 10

---

5. If $\overrightarrow{A} \cdot \overrightarrow{B} = 4$ and $\overrightarrow{A} = m\,\hat{i} + \hat{j} + 4\,\hat{k}$, $\overrightarrow{B} = 2\,\hat{i} + 6\,\hat{j} + 3\,\hat{k}$, then $m =$ ....

(a) 4 (b) 7 (c) -4 (d) -7

---

6. If $\overrightarrow{A} = \hat{i} + \hat{j} - 2\,\hat{k}$, $\overrightarrow{B} = 3\,\hat{i} + 2\,\hat{j} - \hat{k}$, then $(\overrightarrow{A} + 3\,\overrightarrow{B}) \cdot (2\,\overrightarrow{A} - \overrightarrow{B}) =$ ....

(a) zero (b) 4 (c) 5 (d) 6

---

7. If $2\,\overrightarrow{A} = -6\,\hat{i} - 4\,\hat{j}$, $3\,\overrightarrow{B} = 9\,\hat{i} - 15\,\hat{k}$, then $3\,\overrightarrow{A} \cdot 2\,\overrightarrow{B} =$ ....

(a) -54 (b) zero (c) 6 (d) 54

---

8. If $A(2, 1, 3)$, $B(3, 5, -2)$, $C(-1, 4, 0)$, then $\overrightarrow{AB} \cdot \overrightarrow{AC} =$ ....

(a) 5 (b) 24 (c) 17 (d) 13

---

9. $\overrightarrow{A}$, $\overrightarrow{B}$ are two unit vectors enclosing angle of measure 45° , then $\overrightarrow{A} \cdot \overrightarrow{B} =$ ....

(a) 45° (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\sqrt{2}$

---

10. If $\overrightarrow{A}$ and $\overrightarrow{B}$ are two vectors and the measure of the angle between them is 135° , $\|\overrightarrow{A}\| = 6\sqrt{2}$, $\|\overrightarrow{B}\| = 10\sqrt{2}$, then $\overrightarrow{A} \cdot \overrightarrow{B} =$ ....

(a) 120 (b) $\pm 60\sqrt{2}$ (c) $120\sqrt{2}$ (d) $-60\sqrt{2}$

---

11. If $\overrightarrow{A}$ and $\overrightarrow{B}$ are two vectors and the measure of the angle between them is 60° , $\|\overrightarrow{A}\| = 3$ $\|\overrightarrow{B}\| = 6$, then $\overrightarrow{A} \cdot \overrightarrow{B} =$ ....

(a) 6 (b) 12 (c) $6\sqrt{3}$ (d) 18

---

12. ABC is an isosceles triangle , $AB = AC = 6\sqrt{3}$ cm. , $m(\angle A) = 120^\circ$, then $\overrightarrow{BA} \cdot \overrightarrow{BC} =$ ....

(a) $54\sqrt{3}$ (b) 108 (c) 81 (d) 162

13 ABCD is a square of side length 10 cm. , then $\overrightarrow{AB} \cdot \overrightarrow{CA} =$ ....

(a) 100 (b) $100\sqrt{2}$ (c) -100 (d) $-100\sqrt{2}$

14 If $\vec{A}$, $\vec{B}$ are two unit vectors , then $\vec{A} \cdot \vec{B} \in$ ....

(a) $]0 , 1[$ (b) $]-1 , 1[$ (c) $[-1 , 1]$ (d) $\mathbb{R}^+$

15 If $\|\vec{A}\| = 5$, $\|\vec{B}\| = 4$, then the greatest possible value of $(\vec{A} \cdot \vec{B})$ is ....

(a) 9 (b) 10 (c) 20 (d) 40

16 If $\vec{A} = (-1 , 2 , 2)$, $\vec{B}$ is a unit vector then $\vec{A} \cdot \vec{B}$ could be equal to ....

(a) 7 (b) 4 (c) 2 (d) -4

17 If $\|\vec{A}\| = 5$, $\vec{B} = (-1 , 2 , 2)$ and the measure of the angle between the two vectors is 30°
, then $\vec{A} \cdot \vec{B} =$ ....

(a) $(-5 , 10 , 10)$ (b) $7.5\sqrt{3}$ (c) 7.5 (d) -7.5

18 If $\vec{A} \perp \vec{B}$ and $[(2\vec{A} - 4\vec{C}) \cdot 3\vec{B}] = 108$, then $\vec{B} \cdot \vec{C} =$ ....

(a) -7 (b) -8 (c) -9 (d) -12

19 If $\|\vec{A}\| = 4$, $\|\vec{B}\| = 6$ and the measure of the angle between $\vec{A}$ and $\vec{B}$ equals 60°
, then $(3\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) =$ ....

(a) 48 (b) 36 (c) 12 (d) -12

20 $\vec{A}$ and $\vec{B}$ are two unit vectors , θ is the measure of the included angle between them
, then $\vec{A} + \vec{B}$ is a unit vector if $\theta =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) π

21 If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$ and $\|\vec{A}\| = 2$, then $\vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} =$ ....

(a) 8 (b) 4 (c) -4 (d) -8

22 If $\vec{A} \cdot \vec{B} = 13$, then the angle between the two vectors $\vec{A}$ and $\vec{B}$ is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

23 If A , B and C are three non-zero vectors . $\overrightarrow{A} = 3\overrightarrow{C}$, $\overrightarrow{B} = \frac{-1}{3}\overrightarrow{C}$, then measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{B}$ equals ....

(a) 180° (b) 30° (c) 60° (d) zero

---

24 If $\|\overrightarrow{A}\| = 15$, $\|\overrightarrow{B}\| = 20$, $\overrightarrow{A} \cdot \overrightarrow{B} = -150\sqrt{3}$, then measure of the angle between the two vectors = ...

(a) 30° (b) 120° (c) 135° (d) 150°

---

25 If $\|\overrightarrow{A}\| = 6$, $\|\overrightarrow{B}\| = \frac{5}{2}\|\overrightarrow{A}\|$, $\overrightarrow{B} \cdot \overrightarrow{A} = 81$, then measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{B} \approx$ ....

(a) 60° (b) 25° 51̀ (c) 18° 32̀ (d) 64° 9̀

---

26 If θ is the included angle between the two vectors $\overrightarrow{A} = (2, 0, 2)$, $\overrightarrow{B} = (0, 0, 4)$, then θ = ...

(a) 30° (b) 45° (c) 60° (d) 90°

---

27 If $\overrightarrow{A} = (1, 1, -1)$, $\overrightarrow{B} = (1, 2, 6)$, then measure of the angle between the two vectors $\overrightarrow{A}$ and $\overrightarrow{A} + \overrightarrow{B}$ equals ....

(a) zero (b) 45° (c) 60° (d) 90°

---

28 Measure of the angle between the two vectors : $3\hat{i} - \hat{j}$, $-4\hat{i} + 6\hat{j}$ equals .... to the nearest degree.

(a) 140° (b) 41° (c) 142° (d) 143°

---

29 If $\overrightarrow{A} = (\ell, m, n)$, $\overrightarrow{B} = (\grave{\ell}, \grave{m}, \grave{n})$ are two unit vectors , measure of included angle between them is θ , then cos θ = ...

(a) $\overrightarrow{A} \cdot \overrightarrow{B}$ (b) $\frac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{A}\|\|\overrightarrow{B}\|}$ (c) $\ell\grave{\ell} + m\grave{m} + n\grave{n}$ (d) all the previous.

---

30 If $\overrightarrow{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$, $\overrightarrow{B} = \hat{i} + k\hat{j} + 5\hat{k}$ and the measure of the angle between the two vectors $\overrightarrow{A}$, $\overrightarrow{B}$ equals 60° , then k may be equal .... (to nearest hundredth)

(a) 3.8 (b) − 1.27 (c) − 2.12 (d) 2.7

31 If $\overrightarrow{A} = (2\cos\theta , \log_2 x , \sin\theta)$, $\overrightarrow{B} = (\cos\theta , \log_5 27 , 2\sin\theta)$ and $\overrightarrow{A} \cdot \overrightarrow{B} = 11$, then $x \simeq$ .... (to nearest hundredth)

(a) 21.04 (b) 23.15 (c) 31.02 (d) 25.14

---

32 If $\overrightarrow{A} = 3\hat{i} - 4\hat{j}$, $\overrightarrow{B} = 12\hat{i} + 5\hat{j}$, θ is the angle between the two vectors $\overrightarrow{A}$ and $\overrightarrow{B}$, then $\tan\theta =$ ....

(a) $\frac{16}{65}$ (b) $\frac{63}{65}$ (c) $\frac{63}{16}$ (d) $\frac{16}{63}$

---

33 Measure of the angle between the two vectors $\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$ is ....

(a) $\cos^{-1}\left(\frac{1}{\sqrt{15}}\right)$ (b) $\cos^{-1}\left(\frac{4}{\sqrt{15}}\right)$ (c) $\cos^{-1}\left(\frac{4}{15}\right)$ (d) $\frac{\pi}{2}$

---

34 Cosine the angle between the two vectors $\overrightarrow{A} = (1 , -3 , 0)$, $\overrightarrow{B} = (2 , 0 , 1)$ equals ....

(a) $\frac{\sqrt{2}}{5}$ (b) $5\sqrt{2}$ (c) $\frac{5}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

---

35 The measure of the angle between the two vectors $3\hat{i} - \hat{j}$, $-4\hat{i} + 6\hat{j}$ equals ....

(a) $35° 40'$ (b) $108° 42'$ (c) $37° 52' 30''$ (d) $142° 7' 30''$

---

36 If $\|\overrightarrow{A}\| = 2$, $\|\overrightarrow{B}\| = 3$, $\|\overrightarrow{A} - \overrightarrow{B}\| = \sqrt{19}$, then the measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{B} =$ ....

(a) 30° (b) 60° (c) 90° (d) 120°

---

37 If $(\overrightarrow{A} \cdot (\overrightarrow{B} - \overrightarrow{C})) =$ zero , $\overrightarrow{A} = 2\overrightarrow{B}$, $\|\overrightarrow{C}\| = 2\|\overrightarrow{B}\|$, then the measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{C} =$ ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

---

38 If $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are unit vectors , such that $\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C} = \overrightarrow{O}$, then $\overrightarrow{A} \cdot \overrightarrow{B} + \overrightarrow{B} \cdot \overrightarrow{C} + \overrightarrow{C} \cdot \overrightarrow{A} =$ ....

(a) zero (b) -1 (c) -1.5 (d) $-\frac{3\sqrt{2}}{2}$

---

39 The component of vector $\overrightarrow{A}$ in direction of $\overrightarrow{B}$ where θ is the included angle between them = ...

(a) $(\overrightarrow{A} \cdot \overrightarrow{B})\cos\theta$ (b) $\|\overrightarrow{B}\|\,\|\overrightarrow{A}\|$ (c) $\|\overrightarrow{A}\|\cos\theta$ (d) $\|\overrightarrow{A}\|\,\|\overrightarrow{B}\|\cos\theta$

40 If $\|\overrightarrow{A}\| = 4$, $\|\overrightarrow{B}\| = 6$, the measure of the angle between the two vectors is 30°
, then the component of vector $\overrightarrow{B}$ in direction of $\overrightarrow{A}$ is ....

(a) $12\sqrt{3}$ (b) $9\sqrt{3}$ (c) $3\sqrt{3}$ (d) $2\sqrt{3}$

41 If $\overrightarrow{A} = (-1 , 4 , 2)$, $\overrightarrow{B} = (2 , 2 , 1)$, then the component of $\overrightarrow{A}$ in direction of $\overrightarrow{B}$ = ...

(a) $\frac{4}{3}$ (b) $\frac{-8}{3}$ (c) $\frac{8}{3}$ (d) 5

42 The projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $\hat{j}$ is ....

(a) 1 (b) zero (c) 2 (d) − 1

43 The projection of the vector $\overrightarrow{A} = 3\hat{i} - \hat{j} - 2\hat{k}$ in direction of the vector $\overrightarrow{B} = \hat{i} + 2\hat{j} - 3\hat{k}$
is ....

(a) $\frac{\sqrt{14}}{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{14}$ (d) 7

44 If $\overrightarrow{A} = 4\hat{i} - 3\hat{j} + 5\hat{k}$, then the component of $\overrightarrow{A}$ in direction of z-axis equals ....

(a) 4 (b) 3 (c) − 3 (d) 5

45 The projection of vector of $\overrightarrow{A} = (2 , 3 , -1)$ in direction of vector of $\overrightarrow{B} = 3\hat{i} + 4\hat{j}$
equals ....

(a) 18 (b) $\frac{18}{5}$ (c) $\frac{-18}{5}$ (d) $\frac{18}{25}$

46 The algebraic component of the force $\overrightarrow{F} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ in the direction of $\overrightarrow{AB}$
where A (1 , 4 , 0) and B (− 1 , 2 , 3) is ....

(a) 21 (b) 17 (c) $\sqrt{13}$ (d) $\sqrt{17}$

47 If $\overrightarrow{A}$, $\overrightarrow{B}$ are non-zero vectors and $\overrightarrow{A} \cdot \overrightarrow{B} = 0$, then the component of $\overrightarrow{A}$ in direction of $\overrightarrow{B}$
equals ....

(a) $\|\overrightarrow{A}\|$ (b) $\|\overrightarrow{B}\|$ (c) 1 (d) zero

48 *(Trial 2021)* If $\overrightarrow{A}$, $\overrightarrow{B}$ are two vectors and $\|\overrightarrow{A}\| = 5$, the component of $\overrightarrow{B}$ in direction of $\overrightarrow{A}$
is 3 , then $\overrightarrow{A} \cdot \overrightarrow{B}$ = ...

(a) 15 (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) 8

49 If $\|\overrightarrow{A}\| = 6$, $\|\overrightarrow{B}\| = 4$ and the component of $\overrightarrow{A}$ in direction of $\overrightarrow{B}$ equals 3 , then the component of $\overrightarrow{B}$ in direction of $\overrightarrow{A}$ equals ....

(a) -2 (b) 2 (c) -8 (d) 8

50 If the component of $\overrightarrow{A} = k\,\hat{i} + \hat{j} + 4\,\hat{k}$ in direction of $\overrightarrow{B} = 2\,\hat{i} + 6\,\hat{j} + 3\,\hat{k}$ equals 4 , then $k =$ ....

(a) 5 (b) -5 (c) 7 (d) -7

51 If the projection of the vector $m\,\hat{i} - 3\,\hat{j} + 2\,\hat{k}$ in direction of the vector $2\,\hat{i} + \hat{j} + 5\,\hat{k}$ equals $\frac{\sqrt{30}}{2}$, then $m =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

52 ABCD is a square of side length 10 cm. , then $\overrightarrow{BD} \cdot \overrightarrow{CA} =$ ....

(a) $100\sqrt{2}$ (b) zero (c) $-100\sqrt{2}$ (d) 100

53 ABCD is a rectangle in which AB = 6 cm. , BC = 8 cm. , then component of $\overrightarrow{CD}$ in direction of $\overrightarrow{BC} =$ ....

(a) $\overrightarrow{CD} \cdot \overrightarrow{BC}$ (b) $\overrightarrow{AB} \cdot \overrightarrow{BC}$ (c) $\overrightarrow{AB} \cdot \overrightarrow{AD}$ (d) all the previous.

54 ABCD is a trapezium $\overline{AD} \parallel \overline{BC}$, $m(\angle A) = m(\angle B) = 90^\circ$, $AD = \frac{1}{2}\,BC = 24$ cm. , CD = 26 cm. , then $\overrightarrow{DB} \cdot \overrightarrow{BC} =$ ....

(a) 1152 (b) 100 (c) -1152 (d) -100

55 ABCDEF is a regular hexagon , of side length 8 cm. , then $(\overrightarrow{CA} + \overrightarrow{AF}) \cdot \overrightarrow{AD} =$ ....

(a) $128\sqrt{3}$ (b) $-128\sqrt{3}$ (c) 128 (d) -128

56 ABC is a right-angled triangle at $\angle B$, AB = 6 cm. , BC = 8 cm. , D is the midpoint of $\overline{AC}$, then the algebraic projection of vector $\overrightarrow{BD}$ in direction of $\overrightarrow{AB} =$ ....

(a) 3 (b) -3 (c) 4 (d) -4

57 If $\overrightarrow{A} = (1 , -2 , 1)$, $\overrightarrow{B} = (-2 , 1 , 2)$, then the vector component of $\overrightarrow{A}$ in the direction of $\overrightarrow{B} =$ ....

(a) $\left(\frac{4}{9} , \frac{-2}{9} , \frac{-4}{9}\right)$ (b) $\left(\frac{4}{9} , \frac{2}{9} , \frac{4}{9}\right)$ (c) $\left(\frac{-4}{9} , \frac{-2}{9} , \frac{-2}{9}\right)$ (d) $\left(\frac{4}{9} , \frac{2}{9} , \frac{-4}{9}\right)$

58 ABC is a triangle in which A (2 , 3 , 1) , B (3 , 5 , 4) , $\overrightarrow{BC} = (-1, 4, 0)$, then vector component of $\overrightarrow{AC}$ in direction of $\overrightarrow{AB}$ = ...

(a) (3 , 6 , 9) (b) (1 , 2 , 3) (c) $\left(\frac{3}{2}, 3, \frac{9}{2}\right)$ (d) (4 , 2 , – 5)

59 If $\overrightarrow{A} = (4, 2, 5)$, $\overrightarrow{B} = (m, n, \ell)$, $\overrightarrow{A} \cdot \overrightarrow{B} = 15$, $\overrightarrow{A} \,//\, \overrightarrow{B}$, then $m + n + \ell$ = ...

(a) 11 (b) 22 (c) $\frac{11}{3}$ (d) $4\frac{1}{2}$

60 The two vectors $\overrightarrow{A} = (1, 3, -2)$, $\overrightarrow{B} = (-2, -6, 4)$ are ....

(a) parallel and have the same direction. (b) parallel but in opposite directions.
(c) perpendicular. (d) the measure of the angle between them is 60°

61 If $\overrightarrow{A}$, $\overrightarrow{B}$ are non-zero vectors and $\overrightarrow{A} \cdot \overrightarrow{B} = 0$, then the two vectors ....

(a) are parallel. (b) lie in the x y-plane.
(c) are perpendicular. (d) have the same norm.

62 If $\overrightarrow{A} = (k, -3, 1)$; $\overrightarrow{B} = (2, 3, -k)$ are perpendicular , then the value of k = ...

(a) – 9 (b) – 3 (c) 9 (d) 18

63 If $\overrightarrow{A}$, $\overrightarrow{B}$ are perpendicular ; $\overrightarrow{A} = (5, -4, 0)$, then $\overrightarrow{B}$ could be ....

(a) (3 , 4 , 3) (b) (8 , 10 , – 7) (c) (1 , 1 , 5) (d) (0 , 1 , 0)

64 If $\overrightarrow{A} = (3, 4, k)$, $\overrightarrow{B} = (4, 0, -1)$ are perpendicular , then $\|\overrightarrow{A}\|$ = ...

(a) 10 (b) 11 (c) 12 (d) 13

65 If $\overrightarrow{A}$, $\overrightarrow{B}$ are two perpendicular unit vectors , then $(\overrightarrow{A} - 2\overrightarrow{B}) \cdot (3\overrightarrow{A} + 5\overrightarrow{B})$ = ...

(a) – 8 (b) – 7 (c) 24 (d) 0

66 If $\|\overrightarrow{A}\| = 4$, $\|\overrightarrow{B}\| = 2$ and $\left(\overrightarrow{A} - k\overrightarrow{B}\right) \perp \left(\overrightarrow{A} + k\overrightarrow{B}\right)$, then k = ...

(a) $\pm\frac{1}{4}$ (b) $\pm\frac{1}{2}$ (c) ± 2 (d) ± 8

67 If $\overrightarrow{A}$, $\overrightarrow{B}$, $2\overrightarrow{A} + \overrightarrow{B}$ are non-zero vectors and $\|\overrightarrow{A} + \overrightarrow{B}\| = \|\overrightarrow{A}\|$, then $2\overrightarrow{A} + \overrightarrow{B}$, $\overrightarrow{B}$ are ....

(a) parallel. (b) perpendicular.
(c) including an angle of measure 30° (d) including an angle of measure 60°

68 If the vector $\vec{E} = \vec{A} + \vec{B}$ is perpendicular to the vector $\vec{F} = \vec{A} - \vec{B}$, then ....

(a) $\vec{A} \,//\, \vec{B}$ (b) $\vec{A} \perp \vec{B}$ (c) $\vec{B} = \vec{O}$ (d) $\|\vec{A}\| = \|\vec{B}\|$

69 If $\|\vec{A}\| = 2$, $\|\vec{B}\| = 3$, $\|\vec{C}\| = 12$ and $\vec{A}$, $\vec{B}$, $\vec{C}$ are mutually perpendicular , then $\|\vec{A} + \vec{B} + \vec{C}\| =$ ....

(a) 17 (b) 12 (c) $\sqrt{157}$ (d) 13

70 If $\|\vec{A} + \vec{B}\| = \|\vec{A} - \vec{B}\|$, then measure of the angle between $\vec{A}$ and $\vec{B} =$ ....

(a) 30° (b) 60° (c) 90° (d) 45°

71 If $\vec{A}$, $\vec{B}$ are two unit vectors where $\vec{A} \cdot \vec{B} = \frac{-1}{2}$, then measure of the angle between $\vec{A}$ and $(\vec{A} + \vec{B})$ equals ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

72 If $2\vec{A} = 6\hat{j} - 8\hat{i}$, $\vec{B}$ is a unit vector , then $(2\vec{A} \cdot 3\vec{B})$ can not be .... whatever the angle measure between them.

(a) 32 (b) 30 (c) -28 (d) zero

73 If $\vec{A}$, $\vec{B}$ are non-zero vectors where $\|\vec{A} + \vec{B}\| < \|\vec{A} - \vec{B}\|$, then measure of the angle between them must be ....

(a) less than $\frac{\pi}{2}$ (b) more than $\frac{\pi}{2}$ (c) equal to $\frac{\pi}{2}$ (d) equal to π

74 If $\vec{A} \perp \vec{B}$ and $\|\vec{A} + \vec{B}\| = 12$, then $\|\vec{A} - \vec{B}\| =$ ....

(a) 6 (b) 12 (c) $12\sqrt{2}$ (d) $32\sqrt{3}$

75 If $\vec{A}$ and $\vec{B}$ are two unit vectors and $\|\vec{A} + \vec{B}\| = \sqrt{3}$, then $(3\vec{A} - 4\vec{B}) \cdot (2\vec{A} + 5\vec{B}) =$ ....

(a) $\frac{-17}{2}$ (b) $\frac{-21}{2}$ (c) $\frac{-25}{2}$ (d) -14

76 If $\|\vec{A}\| = 11$, $\|\vec{B}\| = 23$, $\|\vec{A} - \vec{B}\| = 30$, then $\|\vec{A} + \vec{B}\| =$ ....

(a) 5 (b) 10 (c) 20 (d) 30

77 If $\vec{A}$, $\vec{B}$ are unit vectors , then $\|\vec{A} - \vec{B}\|^2 \in$ ....

(a) $[0 , 1]$ (b) $[1 , 4]$ (c) $[0 , 4]$ (d) $[2 , 4]$

78 If $\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C} = \overrightarrow{O}$, $\|\overrightarrow{A}\| = 3$, $\|\overrightarrow{B}\| = 5$, $\|\overrightarrow{C}\| = 7$, then measure of angle between $\overrightarrow{A}$ and $\overrightarrow{B}$ is ....

(a) $120°$ (b) $150°$ (c) $60°$ (d) $30°$

---

79 In ΔABC , if a , b , c are the side lengths. , then $\overline{CA} \cdot \overline{CB} =$ ....

(a) ab (b) cos C

(c) $\frac{1}{2}$ a b (d) $\frac{1}{2}(a^2 + b^2 - c^2)$

---

80 If $\|\overrightarrow{A}\| = 2$, $\|\overrightarrow{B}\| = 3$, $\cos\theta = \frac{2}{3}$ where θ is measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{B}$, then $\|2\overrightarrow{A} - 3\overrightarrow{B}\| =$ ....

(a) 8 (b) 7 (c) 6 (d) 5

---

81 If $\overrightarrow{A}$, $\overrightarrow{B}$ and $\overrightarrow{C}$ are three non zero vectors and $\overrightarrow{A} + \overrightarrow{B} = \overrightarrow{C}$ and $\|\overrightarrow{A}\| = \|\overrightarrow{B}\|$, $\|\overrightarrow{C}\| = \sqrt{3}\|\overrightarrow{A}\|$, then measure of the angle between $\overrightarrow{B}$ and $\overrightarrow{C}$ equals ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

---

82 If the vector $(\overrightarrow{A} + \overrightarrow{B}) \perp \overrightarrow{B}$ and the vector $(\overrightarrow{A} + 2\overrightarrow{B}) \perp \overrightarrow{A}$, then $\|\overrightarrow{A}\| =$ ....

(a) $2\|\overrightarrow{B}\|$ (b) $\|\overrightarrow{B}\|$ (c) $\frac{1}{2}\|\overrightarrow{B}\|$ (d) $\sqrt{2}\|\overrightarrow{B}\|$

---

83 If $\overrightarrow{A} \cdot \overrightarrow{B} =$ zero and measure of the angle between the two vectors $\overrightarrow{A}$, $\overrightarrow{A} + \overrightarrow{B}$ equals $60°$, then $\|\overrightarrow{B}\| =$ ....

(a) $\|\overrightarrow{A}\|$ (b) $\sqrt{3}\|\overrightarrow{A}\|$ (c) $\frac{1}{\sqrt{3}}\|\overrightarrow{A}\|$ (d) $2\|\overrightarrow{A}\|$

---

84 If θ is the angle measure between the two unit vectors $\overrightarrow{A} = m\hat{i} - 5\hat{j} + 2\hat{k}$, $\overrightarrow{B} = \hat{i} + 3\hat{j} + m\hat{k}$, where $90° < \theta < 180°$, then m ∈ ....

(a) $]\frac{14}{3}, 5[$ (b) $]0, 3[$ (c) $]\frac{7}{2}, 4[$ (d) $]\frac{7}{3}, 5[$

---

85 The work done by the force $\overrightarrow{F} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ to move a body from a point A (1 , 2 , 3) to the point B (3 , 4 , 5) is .... work unit.

(a) 2 (b) 3 (c) $\sqrt{17}$ (d) $2\sqrt{51}$

---

86 The work done by force $\overrightarrow{F} = 3\hat{i} + 7\hat{k}$ to move a body from point A (1 , 1 , 2) to point B (7 , 3 , 5) equals .... work unit.

(a) 40 (b) 39 (c) 32 (d) 17

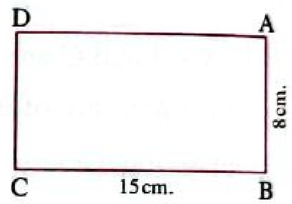
87 Force $\vec{F} = 5\hat{i} - 4\hat{j} + \hat{k}$ acted on a body and moved it displacement $\vec{S} = 2\hat{i} + \hat{j} - 3\hat{k}$, the force is measured in newton and the displacement in meter, then the work done by force = ... Joule.

(a) -3 (b) 3
(c) 7 (d) 10

88 Force of magnitude 15 newton acted on a body, so it moved from A (2, 3, −1) to B (3, 5, 1) in the opposite direction as the force, then the work done by force = ...

(a) 45 (b) 30
(c) −45 (d) ± 15

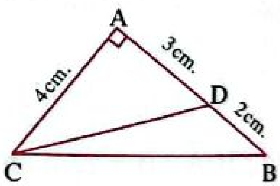
89 **In the opposite figure :**



$\overrightarrow{CA} \cdot (\overrightarrow{CD} + \overrightarrow{DA}) =$ ....

(a) −225 (b) −64
(c) 225 (d) 289

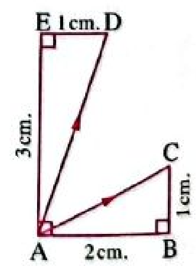
90 **In the opposite figure :**



ΔABC is a right-angled at A, $D \in \overline{BA}$, then $\overrightarrow{CD} \cdot \overrightarrow{DB} =$ ....

(a) 3 (b) 4
(c) 5 (d) 6

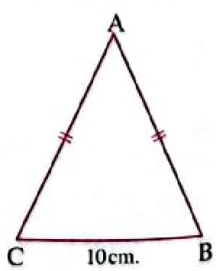
91 **In the opposite figure :**



$\overrightarrow{AD} \cdot \overrightarrow{AC} =$ ....

(a) 5 (b) 6
(c) 8 (d) 10

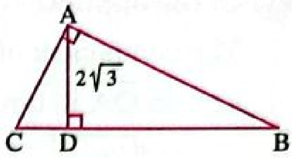
92 **In the opposite figure :**



$\overrightarrow{CA} \cdot \overrightarrow{CB} =$ ....

(a) 100 (b) 75
(c) 50 (d) 25

93 **In the opposite figure :**



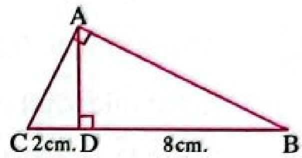
$\overrightarrow{AD} \cdot (\overrightarrow{AB} + \overrightarrow{AC}) =$ ....

(a) 36 (b) 24

(c) 12 (d) 6

---

94 **In the opposite figure :**



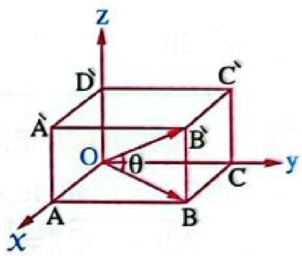
$\overrightarrow{AD} \cdot (\overrightarrow{CD} + \overrightarrow{CA}) =$ ....

(a) -16 (b) -10

(c) -8 (d) -4

---

95 **In the opposite figure :**



$ABCOA'B'C'D'$ is a cuboid

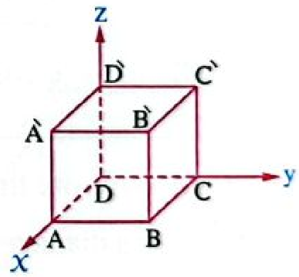
, $B'(3, 5, 4)$, then $\theta \approx$ .... (to the nearest degree)

(a) 27° (b) 34°

(c) 48° (d) 64°

---

96 **In the opposite figure :**



$ABCDA'B'C'D'$ is a cube of side length unity

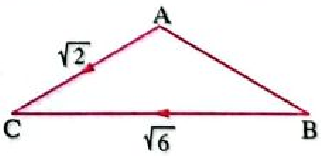
, then $\overrightarrow{AB'} \cdot \overrightarrow{BD} =$ ....

(a) -1 (b) zero

(c) 1 (d) $\frac{1}{2}$

---

97 **In the opposite figure :**



If $\|\overrightarrow{BC}\| = \sqrt{6}$, $\|\overrightarrow{AC}\| = \sqrt{2}$

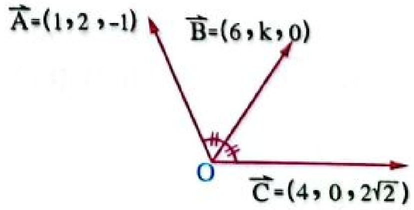
, $\overrightarrow{BA} = (-1, 0, 1)$

, then $\overrightarrow{BA} \cdot \overrightarrow{BC} =$ ....

(a) $\sqrt{3}$ (b) -3 (c) 3 (d) $\frac{\sqrt{3}}{2}$

---

98 **In the opposite figure :**



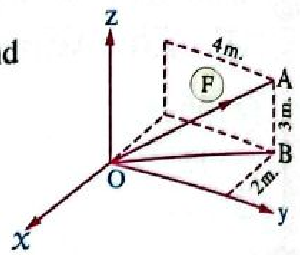
The value of k = ...

(a) -3 (b) 3

(c) $\sqrt{3}$ (d) 2

99 **In the opposite figure :**

The component of the force $\overrightarrow{F}$ whose magnitude $12\sqrt{29}$ newtons and acts on $\overline{OA}$ in direction of $\overrightarrow{OB}$ is .... newton.



(a) $24\sqrt{29}$ (b) $24\sqrt{5}$

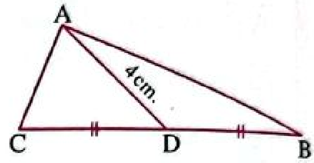
(c) $-48\sqrt{29}$ (d) $6\sqrt{29}$

---

100 **In the opposite figure :**

D is the midpoint of $\overline{BC}$, AD = 4 cm.

, then $\overrightarrow{AD} \cdot (\overrightarrow{AB} + \overrightarrow{AC}) =$ ....



(a) 16 (b) 24

(c) 28 (d) 32

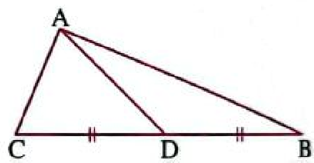
---

101 **In the opposite figure :**

ABC is a triangle , D is the midpoint if $\overline{BC}$

If $\|\overrightarrow{AB} + \overrightarrow{AC}\| = \|\overrightarrow{AB} - \overrightarrow{AC}\|$

, then m (∠ BAC) = ...



(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

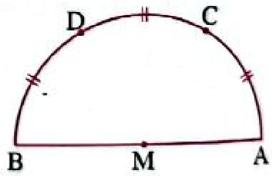
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

---

102 **In the opposite figure :**

If $\overline{AB}$ is a diameter in circle M , its radius length = 10 cm.

, m $(\widehat{AC})$ = m $(\widehat{CD})$ = m $(\widehat{DB})$, then



First : $\overrightarrow{CA} \cdot \overrightarrow{CB} =$ ....

(a) Zero (b) 1 (c) 100 (d) 400

Second : $\overrightarrow{BC} \cdot \overrightarrow{BD} =$ ....

(a) Zero (b) 100 (c) 150 (d) 300

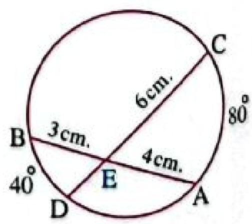
---

103 **In the opposite figure :**

If $\overline{AB}$, $\overline{CD}$ are two intersecting chords in the circle at E ,

EC = 6 cm. , AE = 4 cm. , EB = 3 cm. ,

m $(\widehat{AC}) = 80°$, m $(\widehat{DB}) = 40°$, then $\overrightarrow{EB} \cdot \overrightarrow{ED} =$ ....



(a) 2 (b) 3

(c) 8 (d) 24

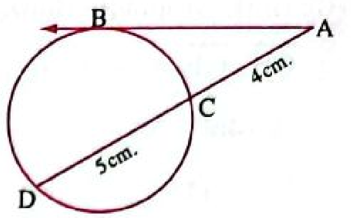
104 In the opposite figure :

If $m(\overset{\frown}{BC}) = x^\circ$, $m(\overset{\frown}{BD}) = (x + 60)^\circ$

, then $\overrightarrow{AB} \cdot \overrightarrow{AD} = \dots\dots\dots$

(a) 27 (b) $27\sqrt{3}$

(c) 54 (d) $54\sqrt{3}$

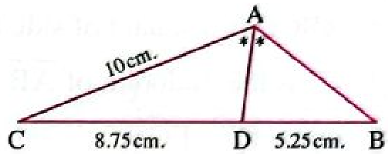


105 In the opposite figure :

$\overrightarrow{AB} \cdot \overrightarrow{AC} = \dots\dots\dots$

(a) -5.25 (b) 5.25

(c) 30 (d) -30



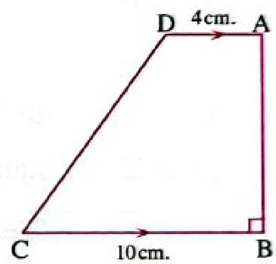
106 In the opposite figure :

ABCD is a trapezium which a right-angled at B

, $\overrightarrow{AD} \parallel \overrightarrow{BC}$, then $\overrightarrow{CD} \cdot \overrightarrow{CB} = \dots\dots\dots$

(a) 20 (b) 40

(c) 60 (d) 80



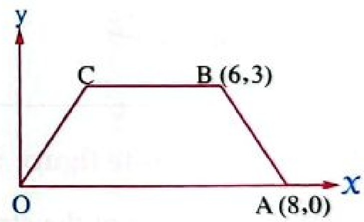
107 In the opposite figure :

OABC is an isosceles trapezium

, then $\overrightarrow{OB} \cdot \overrightarrow{OC} = \dots\dots\dots$

(a) 16 (b) 18

(c) 21 (d) 24

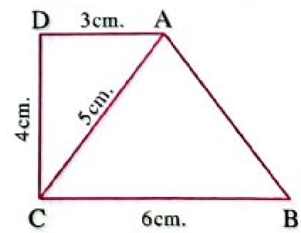


108 In the opposite figure :

$\overrightarrow{CA} \cdot \overrightarrow{CB} = \dots\dots\dots$

(a) 9 (b) 18

(c) 27 (d) 30



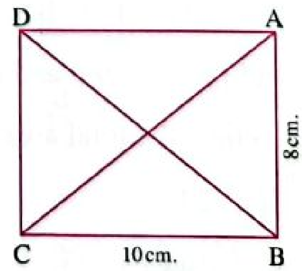
109 In the opposite figure :

ABCD is a rectangle

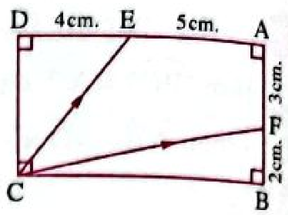
, then $\overrightarrow{CA} \cdot \overrightarrow{BD} = \dots\dots\dots$

(a) -36 (b) -16

(c) 16 (d) 36



110 **In the opposite figure :**



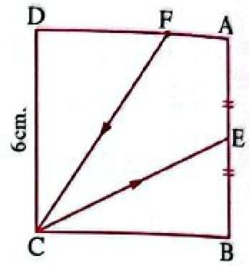
$\overrightarrow{CF} \cdot \overrightarrow{CE} = \dots\dots\dots$

(a) 46 (b) 44

(c) -44 (d) -46

---

111 **In the opposite figure :**



ABCD is a square of side length 6 cm.

, E is the midpoint of $\overline{AB}$, FD = 2 AF

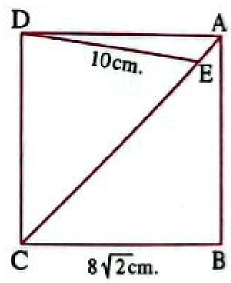
, then $\overrightarrow{CE} \cdot \overrightarrow{FC} = \dots\dots\dots$

(a) 42 (b) 40

(c) -40 (d) -42

---

112 **In the opposite figure :**



If ABCD is a square of side length $8\sqrt{2}$ cm.

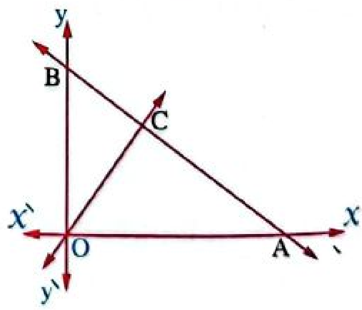
, then $\overrightarrow{CB} \cdot \overrightarrow{AE} = \dots\dots\dots$

(a) -24 (b) -18

(c) -16 (d) -12

---

113 **In the opposite figure :**



If the equation of the straight line $\overleftrightarrow{AB}$ is $\frac{x}{12} + \frac{y}{9} = 1$

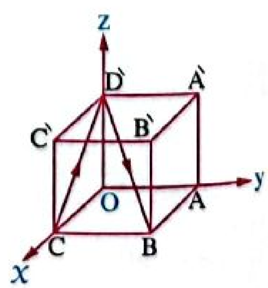
and AB = 3 BC , then $\overrightarrow{CO} \cdot \overrightarrow{CA} = \dots\dots\dots$

(a) 4 (b) 16

(c) 32 (d) 36

---

114 **In the opposite figure :**



$ABCO\,A'B'C'D'$ is a cube of side length

(ℓ) length unit and $\overrightarrow{CD'} \cdot \overrightarrow{D'B} = -32$

, then the total area of the cube = ... square unit.

(a) 24 (b) 96

(c) 144 (d) 198

115 **In the opposite figure :**

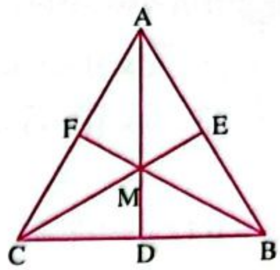
ABC is an equilateral triangle of side length 4 cm.

, M is the point of intersection of its medians

, then $\overrightarrow{MB} \cdot \overrightarrow{CM} =$ ....

(a) $\frac{-8}{3}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{8}{3}$ (d) $\frac{16}{3}$



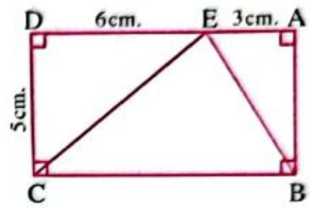
116 **In the opposite figure :**

ABCD is a rectangle , $E \in \overline{AD}$

, then $\overrightarrow{EB} \cdot \overrightarrow{EC} =$ ....

(a) 7 (b) 8

(c) 9 (d) 10

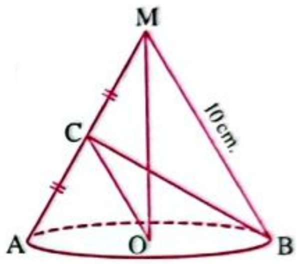


117 **In the opposite figure :**

A right-circular cone , the circumference of its base = 12 π cm. , then $\overrightarrow{BC} \cdot \overrightarrow{CO} =$ ....

(a) -43 (b) -40

(c) -37 (d) -33



Fifth Questions on vector product

Choose the correct answer from the given ones :

1 If $\overrightarrow{A} = (3, 0, 4)$, $\overrightarrow{B} = \hat{i} - 2\hat{j} + 3\hat{k}$, then $\overrightarrow{A} \times \overrightarrow{B} =$ ....

(a) $(8, -5, -6)$ (b) $(8, 5, 6)$ (c) $(8, -5, 6)$ (d) $(-8, -5, -6)$

2 If $\overrightarrow{A} = \hat{i} - 3\hat{j} + \hat{k}$, $\overrightarrow{B} = \hat{i} + \hat{j} + \hat{k}$, then $\| \overrightarrow{A} \times \overrightarrow{B} \| =$ ....

(a) $4\sqrt{2}$ (b) $3\sqrt{2}$ (c) $2\sqrt{5}$ (d) $2\sqrt{3}$

3 $\hat{i} \times \hat{j} =$ ....

(a) $\overrightarrow{0}$ (b) zero (c) 1 (d) $\hat{k}$

4 If $\{\hat{i}, \hat{j}, \hat{k}\}$ is a set of right-hand system of unit vectors , then ....

(a) $\hat{i} \cdot \hat{j} = 1$ (b) $\hat{i} \cdot \hat{i} = 1$ (c) $\hat{i} \times \hat{j} = 1$ (d) $\hat{i} \times (\hat{j} \times \hat{k}) = 1$

5. If $\vec{A}$, $\vec{B}$ are two vectors non-parallel in space , then ....

(a) $\vec{A} \times \vec{B} = k$ where $k \in \mathbb{R}$
(b) $\vec{A} \times \vec{B} = \vec{C}$ where $\vec{C}$ is non-zero vector.
(c) $\vec{A} \times \vec{B} = \vec{O}$ if $\vec{A} \perp \vec{B}$
(d) All the previous are true.

6. If $(2\hat{i} + 6\hat{j} - 27\hat{k}) \times (\hat{i} + \ell\hat{j} + m\hat{k}) = \vec{O}$, then $2\ell m =$ ....

(a) -27 (b) 81 (c) -81 (d) 27

7. If $\vec{A} = (2 , 3 , 0)$, $\vec{B} = (-1 , 3 , 1)$, $\vec{C} = (1 , 2 , 5)$
, then : $(\vec{A} - \vec{B}) \times (\vec{C} - \vec{A}) =$ ....

(a) $(-1 , 14 , -3)$ (b) $(-1 , -14 , -3)$ (c) $(2 , -3 , 14)$ (d) $(14 , -3 , 1)$

8. If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = 2\hat{i} - \hat{j} - \hat{k}$, then $\vec{A} \times (\vec{A} - \vec{B}) =$ ....

(a) $\hat{i} + \hat{k}$ (b) $-3\hat{j} + 3\hat{k}$ (c) $-3\hat{i} - 3\hat{j}$ (d) $3\hat{i} - 2\hat{j}$

9. If $\vec{A} = (1 , 0 , 2)$, $\vec{B} = (2 , -1 , -2)$, then $(\vec{A} \times \vec{B}) \cdot (\vec{B} \times \vec{A}) =$ ....

(a) 41 (b) 31 (c) -31 (d) -41

10. If $\vec{A}$, $\vec{B}$ are non-zero vectors and the measure of the angle between them is 30°
, $\|\vec{A}\| = 5$, $\|\vec{B}\| = 4$, then $\|\vec{A} \times \vec{B}\| =$ ....

(a) $20\vec{C}$ (b) $10\vec{C}$ (c) 10 (d) -20

11. If $\|\vec{A}\| = 5$, $\|\vec{B}\| = 8.5$, $\theta = 30°$, $\vec{C}$ is a unit vector perpendicular to each of $\vec{A}$ and $\vec{B}$
, then $\vec{A} \times \vec{B} =$ .... $\vec{C}$

(a) 21.45
(b) -21.25
(c) ± 21.25
(d) non of the previous.

12. In the cartesian plane x y if θ is the measure of the angle between $\vec{A}$ and $\vec{B}$
, then $\dfrac{\|\vec{A} \times \vec{B}\|}{\vec{A} \cdot \vec{B}} =$ ....

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

13. If $\vec{A} \parallel \vec{B}$, then $\vec{A} \times \vec{B} =$ ....

(a) $\vec{O}$ (b) 1 (c) $\|\vec{A}\|$ (d) $\|\vec{B}\|$

14 If $\|\overrightarrow{A} \times \overrightarrow{B}\| = 36\sqrt{3}$, $\|\overrightarrow{A}\| = 8$, $\|\overrightarrow{B}\| = 9$, then the measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{B}$ could be ....

(a) 30° (b) 120° (c) 150° (d) 45°

---

15 If θ is the angle between the two vectors $(\hat{i} + \hat{j} + \hat{k})$ and $(2\hat{i} - \hat{j} + 2\hat{k})$, then $\sin\theta =$ ....

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{\frac{2}{3}}$

(c) $\frac{\sqrt{2}}{3}$ (d) non of the previous.

---

16 Sine the angle between the two vectors $\overrightarrow{A} = (1 , 1 , 1)$, $\overrightarrow{B} = (-2 , 2 , -2)$ equals ....

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-2\sqrt{2}}{3}$

---

17 If $\overrightarrow{A} \cdot \overrightarrow{B} = \|\overrightarrow{A} \times \overrightarrow{B}\|$, then the measure of the angle between the two vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ equals ....

(a) 30° (b) 45° (c) 60° (d) 135°

---

18 If $\overrightarrow{A}$, $\overrightarrow{B}$ are non-zero vectors and $\overrightarrow{A} \cdot \overrightarrow{B} < 0$, $|\overrightarrow{A} \cdot \overrightarrow{B}| = \|\overrightarrow{A} \times \overrightarrow{B}\|$, then measure of the angle between the two vectors $\overrightarrow{A}$, $\overrightarrow{B}$ equals ....

(a) π (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

---

19 If $\overrightarrow{A} \cdot \overrightarrow{B} = \sqrt{3}\,\|\overrightarrow{A} \times \overrightarrow{B}\|$, then the measure of the angle between the two vectors $\overrightarrow{A}$, $\overrightarrow{B} =$ ....

(a) 30° (b) 45° (c) 60° (d) 90°

---

20 If $\overrightarrow{A} \times \overrightarrow{B} = -65\,\overrightarrow{C}$ and $\|\overrightarrow{A}\| = 5$, $\|\overrightarrow{B}\| = 26$, then the measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{B} =$ ....

(a) 30° or 150° (b) 60° or 120° (c) 150° (d) 120°

---

21 If $\overrightarrow{A}$, $\overrightarrow{B}$ are two unit vectors , measure of the angle between them $= \theta$, then $\|(\overrightarrow{A} - \overrightarrow{B}) \times (\overrightarrow{A} + \overrightarrow{B})\| =$ ....

(a) zero (b) 1 (c) $2\sin\theta$ (d) $2\cos\theta$

22 If $\overrightarrow{A}$, $\overrightarrow{B}$ are two unit vectors , θ is the measure of the angle between them , then $2\|\overrightarrow{A}\times\overrightarrow{B}\|(\overrightarrow{A}\cdot\overrightarrow{B}) =$ ....

(a) $\sin\theta\cos\theta$ (b) $2\sin\theta$ (c) $\cos 2\theta$ (d) $\sin 2\theta$

23 If $\overrightarrow{C}$ is a unit vector perpendicular to $\overrightarrow{A}$ and $\overrightarrow{B}$, $\|\overrightarrow{A}\| = 9$, $\|\overrightarrow{B}\| = 16$ and $\overrightarrow{A}\cdot\overrightarrow{B} = -72\sqrt{3}$, then $\overrightarrow{A}\times\overrightarrow{B} =$ ....

(a) $\pm 26\overrightarrow{C}$ (b) $\pm 36\overrightarrow{C}$ (c) $\pm 72\overrightarrow{C}$ (d) $\pm 72\sqrt{3}\overrightarrow{C}$

24 ***(2nd Session 2021)*** If $\overrightarrow{u}$ is the perpendicular unit vector to the plane containing the two vectors $\overrightarrow{A}$, $\overrightarrow{B}$ where $\overrightarrow{u} = \left(\frac{3}{5}, 0, \frac{4}{5}\right)$ and $\|\overrightarrow{A}\times\overrightarrow{B}\| = 5$, then $(3\overrightarrow{A}+\overrightarrow{B})\times(4\overrightarrow{A}+2\overrightarrow{B}) =$ ....

(a) (3 , 0 , 4) (b) (30 , 0 , 40) (c) (−30 , 0 , −40) (d) (6 , 0 , 8)

25 If $\|\overrightarrow{A}\| = 1$, $\|\overrightarrow{B}\| = 5$ and $\overrightarrow{A}\times\overrightarrow{B} = 2\hat{i}+\hat{j}-2\hat{k}$, where the angle between the two vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ is acute , then $\overrightarrow{A}\cdot\overrightarrow{B} =$ ....

(a) 3 (b) 4 (c) 6 (d) 12

26 If $\overrightarrow{A}\cdot\overrightarrow{B} = \sqrt{3}$, $\overrightarrow{A}\times\overrightarrow{B} = \hat{i}+2\hat{j}+2\hat{k}$, then measure of the angle between the two vectors $\overrightarrow{A}$ and $\overrightarrow{B} =$ ....°

(a) 30 (b) 45 (c) 60 (d) 90

27 If $\overrightarrow{A}$, $\overrightarrow{B} \in \mathbb{R}^3$ are non-zero vectors , then $\|\overrightarrow{A}\times\overrightarrow{B}\|^2 + (\overrightarrow{A}\cdot\overrightarrow{B})^2 =$ ....

(a) 1 (b) $\|\overrightarrow{A}\|^2 + \|\overrightarrow{B}\|^2$ (c) $\|\overrightarrow{A}+\overrightarrow{B}\|^2$ (d) $\|\overrightarrow{A}\|^2\|\overrightarrow{B}\|^2$

28 If $\|\overrightarrow{A}\times\overrightarrow{B}\|^2 + (\overrightarrow{A}\cdot\overrightarrow{B})^2 = 144$, $\|\overrightarrow{A}\| = 4$, then $\|\overrightarrow{B}\| =$ ....

(a) 16 (b) 8 (c) 3 (d) 5

29 If $\overrightarrow{A}$, $\overrightarrow{B}$ are two non-zero parallel vectors , then which of the following statements is not true ?

(a) $\overrightarrow{A} = k\overrightarrow{B}$, $k \neq 0$ (b) $\frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z}$ (c) $\overrightarrow{A}\cdot\overrightarrow{B} =$ zero (d) $\overrightarrow{A}\times\overrightarrow{B} = \overrightarrow{\text{zero}}$

30 If $\overrightarrow{A} = (4, -k, 6)$, $\overrightarrow{B} = (2, 2, m)$ and $\overrightarrow{A} \parallel \overrightarrow{B}$, then $k + m =$ ....

(a) −3 (b) −2 (c) −1 (d) zero

31 If $\overrightarrow{A} = (4, -5, 1)$, $\overrightarrow{B} = (2, -k, -2)$, $\overrightarrow{C} = (-4, 4, m-2)$ and $\overrightarrow{AB} \parallel \overrightarrow{C}$, then $k - m =$ ....

(a) 5 (b) 7 (c) 8 (d) 9

32 *(1st Session 2021)* If $\|\overrightarrow{A}\| = \sqrt{13}$, $\overrightarrow{A} \parallel \overrightarrow{DC}$ and in its direction , such that $D(1, 3, -2)$, $C(1, -1, 4)$ and $\overrightarrow{B} = (-2, 3, 5)$, then $\overrightarrow{A} \times \overrightarrow{B} =$ ....

(a) $-19\hat{i} + 6\hat{j} + 4\hat{k}$ (b) $-28\hat{i} - 12\hat{j} - 8\hat{k}$

(c) $-28\hat{i} + 12\hat{j} - 8\hat{k}$ (d) $-19\hat{i} - 6\hat{j} - 4\hat{k}$

33 If $(2, 5, 3)$, $(4, 6, -1)$, $(8, 8, k)$ are three collinear points then $k =$ ....

(a) 8 (b) 7 (c) -9 (d) -7

34 If $m\overrightarrow{A} = n\overrightarrow{B}$ where $m, n \in \mathbb{Z}^+$, then $\frac{\overrightarrow{A} \cdot \overrightarrow{B} + \|\overrightarrow{A} \times \overrightarrow{B}\|}{\|\overrightarrow{A}\| \|\overrightarrow{B}\|} =$ ....

(a) $m + n$ (b) $m - n$ (c) 1 (d) zero

35 If $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{O}$, $\overrightarrow{A} \cdot \overrightarrow{C} = 0$, then $\overrightarrow{B} \cdot \overrightarrow{C} =$ ....

(a) 0 (b) 1 (c) $\overrightarrow{A}$ (d) $\|\overrightarrow{B}\|$

36 The unit vector perpendicular to each of vectors $(\hat{i} + \hat{j})$ and $(\hat{j} + \hat{k})$ is ....

(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ (c) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (d) $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

37 If $\overrightarrow{A}$, $\overrightarrow{B}$ are two unit perpendicular vectors , then $\|(\overrightarrow{A} - \overrightarrow{B}) \times (\overrightarrow{A} + \overrightarrow{B})\| =$ ....

(a) zero (b) 1 (c) -1 (d) 2

38 If $\|\overrightarrow{A} \times \overrightarrow{B}\| = 4$, then the unit vector perpendicular to each of $\overrightarrow{A}$ and $\overrightarrow{B} =$ ....

(a) $\pm \frac{1}{8}(\overrightarrow{A} \times \overrightarrow{B})$ (b) $\pm \frac{1}{4}(\overrightarrow{A} \times \overrightarrow{B})$ (c) $\pm \frac{1}{2}(\overrightarrow{A} \times \overrightarrow{B})$ (d) $\pm 4(\overrightarrow{A} \times \overrightarrow{B})$

39 The vector whose magnitude 3 units and perpendicular to each of the two vectors $\overrightarrow{A} = 3\hat{i} + \hat{j} - 4\hat{k}$, $\overrightarrow{B} = 6\hat{i} + 5\hat{j} - 2\hat{k}$ from the following is ....

(a) $(18, -18, 9)$ (b) $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$ (c) $(-18, 18, -9)$ (d) $(2, -2, 1)$

40 If A (0 , 0 , 1) , B (1 , 0 , 0) , C (0 , 1 , 0) , then the orthogonal unit vector to the plane ABC from the following is ....

(a) (1 , 1 , 1)
(b) $(\sqrt{2}, \sqrt{2}, \sqrt{2})$
(c) $(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}})$
(d) non of the previous.

---

41 If $\overrightarrow{A} \perp \overrightarrow{B}$, $\overrightarrow{A} \perp \overrightarrow{C}$, $\overrightarrow{B} = (2, 3, 2)$, $\overrightarrow{C} = (1, 2, 1)$ and $\|\overrightarrow{A}\| = 4\sqrt{2}$, then $\overrightarrow{A}$ can be ....

(a) (2 , 3 , 1)
(b) (– 4 , 0 , 4)
(c) (4 , 4 , 0)
(d) (0 , – 4 , 4)

---

42 If OACB is parallelogram and $\overrightarrow{OC} = \overrightarrow{d_1}$, $\overrightarrow{AB} = \overrightarrow{d_2}$, then $\overrightarrow{OA} =$ ....

(a) $\overrightarrow{d_1} + \overrightarrow{d_2}$
(b) $\overrightarrow{d_1} - \overrightarrow{d_2}$
(c) $\frac{1}{2}(\overrightarrow{d_2} - \overrightarrow{d_1})$
(d) $\frac{1}{2}(\overrightarrow{d_1} - \overrightarrow{d_2})$

---

43 ABCDEF is a uniform hexagon with side length 2 cm. , then $\|\overrightarrow{AB} \times \overrightarrow{AF}\| =$ ....

(a) $2\sqrt{3}$
(b) $\sqrt{3}$
(c) 2
(d) 1

---

44 If $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{C}$, then ....

(a) $\overrightarrow{B} \times \overrightarrow{C} = \overrightarrow{A}$
(b) $\overrightarrow{A} \times \overrightarrow{C} = \overrightarrow{B}$
(c) $\overrightarrow{A} \times (\overrightarrow{B} + \overrightarrow{A}) = \overrightarrow{C}$
(d) $\overrightarrow{B} \times \overrightarrow{A} = \overrightarrow{C}$

---

45 If $\overrightarrow{A}$ is a non-zero vector , then the expression

$\|\overrightarrow{A} \times \hat{i}\|^2 + \|\overrightarrow{A} \times \hat{j}\|^2 + \|\overrightarrow{A} \times \overrightarrow{k}\|^2 =$ ....

(a) $\|\overrightarrow{A}\|^2$
(b) zero
(c) $2\|\overrightarrow{A}\|^2$
(d) $3\|\overrightarrow{A}\|^2$

---

46 If $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are unit vectors and $\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{A} \cdot \overrightarrow{C} = 0$, measure of the angle between $\overrightarrow{B}$ and $\overrightarrow{C}$ equals $\frac{\pi}{4}$, then $\overrightarrow{A} =$ ....

(a) $\pm 2(\overrightarrow{B} \times \overrightarrow{C})$
(b) $\pm 3(\overrightarrow{B} \times \overrightarrow{C})$
(c) $\pm\sqrt{2}(\overrightarrow{B} \times \overrightarrow{C})$
(d) $\pm\sqrt{3}(\overrightarrow{B} \times \overrightarrow{C})$

---

47 If $\overrightarrow{A} = (1, 6, 2)$, $\overrightarrow{B} = (k, 3, m)$, $\overrightarrow{C} = (k, m, k+m)$ and $\overrightarrow{A} // \overrightarrow{B}$, then $\|\overrightarrow{C}\| =$ ....

(a) $\sqrt{14}$
(b) $2\sqrt{7}$
(c) $\frac{1}{2}\sqrt{14}$
(d) $4\sqrt{7}$

48 If ABCD is a parallelogram , then $\overrightarrow{AD} \times \overrightarrow{BC} =$ ....

(a) $\overrightarrow{AB} \times \overrightarrow{DC}$ (b) $\overrightarrow{AC} \times \overrightarrow{BD}$ (c) area of ▱ ABCD (d) $\overrightarrow{AB} \times \overrightarrow{AD}$

49 If the area of the parallelogram ABCD equals 12 cm², then $\| \overrightarrow{AC} \times \overrightarrow{BD} \| =$ ....

(a) 6 (b) 12 (c) 24 (d) 48

50 The area of Δ ABC is 24 cm², then $\| \overrightarrow{BA} \times \overrightarrow{BC} \| =$ ....

(a) 12 (b) 24 (c) 36 (d) 48

51 If ABC is triangle , then $\| \overrightarrow{AB} \times \overrightarrow{AC} \| =$ ....

(a) $\overrightarrow{AB} \times (\overrightarrow{AC} \times \overrightarrow{BC})$ (b) $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{BC})$

(c) $\| \overrightarrow{CB} \times \overrightarrow{CA} \|$ (d) $\overrightarrow{CB} \cdot \overrightarrow{AC}$

52 In Δ ABC , $\overrightarrow{AB} = (1 , 2 , 3)$, $\overrightarrow{BC} = 2\hat{k}$, then the area of the parallelogram ABCD = ... area units.

(a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) $4\sqrt{5}$ (d) 6

53 If $\overline{AD}$ is a median of Δ ABC , then $\| \overrightarrow{AD} \times \overrightarrow{AB} \| =$ .... × the area of Δ ABC

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

54 Area of Δ ABC where : A (5 , 1 , − 2) , B (4 , − 4 , 3) , C (2 , 4 , 0) ≈ .... square unit.

(a) 16.7 (b) 18.2 (c) $8\sqrt{3}$ (d) 14.27

55 Δ ABC in which M ∈ $\overline{AB}$, N ∈ $\overline{AC}$ where $\frac{AM}{MB} = \frac{CN}{NA} = \frac{2}{3}$ and $\overrightarrow{AN} \times \overrightarrow{MN} = \overrightarrow{AC} \times (k \overrightarrow{BC})$, then k = ... where $k \in \mathbb{R}^+$

(a) $\frac{25}{6}$ (b) $\frac{6}{25}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

56 If ABCD is a parallelogram , M is the point of intersection of its diagonals , then $\| \overrightarrow{MD} \times \overrightarrow{MC} \| =$ ....

(a) $\vec{O}$ (b) $\| \overrightarrow{AB} \times \overrightarrow{AD} \|$

(c) $\| \overrightarrow{MC} \times \overrightarrow{MB} \|$ (d) $\overrightarrow{MD} \cdot \overrightarrow{MC}$

57 ABCD is a parallelogram in which $\overrightarrow{AB} = (2, 2, -1)$, $\overrightarrow{AD} = (-1, 2, -3)$, then the surface area of the parallelogram = ... square units.

(a) 6 (b) $7\sqrt{2}$ (c) $3\sqrt{11}$ (d) $\sqrt{101}$

58 Area of ▱ ABCD where A (2 , 1 , 3) , B (1 , 4 , 5) , C (2 , 5 , 3) = ... square unit.

(a) $3\sqrt{2}$ (b) $4\sqrt{5}$ (c) $2\sqrt{5}$ (d) $2\sqrt{7}$

59 Area of parallelogram in which its diagonals are $\overrightarrow{M} = 2\hat{i} - \hat{j}$ and $\overrightarrow{N} = 4\hat{i} - 5\hat{j}$ equals .... square unit.

(a) $3\sqrt{2}$ (b) $\frac{3\sqrt{2}}{2}$ (c) $6\sqrt{2}$ (d) 3

60 If the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$ are two adjacent sides in a parallelogram ABCD , then $\| \overrightarrow{AC} \times \overrightarrow{BD} \| =$ ....

(a) $20\sqrt{5}$ (b) $22\sqrt{5}$ (c) $24\sqrt{5}$ (d) $26\sqrt{5}$

61 The area of the rhombus whose diagonals are $\overrightarrow{F_1} = 5\hat{i}$, $\overrightarrow{F_2} = 2\hat{j}$ equals ....

(a) 5 (b) 10 (c) 25 (d) 100

62 If $\overrightarrow{A}$ and $\overrightarrow{B}$ are non-zero vectors where $\| \overrightarrow{A} \| = 2\sqrt{2}$, $\| \overrightarrow{B} \| = 3$, measure of the angle between them equals $\frac{\pi}{4}$, then area of the parallelogram in which $5\overrightarrow{A} + 2\overrightarrow{B}$ and $\overrightarrow{A} + 3\overrightarrow{B}$ are two adjacent sides in it equals .... square units.

(a) 72 (b) 92 (c) 78 (d) 112

63 Area of parallelogram ABCD whose diagonals intersect at M equals ....

(a) $\| \overrightarrow{AM} \times \overrightarrow{BM} \|$ (b) $\| \overrightarrow{AB} \times \overrightarrow{AM} \|$

(c) $\| \overrightarrow{AB} \times \overrightarrow{AC} \|$ (d) $\| \overrightarrow{AC} \times \overrightarrow{BD} \|$

64 $\overrightarrow{A} \cdot (\overrightarrow{B} + \overrightarrow{C}) \times (\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C}) =$ ....

(a) zero (b) 1

(c) $\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C}$ (d) $\overrightarrow{A} \cdot (\overrightarrow{B} + \overrightarrow{C})$

65 $\hat{i} \cdot \hat{j} \times \hat{i} =$ ....

(a) $\vec{O}$ (b) zero (c) 1 (d) $\hat{k}$

66 *(Trial 2021)* If $\{\hat{i}, \hat{j}, \hat{k}\}$ is a set of a right hand system of unit vectors , then $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j} =$ ....

(a) -1 (b) zero (c) 1 (d) 2

67 If $\vec{A}, \vec{B}, \vec{C}$ are unit vectors , then $\vec{A} \cdot \vec{B} \times \vec{C}$ could be equal ....

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{-1}{2}$ (d) $\frac{-3}{2}$

68 If $\vec{A}, \vec{B}, \vec{C}$ are unit vectors , then the greatest value for the expression $\vec{A} \cdot \vec{B} \times \vec{C}$ equals ....

(a) 1 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$

69 If $\vec{A} = (1, -1, 2)$, $\vec{B} = (3, -2, 0)$, $\vec{C} = (0, 2, 4)$, then $\vec{A} \cdot \vec{B} \times \vec{C} =$ ....

(a) 10 (b) 12 (c) 14 (d) 16

70 $\vec{A} \cdot (\vec{B} \times \vec{C}) =$ ....

(a) $\vec{A} \cdot (\vec{C} \times \vec{B})$ (b) $\vec{C} \cdot (\vec{B} \times \vec{A})$ (c) $\vec{A} \times \vec{B} \times \vec{C}$ (d) $\vec{C} \cdot (\vec{A} \times \vec{B})$

71 $\vec{B} \cdot (\vec{A} \times \vec{C}) =$ ....

(a) $(\vec{B} \times \vec{A}) \cdot \vec{C}$ (b) $\vec{A} \cdot (\vec{B} \times \vec{C})$ (c) $(\vec{A} \times \vec{B}) \times \vec{C}$ (d) $\vec{C} \cdot (\vec{A} \times \vec{C})$

72 $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j}) =$ ....

(a) 1 (b) 3 (c) -3 (d) zero

73 If $\vec{A} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{B} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{C} = 3\hat{i} - 4\hat{j} - 12\hat{k}$, then the projection of the vector $(\vec{A} \times \vec{B})$ on the vector $\vec{C}$ is ....

(a) 14 (b) -14 (c) 12 (d) 15

74 If $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are three non-zero vectors , then $\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C} = \|\overrightarrow{A}\| \|\overrightarrow{B}\| \|\overrightarrow{C}\|$ if ....

ⓐ $\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{B} \cdot \overrightarrow{C} = \text{zero}$ ⓑ $\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{A} \cdot \overrightarrow{C} = \text{zero}$

ⓒ $\overrightarrow{B} \cdot \overrightarrow{C} = \overrightarrow{A} \cdot \overrightarrow{C} = \text{zero}$ ⓓ $\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{B} \cdot \overrightarrow{C} = \overrightarrow{C} \cdot \overrightarrow{A} = \text{zero}$

75 The volume of the parallelepiped in which 3 adjacent edges represented by the vectors $\overrightarrow{A} = (3 , -4 , 0)$, $\overrightarrow{B} = (0 , -4 , 3)$, $\overrightarrow{C} = (0 , 0 , 5)$ equals .... cubic unit.

ⓐ 12 ⓑ 50 ⓒ 60 ⓓ 125

76 Volume of the triangular pyramid in which the vectors $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ represents three non-coplanar edges equals .... cubic units.

ⓐ $\frac{1}{2} |\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C}|$ ⓑ $\frac{1}{3} |\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C}|$

ⓒ $\frac{1}{4} |\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C}|$ ⓓ $\frac{1}{6} |\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C}|$

77 *(Trial 2021)* If $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are three connected edges in a parallelepiped where $\|\overrightarrow{A}\| = 2$ The direction cosines of the vector $\overrightarrow{A}$ are $(135° , 60° , 120°)$ and $\overrightarrow{B} = \left(1 , \sqrt{2} , 0\right)$, $\overrightarrow{C} = \left(\sqrt{2} , 3 , 5\right)$, then the volume of the parallelepiped = ... cubic units

ⓐ 16 ⓑ $6\sqrt{2}$ ⓒ 11 ⓓ $16\sqrt{2}$

78 A parallelepiped formed from the vectors $\overrightarrow{A} = \hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{B} = 5\hat{i} + 2\hat{j} + \ell\hat{k}$, $\overrightarrow{C} = \ell\hat{i} + 4\hat{j} + \hat{k}$, then smaller volume of this parallelepiped at ℓ = ...

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

79 If $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{A} \times \overrightarrow{C}$, then ....

ⓐ $\overrightarrow{B} = \overrightarrow{C}$ ⓑ $\overrightarrow{A}$ and $\overrightarrow{B}$ are parallel.

ⓒ $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are mutually perpendicular. ⓓ $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ in the same plane.

80 If $\overrightarrow{A} = (2 , 1 , -2)$, $\overrightarrow{A} + \overrightarrow{B} = \overrightarrow{A} \times \overrightarrow{B}$, then $\overrightarrow{B}$ = ...

ⓐ $(2 , -1 , -2)$ ⓑ $(2 , 1 , -2)$

ⓒ $(-2 , -1 , 2)$ ⓓ $(-2 , -1 , 3)$

81 The points A , B , C , D are coplanar , then ....

ⓐ $\vec{A} \cdot (\vec{B} \times \vec{C}) = \text{zero}$
ⓑ $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = \text{zero}$
ⓒ $\vec{A} \times \vec{B} = \vec{C} \times \vec{D}$
ⓓ $\overrightarrow{AB} \cdot (\overrightarrow{BC} \times \overrightarrow{BD}) \neq \text{zero}$

---

82 If the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are in the same plane , then $\vec{A} \cdot \vec{B} \times \vec{C} =$ ....

ⓐ 1
ⓑ zero
ⓒ -1
ⓓ $(\vec{B} \times \vec{A}) \times \vec{C}$

---

83 If the vectors $\vec{A} = (1 , 1 , 1)$, $\vec{B} = (4 , 3 , 4)$, $\vec{C} = (1 , a , b)$ lies in the same plane and $\|\vec{C}\| = \sqrt{3}$, then $a^2 - b =$ ....

ⓐ -2
ⓑ -1
ⓒ zero
ⓓ 1

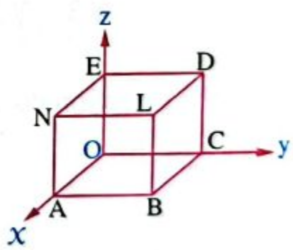
---

84 If $m , n , \ell \in \mathbb{R}^+$ and the vectors $m\hat{i} + m\hat{j} + \ell\hat{k}$, $\hat{i} + \hat{k}$, $\ell\hat{i} + \ell\hat{j} + n\hat{k}$ lies in the same plane , then the equation $m x^2 + 2\ell x + n = 0$ has ....

ⓐ two equal real roots.
ⓑ two unequal roots.
ⓒ non-real roots.
ⓓ two unequal real roots.

---

85 **In the opposite figure :**



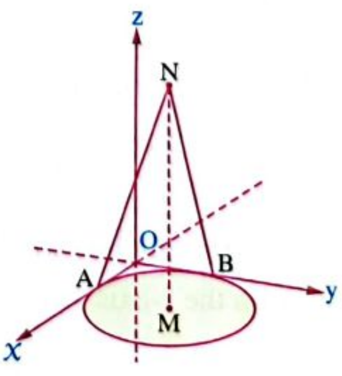
A cube with volume 8 cubic units ,

then $\overrightarrow{AC} \times \overrightarrow{AE} =$ ....

ⓐ $-2\hat{i} + 2\hat{j}$
ⓑ $-2\hat{i} + 2\hat{k}$
ⓒ $4\hat{i} + 4\hat{j} + 4\hat{k}$
ⓓ $2\hat{i} + 2\hat{j} + 2\hat{k}$

---

86 **In the opposite figure :**



If circle M touches the x-axis and y-axis at A and B respectively A (2 , 0 , 0)

$\overrightarrow{MN}$ // the z-axis , MN = 4 length units ,

then $\overrightarrow{NA} \times \overrightarrow{NB} =$ ....

ⓐ $-8\hat{i} + 8\hat{j} + 4\hat{k}$
ⓑ $8\hat{i} - 8\hat{j} + 4\hat{k}$
ⓒ $8\hat{i} + 8\hat{j} - 4\hat{k}$
ⓓ $8\hat{i} + 8\hat{j} + 4\hat{k}$

Sixth Questions on the equation of the straight line in the space

Choose the correct answer from the given ones :

1. The direction vector of the straight line passes through the two points $(2, -3, 4)$, $(5, -2, 1)$ could be ....

(a) $(3, 1, -3)$ (b) $(3, -1, -3)$ (c) $(-3, 1, -3)$ (d) $(3, 1, 3)$

2. The direction cosines of the straight line passes through two points $(10, 9, 1)$ and $(4, 7, -2)$ could be ....

(a) $(6, 2, 3)$ (b) $(2, 4, -13)$ (c) $\left(\frac{-6}{7}, \frac{-2}{7}, \frac{-3}{7}\right)$ (d) $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}, 0\right)$

3. If direction ratios of the straight line in space are $(-2, 6, -4)$, then its direction cosines are ....

(a) $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$ (b) $\left(\frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right)$

(c) $\left(\frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}\right)$ (d) $\left(\frac{-1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{-2}{\sqrt{14}}\right)$

4. Which of the following gives the direction cosines of a straight line ?

(a) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (b) $(1, 1, 1)$ (c) $(1, -1, -1)$ (d) $(1, -1, 1)$

5. If the direction cosines of a straight line is $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$, then ....

(a) $c > 0$ (b) $0 < c < 1$ (c) $c = \pm\sqrt{3}$ (d) $c > 2$

6. The direction cosines of the straight line perpendicular to the xz–plane could be ....

(a) $(1, 0, 0)$ (b) $(0, 1, 0)$ (c) $(0, 0, 1)$ (d) $(1, 0, 1)$

7. The direction cosines of the x-axis is ....

(a) $(1, 0, 0)$ (b) $(0, 1, 0)$ (c) $(0, 0, 1)$ (d) $(1, 1, 1)$

8. The straight line which makes a direction angle of measure 60° with the y-axis and 60° with the z-axis, then it makes a direction angle with x-axis of measure ....

(a) 60° (b) 30° (c) 45° (d) 75°

9 If the direction angles of a straight line are $\theta_x , \theta_y , \theta_z$, then $\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z =$ ....

(a) -2 (b) -1 (c) 1 (d) 2

10 If the direction angles of a straight line are $\theta_x , \theta_y , \theta_z$, then $\cos 2\theta_x + \cos 2\theta_y + \cos 2\theta_z =$ ....

(a) -2 (b) -1 (c) 1 (d) 2

11 If ℓ , m , n are the direction cosines of a straight line in space , then ....

(a) $\ell = m = n = 1$ (b) $\ell + m + n = 1$

(c) $\ell^2 + m^2 + n^2 = \text{zero}$ (d) $\ell^2 + m^2 = 1 - n^2$

12 The equation of the straight line passing through the point A $(-1 , 0 , 2)$ and has direction vector $\vec{d} = (1 , -1 , 3)$ is ....

(a) $\frac{x-1}{2} = \frac{y}{1} = \frac{z-1}{-1}$ (b) $\frac{x+1}{1} = \frac{y}{-1} = \frac{z-2}{3}$

(c) $\frac{x-1}{3} = \frac{y}{-1} = \frac{z}{1}$ (d) $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z}{2}$

13 The equation of the straight line which passes through the point $(1 , 2 , 3)$ and parallel to the straight line : $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z+10}{3}$ is ....

(a) $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z+10}{3}$ (b) $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{3}$

(c) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ (d) no thing from the previovs.

14 The equation of the straight line passes through the two points A $(2 , 1 , -3)$, B $(1 , 2 , -5)$ is ....

(a) $\vec{r} = (-1 , 2 , -2) + t(2 , 1 , -3)$ (b) $\vec{r} = (1 , 2 , -5) + t(2 , 1 , -3)$

(c) $\vec{r} = (3 , 2 , 4) + t(-1 , 1 , 2)$ (d) $\vec{r} = (2 , 1 , -3) + t(-1 , 1 , -2)$

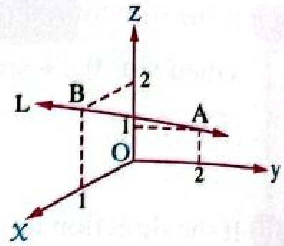
15 The equation of the straight line passing through the two points $(-2 , 4 , 2)$, $(7 , -2 , 5)$ is ....

(a) $\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z-2}{1}$ (b) $\frac{x}{7} = \frac{y}{-2} = \frac{z}{5}$

(c) $\frac{x}{-2} = \frac{y}{4} = \frac{z}{2}$ (d) $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+1}{1}$

16 **In the opposite figure :**

The equation of the straight line is ....



(a) $x = \frac{y-2}{2} = z - 1$ (b) $x = \frac{y+2}{2} = \frac{z-1}{2}$

(c) $\frac{x}{2} = \frac{y-2}{-2} = z - 1$ (d) $x = \frac{y-2}{-2} = z - 1$

---

17 The parametric equations of the straight line passes through the two points A $(-1, 0, 3)$, B $(1, -1, 0)$ is ....

(a) $x = -1 + t, y = -t, z = 3 - 3t$ (b) $x = -1 + 2t, y = -t, z = -3t$

(c) $x = 1 + 2t, y = -1 - t, z = -3t$ (d) $x = 1 + 2t, y = -1 + t, z = 3 - 3t$

---

18 The cartesion equation of the straigth line whose vector equation is $\vec{r} = (1, 3, 9) + k(5, 4, 2)$ is ....

(a) $x - 5 = \frac{y-4}{3} = \frac{z-2}{9}$ (b) $\frac{x-1}{5} = \frac{y-3}{4} = \frac{z-9}{2}$

(c) $x + 5 = \frac{y+4}{3} = \frac{z+2}{9}$ (d) $\frac{x+1}{5} = \frac{y+3}{4} = \frac{z-9}{2}$

---

19 The vector equation of the straight line whose cartesian equation : $\frac{x+3}{2} = \frac{2y-1}{5} = \frac{3z+2}{4}$ is ....

(a) $\vec{r} = \left(-3, \frac{1}{2}, \frac{-2}{3}\right) + t\left(2, \frac{5}{2}, \frac{4}{3}\right)$ (b) $\vec{r} = (-3, 1, -2) + t(2, 5, 4)$

(c) $\vec{r} = (2, 5, 4) + t(-3, 1, -2)$ (d) $\vec{r} = \left(2, \frac{5}{2}, \frac{4}{3}\right) + t\left(-3, \frac{-1}{2}, \frac{-2}{3}\right)$

---

20 ***(2nd Session 2021)*** The vector equation of the straight line passing through the point A $(2, -1, 4)$ and parallel to the bisector of the angle between $\overrightarrow{Oy}$, $\overrightarrow{OZ}$ in the plane y z is ....

(a) $\vec{r} = (2, -1, 4) + t(0, 1, 1)$ (b) $\vec{r} = (2, -1, 4) + t(0, -1, 1)$

(c) $\vec{r} = (2, -1, 4) + t(-1, 0, 1)$ (d) $\vec{r} = (2, -1, 4) + t(1, 0, -1)$

---

21 ***(1st Session 2021)*** ABC is a triangle, where A $(1, 2, 4)$, B $(-2, 0, 5)$, C $(1, 4, 0)$, if M is the intersection point of its medians, then the equation of straight line $\overleftrightarrow{AM}$ is ....

(a) $\vec{r} = (1, 2, 4) + t(-1, 4, 5)$ (b) $\vec{r} = (1, 2, 4) + t(-1, 1, 1)$

(c) $\vec{r} = (1, 2, 4) + t(-1, 0, -1)$ (d) $\vec{r} = (0, 2, 3) + t(1, 2, 4)$

22 The equation of the straight line passing through the point (a , b , c) parallels to the z-axis is ....

(a) $x = 0 , y = 0$ (b) $z = c$ (c) $z = 0$ (d) $x = a , y = b$

23 The equation of the straight line passes through the origin and makes equal angles with the coordinate axes is all the following except ....

(a) $x = y = z$

(b) $\frac{x}{\sqrt{2}} = \frac{y}{\sqrt{2}} = \frac{z}{\sqrt{2}}$

(c) $\frac{x-4}{2} = \frac{y-4}{2} = \frac{z-4}{2}$

(d) $\frac{x-1}{\sqrt{3}} = \frac{1-y}{\sqrt{3}} = \frac{z-1}{\sqrt{3}}$

24 The equation of the straight line passing through the point (2 , 3 , – 5) and makes equal angles with the coordinates axes is ....

(a) $\frac{x}{2} = \frac{y}{3} = \frac{z}{-5}$

(b) $\frac{x-2}{5} = \frac{3-y}{-5} = \frac{z+5}{5}$

(c) $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{-5}$

(d) $\frac{x-2}{\sqrt{3}} = \frac{y-3}{\sqrt{3}} = \frac{z+5}{3}$

25 The equation of the straight line passing through the point (2 , – 1 , 3) and perpendicular to the two straight lines $\vec{r_1} = (1 , 1 , -1) + t_1 (2 , -2 , 1)$, $\vec{r_2} = (2 , -1 , 3) + t_2 (1 , 2 , 2)$ is ....

(a) $\frac{x-2}{2} = -y-1 = \frac{z-3}{-2}$

(b) $\frac{x-1}{2} = y+1 = \frac{z-3}{2}$

(c) $\frac{x-2}{2} = y+1 = \frac{z-3}{-2}$

(d) $\frac{x-1}{-2} = -y-1 = \frac{z-3}{2}$

26 The equation of the y-axis in space is ....

(a) $x = 0 , z = 0$ (b) $z = 0 , y = 0$ (c) $x = 0$ (d) $z = 0$

27 The vector $\vec{A} = (-1 , 1 , 1)$ is perpendicular to the straight line that has the equation ....

(a) $\frac{x+1}{-2} = \frac{y-2}{3} = \frac{z-1}{-1}$

(b) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{1-z}{-3}$

(c) $\frac{x+1}{-2} = \frac{y+2}{-1} = \frac{z+2}{3}$

(d) $\frac{x-1}{2} = \frac{y-2}{4} = \frac{2-z}{2}$

28 The point lying on the straight line $\vec{r} = (2 , -1 , 3) + t (1 , 2 , -1)$ is ....

(a) (1 , 1 , 1) (b) (0 , 2 , – 2) (c) (3 , 1 , 2) (d) (4 , – 3 , 0)

29 The straight line with equation $\vec{r} = (3, 2, 1) + t(2, 3, 1)$ intersects the x y-plane at the point ....

(a) $(1, -1, 0)$ (b) $(3, 2, 0)$ (c) $(2, 1, 0)$ (d) $(5, 5, 0)$

30 Which point lies on the straight line : $\frac{x}{3} = \frac{y+1}{1} = \frac{z-3}{2}$ such that its x-coordinate is twice its y-coordinate ?

(a) $(-6, -3, -1)$ (b) $(4, 2, -1)$ (c) $(6, 3, -1)$ (d) $(2, 1, -1)$

31 If the three points $(5, 2, 4)$, $(6, -1, 2)$, $(8, -7, k)$ belong to the same straight line, then $k =$ ....

(a) 2 (b) -1 (c) 3 (d) -2

32 The point (x, y, z) moves parallel to y-axis which of the following variables x, y, z has a fixed value ?

(a) z, x (b) x, y (c) y (d) z

33 If the point $(2, 3, -3)$ moves parallel to the x-axis, then its path is a straight line whose equation is ....

(a) $\frac{x}{2} = \frac{y}{3} = \frac{z}{-3}$ (b) $y = 3, z = -3$

(c) $2x + 3y - 3z = 0$ (d) $x = 0, 3y - 3z = 0$

34 The direction vector of the straight line : $x = \frac{y+5}{-2} = \frac{z+1}{3}$ of the following is ....

(a) $(4, 5, 6)$ (b) $(3, -5, -1)$ (c) $(-3, 5, 1)$ (d) $(1, -2, 3)$

35 Which of the following represents direction vector of the y-axis ?

(a) $(1, 0, 1)$ (b) $(0, -1, 0)$ (c) $(2, 0, 0)$ (d) $(1, 1, 1)$

36 The straight line passes through the origin point and the point $(3, -1, 2)$, then $\cos \theta_z =$ .... where θ_z is measure of the angle which the straight line makes with the positive direction of z-axis.

(a) $\frac{1}{2}$ (b) $\frac{-1}{\sqrt{14}}$ (c) $\frac{2}{\sqrt{14}}$ (d) $\sqrt{14}$

37 The direction vector of the straight line $\frac{2x-3}{4} = \frac{y+5}{7} = \frac{3z-8}{6}$ of the following is ....

(a) $(4, 7, 6)$ (b) $\left(\frac{3}{2}, -5, \frac{8}{3}\right)$ (c) $(2, 7, 2)$ (d) $(3, 4, -2)$

38 The direction vector of the straight line $\ell : \frac{x-2}{3} = \frac{y+3}{2}$, $z = 4$ of the following is ....

(a) (3 , 2 , 4) (b) (3 , 2 , 0) (c) (4 , 2 , 3) (d) (2 , 3 , 4)

39 The direction vector of the straight line $2x + 3y = 5$, $z = 4$ of the following is ....

(a) (2 , 3 , 4) (b) (3 , 2 , 4) (c) (3 , − 2 , 0) (d) (− 1 , 1 , 0)

40 The direction vector of the straight line : $\frac{y+5}{-2} = \frac{x-3}{4} = \frac{z+1}{3}$ of the following is ....

(a) (− 2 , 4 , 3) (b) (3 , − 5 , − 1) (c) (− 3 , 5 , 1) (d) (4 , − 2 , 3)

41 The direction vector of the straight line : $\frac{2x-3}{5} = \frac{1-z}{2} = \frac{3y-5}{2}$ of the following is ....

(a) (5 , 3 , 2) (b) (5 , 2 , 3) (c) $\left(\frac{5}{2}, \frac{2}{3}, -2\right)$ (d) $\left(\frac{5}{2}, -2, \frac{2}{3}\right)$

42 If $\overrightarrow{A} = (-2, k, -3)$ is parallel to the straight line $\frac{x+2}{4} = \frac{y}{8} = \frac{z-1}{6}$, then k = ...

(a) − 4 (b) − 3 (c) − 2 (d) − 1

43 The direction cosines of the straight line $\frac{\sqrt{2}\,x - 3\sqrt{2}}{1} = \frac{2\sqrt{2} - \sqrt{2}\,y}{2}$, $z + 1 = 0$ could be ....

(a) $\left(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, \text{zero}\right)$ (b) $\left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \text{zero}\right)$

(c) $\left(\frac{2}{\sqrt{5}}, \text{zero}, \frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, \text{zero}\right)$

44 By using different real values for t in the equation of the straight line $\overrightarrow{r} = (2, 1, -3) + t(1, -2, 4)$ you get ....

(a) different direction vectors to the same straight line.

(b) different points on the straight line.

(c) different direction vectors perpendicular to the straight line.

(d) different direction cosines to the same straight line.

45 If the straight line $\overrightarrow{r} = (1, 2, 3) + t(-2, 1, -1)$ intersects the plane $z = 2$ at point A and the plane $x = 3$ at point B , then length of $\overline{AB}$ = ... length units.

(a) 4 (b) 5 (c) $2\sqrt{6}$ (d) $6\sqrt{2}$

46 If the straight line whose equation $\vec{r} = (1, 2, 5) + t(2, a, 4)$ makes with the coordinate axes angles of measures θ, α, β such that $\cos\theta + \cos\alpha + \cos\beta = \frac{4}{\sqrt{6}}$, then a = ... (where $a \in \mathbb{Z}$)

ⓐ -2 ⓑ 1 ⓒ 2 ⓓ 4

47 The projection of the point $(1, 2, 3)$ on the straight line $\vec{r} = (6, 7, 7) + t(3, 2, -2)$ is the point ....

ⓐ $(3, 5, 9)$ ⓑ $(6, 7, 7)$ ⓒ $(9, 9, 5)$ ⓓ $(5, 3, 9)$

48 If $A(1, 2, -3)$, $B(-1, 1, 4)$, then the equation of the straight line $\overleftrightarrow{AB}$ by reflection in the x-axis is ....

ⓐ $\frac{x-1}{-2} = \frac{y+2}{1} = \frac{z-3}{-7}$ ⓑ $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-7}$

ⓒ $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{7}$ ⓓ $\frac{x+1}{2} = \frac{y+2}{1} = \frac{z-3}{7}$

49 The measure of the angle between the two the straight lines whose direction cosines are $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ equals ....

ⓐ $60°$ ⓑ $30°$ ⓒ $90°$ ⓓ $120°$

50 The measure of the angle between the two lines which have direction ratios $(1, 1, 2)$ and $(1, 1, 4)$ equals ....

ⓐ $\cos^{-1}\left(\frac{-5\sqrt{3}}{9}\right)$ ⓑ $\cos^{-1}\left(\frac{5\sqrt{3}}{9}\right)$ ⓒ $\sin^{-1}\left(\frac{5\sqrt{3}}{9}\right)$ ⓓ $\cos^{-1}\left(\frac{\sqrt{3}}{9}\right)$

51 If (a, b, c) and $(\grave{a}, \grave{b}, \grave{c})$ are two direction vectors of two perpendicular straight lines, then ....

ⓐ $a\grave{a} = b\grave{b} = c\grave{c} = 1$ ⓑ $a\grave{a} + b\grave{b} + c\grave{c} = 0$

ⓒ $\frac{a}{\grave{a}} = \frac{b}{\grave{b}} = \frac{c}{\grave{c}}$ ⓓ $a + b + c = \grave{a} + \grave{b} + \grave{c}$

52 If the straight line : $\frac{x+2}{6} = \frac{y-1}{m} = \frac{z-1}{3}$ is perpendicular to the straight line $\frac{x-9}{-2} = \frac{y+8}{1}$, $z = 3$, then m = ...

ⓐ -12 ⓑ 12 ⓒ 6 ⓓ 0

53 *(1st Session 2021)* If the two straight lines $L_1 : \vec{r} = t_1 (-2, m, 7)$ and $L_2 : \frac{x-1}{n} = \frac{1-y}{4} = \frac{z-2}{2}$ are perpendicular, then : $n + 2m = \dots\dots$

(a) 7 (b) -7 (c) 14 (d) -14

54 *(2nd Session 2021)* If the two straight lines $L_1 : \vec{r_1} = (1, 2, 3) + t_1 (-1, 3, 4)$ and $L_2 : \vec{r_2} = (-2, 5, -1) + t_2 (m, n, 1)$ are perpendicular, then $3n - m = \dots\dots$

(a) -4 (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) 4

55 The measure of the angle between the two lines $\vec{r_1} = (-2, 5, -7) + k(-6, 6, 8)$, $\vec{r_2} = (1, -2, 3) + \acute{k}(4, 12, -6)$ equals $\dots\dots$

(a) zero (b) 30° (c) 60° (d) 90°

56 The measure of the angle between the two straight lines $\frac{x-3}{2} = \frac{z+1}{-2}$, $y = 1$ and $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z+1}{-2}$ equals $\dots\dots$

(a) 15° (b) 30° (c) 45° (d) 60°

57 The measure of the angle between the straight line $\frac{x-1}{\sqrt{2}} = \frac{y-\sqrt{2}}{1} = \frac{z+1}{1}$ with the positive direction of z-axis is $\dots\dots$

(a) 30° (b) 45° (c) 60° (d) 120°

58 The measure of the included angle between the two straight lines $L_1 : x = 2y + 1$, $2y = 1 - z$ and $L_2 : 2x + y + z = 0$, $z + 2 = 0$ equals $\dots\dots$

(a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

59 The measure of the angle between the two straight lines : $2x = 3y = -z$ and $6x = -y = -4z$ equals $\dots\dots$

(a) zero (b) 45° (c) 90° (d) 120°

60 If the measure of the angle between the two lines : $\frac{x}{a} = \frac{y}{2} = \frac{z}{1}$, $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$ equals 60°, then the value of $a = \dots\dots$

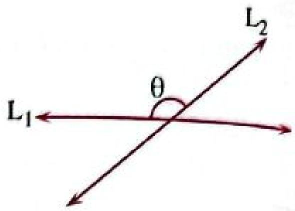
(a) 1 or $\frac{2}{5}$ (b) 1 or $\frac{-13}{5}$ (c) $\frac{2}{5}$, $\frac{-13}{5}$ (d) $\pm \frac{13}{5}$

61 If θ is the angle measure between the two straight lines $x + 1 = \frac{y-1}{2} = \frac{z-2}{2}$, $\vec{r} = (0 , 2 , 3) + t(-1 , 1 , a)$ and $\tan \theta = \sqrt{2}$, then $a =$ .... (where $a > 0$)

(a) 1 (b) 2 (c) 3 (d) 5

---

62 **In the opposite figure :**



If $L_1 : x = 0$, $y = z$

$L_2 : y = 0$, $x = z$, then $\theta =$ ....

(a) $120°$ (b) $135°$

(c) $150°$ (d) $165°$

---

63 The two straight lines : $\vec{r_1} = \vec{A_1} + t_1 \vec{d_1}$, $\vec{r_2} = \vec{A_2} + t_2 \vec{d_2}$ are parallel if ....

(a) $\vec{d_1} = \ell \vec{d_2}$, $\ell \in \mathbb{R}^*$ (b) $\vec{d_2} // \left(\vec{A_2} - \vec{A_1}\right)$

(c) $\vec{d_1} // \left(\vec{A_2} - \vec{A_1}\right)$ (d) $\vec{d_1} \cdot \vec{d_2} = \text{zero}$

---

64 If $\ell_1 : \frac{x-3}{2} = \frac{-y-1}{6} = \frac{z}{k}$ parallel to $\ell_2 : \frac{x+2}{6} = \frac{y-z}{m} = \frac{z-1}{3}$, then $k + m =$ ....

(a) -17 (b) -10 (c) 10 (d) 17

---

65 If the straight lines $L_1 : x = 2t_1 - 1$, $y = t_1 + 1$, $z = t_1 - 1$, $L_2 : x = a t_2 - 1$, $y = 2t_2 + 1$, $z = b t_2 - 2$ are parallel , then $a + b =$ ....

(a) 4 (b) 2 (c) 6 (d) -2

---

66 Which of the following straight lines passes through the point $(-2 , 3 , 5)$ and is parallel to the straight line $L : \frac{x-1}{2} = \frac{y+1}{4} = \frac{z-3}{3}$?

(a) $x = 2 + 2k$, $y = 3 + 4k$, $z = 5 + 3k$

(b) $\frac{x+2}{2} = \frac{y-3}{4} = \frac{z-5}{3}$

(c) $x = 1 + 2k$, $y = -1 + 4k$, $z = 3 + 3k$

(d) $x = 2 + 2k$, $y = -3 + 4k$, $z = -5 + 3k$

---

67 Point of intersection of the two straight lines : $x = y = z$ and $2x = 3y = z$ is ....

(a) $(1 , 1 , 1)$ (b) $(2 , 3 , 1)$ (c) $(-2 , -3 , -1)$ (d) $(0 , 0 , 0)$

68 If the two straight lines $L_1 : \overrightarrow{r_1} = (1 , 2 , 3) + t_1 (1 , 2 , 3)$ and $L_2 : \overrightarrow{r_2} = (-1 , 2 , 0) + t_2 (m , 2 , -1)$ are intersecting , then $m =$ ....

(a) $\frac{-8}{3}$ (b) $\frac{-5}{3}$ (c) zero (d) -3

69 The two straight lines $x = y = z$, $2x = 3y = z$ are ....

(a) skew. (b) intersecting and perpendicular.
(c) parallel. (d) intersecting but not perpendicular.

70 The two straight lines $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{7}$ and $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{7}$ are ....

(a) perpendicular. (b) intersecting. (c) skew. (d) parallel.

71 If $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$, $\frac{x}{2} = \frac{y+2}{2} = \frac{z-3}{-2}$ are two straight lines , then they are ....

(a) perpendicular. (b) coincident. (c) parallel. (d) intersecting.

72 The two straight lines : $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{6}$ are ....

(a) intersecting. (b) coincident. (c) parallel. (d) skew.

73 The points $(5 , 2 , 4)$, $(6 , -1 , 2)$, $(8 , -7 , m)$ are collinear , if $m =$ ....

(a) -2 (b) 2 (c) 3 (d) -1

74 The distance between the point $(3 , 4 , 5)$ from the y-axis equals .... length units.

(a) 3 (b) 5 (c) $\sqrt{34}$ (d) $\sqrt{41}$

75 The shortest distance between the point (a , b , c) and the x-axis equals ....

(a) $|c|$ (b) $|a|$ (c) $|b|$ (d) $\sqrt{b^2 + c^2}$

76 The distance between the point $(3 , 4 , 5)$ and the straight line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ is ....

(a) zero (b) 1 (c) 2 (d) 5

77 The length of the perpendicular drawn from the point $(-1 , 0 , 1)$ to the straight line $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z+1}{-1}$ equals .... length units.

(a) $\sqrt{30}$ (b) $\sqrt{6}$ (c) $\frac{\sqrt{30}}{6}$ (d) $2\sqrt{6}$

78 *(2nd Session 2021)* The perpendicular distance between the point (2 , 4 , 7) and the straight line $2x - 4 = \frac{2y-8}{3} = \frac{2z-14}{5}$ equals .... length units.

(a) zero (b) 1 (c) 2 (d) 5

79 The length of the perpendicular drawn from the point A (1 , 0 , 2) to the straight line $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-3}{-2}$ equals ....

(a) $\frac{\sqrt{26}}{4}$ (b) $\frac{\sqrt{26}}{5}$ (c) $\frac{\sqrt{26}}{3}$ (d) $\frac{\sqrt{26}}{6}$

80 If the length of the perpendicular drawn from the point (2 , − 1 , m) on the straight line $x - 1 = 3 - y = z$ equals 5 length unit , then number of possible values of m equals ....

(a) zero (b) 1 (c) 2 (d) infinite number.

81 *(1st Session 2021)* If the perpendicular distance between the point (− 1 , 2 , m) and the straight line : $\vec{r} = (-1 , 3 , 0) + t\,(0 , -3 , 0)$ is 8 length unit , then the value of m equals .... , where $m \in \mathbb{R}^+$

(a) 4 (b) 16 (c) 8 (d) 2

82 If the point (1 , − 1 , 2) is the project of the point (0 , 3 , 1) on the straight line $\frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$, then a + b = ...

(a) 8 (b) 13 (c) 14 (d) 15

83 If B , C ∈ the straight line L where L : $\frac{x+2}{3} = \frac{z}{4}$, y = 1 , BC = 5 length units , A = (1 , − 1 , 2) then area of Δ ABC = ... square units.

(a) $5\sqrt{17}$ (b) $2\sqrt{34}$ (c) 6 (d) $\sqrt{34}$

84 If a straight line with direction vector (2 , 0 , − 2) intersects the sphere : $(x-2)^2 + (y+1)^2 + (z-3)^2 = 4$ at two points A and B where A = (0 , − 1 , 3) , then the length of $\overline{AB}$ = ... length units.

(a) 2 (b) $2\sqrt{2}$ (c) $2\sqrt{6}$ (d) $\sqrt{6}$

85 The distance between the two parallel straight lines : $L_1 : \frac{x-2}{2} = \frac{y+1}{1} = \frac{z}{3}$ and $L_2 : \frac{x-2}{2} = \frac{y-1}{1} = \frac{z-3}{3}$ equals approximately .... length units.

(a) 3.1 (b) 1.7 (c) 2.1 (d) 3.8

86 The radius of the sphere $(x+1)^2 + (y-2)^2 + (z-5)^2 = r^2$ which the straight line $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-5}{6}$ touches it. equals approximately .... length units.

(a) 8.56 (b) 8.92 (c) 12.93 (d) 4.46

87 The length of the intercepted part of the straight line $\frac{x-5}{1} = \frac{y-3}{3} = \frac{z-1}{-2}$ by the sphere $x^2 + y^2 + z^2 - 2x - 4y - 2z - 39 = 0$ equals approximately .... length units.

(a) 5.6 (b) 22.4 (c) 11.2 (d) 8.3

88 The smallest distance between the straight line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{4}$ and the x-axis = ... length units.

(a) $\sqrt{2}$ (b) 2.2 (c) 3.6 (d) 4.8

Seventh Questions on equation of the plane in the space

Choose the correct answer from those given :

1 The plane is determined by ....

(a) three distinct non collinear points. (b) a straight line and a point outside it.
(c) two intersecting straight line. (d) all the previous.

2 The equation of the plane x y is ....

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $x + y = 0$

3 The two straight lines $\overleftrightarrow{xx}$ and $\overleftrightarrow{yy}$ form the cartesian plane whose equation ....

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) otherwise.

4 The equation of the plane passing through point (1 , −2 , 5) and the vector (2 , 1 , 3) is perpendicular to it is ....

(2,1,3). (x,y,z)

(a) $2x + y + 3z = 1$ (b) $\boxed{2x + y + 3z = 15}$
(c) $x - 2y + 5z = 15$ (d) $x + y + z = 4$

=

$(2, 1, 3).(x, y, z) = 13$

5 The plane perpendicular to the vector (2 , 1 , 3) and passes through the point (1 , 2 , 3) has the equation ....

(a) $2x + y + 3z = 0$
(b) $2x + y + 3z = 13$
(c) $2x + y + 3z = 14$
(d) $2x + y + 3z = 7$

6 The equation of the plane which is perpendicular to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passes through the point (3 , 2 , 1) is ....

$(3, 1, 2) . (x, y, z) = 13$

(a) $3x + y + 2z = 0$
(b) $3x + y + 2z = 13$
(c) $3x + y + 2z = 14$
(d) $3x + y + 2z = 7$

7 The equation of the plane passing through the point (a , b , c) and parallel to the plane $x + y + z = 0$ is ....

$(1, 1, 1) . (x, y, z) = a+b+c$

(a) $ax + by + cz = 1$
(b) $x + y + z + a + b + c = 0$
(c) $x + y + z = a + b + c$
(d) $ax + by + cz = a + b + c$

8 The equation of the plane passes through the point (4 , 0 , 1) parallel to the plane $4x + 3y - 12z + 6 = 0$ is ....

$(4, 3, -12) . (x, y, z) = 4$

(a) $4x + 3y - 12z = 6$
(b) $\vec{r} . (4, 3, -12) = 4$
(c) $4x + 3y - 12z = 0$
(d) $\frac{x}{4} + \frac{y}{3} + \frac{z}{-12} = 1$

9 *(2nd Session 2021)* The general form of the equation of the plane which passing through the point (–2 , 2 , –1) and parallel to the plane whose equation : $(2, 3, -5) . \vec{r} = 1$ is ....

(a) $2x + 3y - 5z = -7$
(b) $2x + 2y - z = 1$
(c) $2x - 3y + 5z = -7$
(d) $2x + 3y - 5z = 7$

10 The equation of the plane which contains the two points A (0 , 1 , 2) , B (– 1 , 0 , 3) and perpendicular to the plane $2x + 3y + z = 5$ is ....

$AB = (-1, -1, 1)$

$\begin{vmatrix} 2 & 3 & 1 \\ -1 & -1 & -1 \end{vmatrix}$

$-2i + j + k$

(a) $4x - 3y + z + 1 = 0$
(b) $4x - 3y + z + 31 = 0$
(c) $-4x + 3y - z + 32 = 0$
(b) $3x + 4y - 18z + 32 = 0$

11 The equation of the straight line passing through the origin perpendicular to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is ....

(a) $\vec{r} = (0, 0, 0) + t(a, b, c)$
(b) $\vec{r} = (0, 0, 0) + t\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$
(c) $\vec{r} = (a, b, c) + t(0, 0, 0)$
(b) $\vec{r} = (a, b, c) + t(1, 1, 1)$

12 The equation of the straight line which passes through the point (1 , 1 , 1) and perpendicular to the plane : $-3x+2y+z+5=0$ is ....

ⓐ $\frac{x-1}{-1}=\frac{y-1}{1}=\frac{z-1}{1}$ ⓑ $\frac{x-1}{-3}=\frac{y-1}{2}=\frac{z-1}{1}$

ⓒ $\frac{x-1}{3}=\frac{y-1}{3}=\frac{z-1}{1}$ ⓑ $\frac{x-1}{1}=\frac{y-1}{3}=\frac{z-1}{2}$

13 The equation of the plane which contains the line $\vec{r}=\hat{i}+\hat{j}+t(2\hat{i}+\hat{j}+4\hat{k})$ is ....

ⓐ $\vec{r}\cdot(-\hat{i}-2\hat{j}+\hat{k})=3$ ⓑ $\vec{r}\cdot(-\hat{i}+2\hat{j}+\hat{k})=6$

ⓒ $\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k})=3$ ⓓ $\vec{r}\cdot(-\hat{i}-2\hat{j}+\hat{k})=6$

14 The equation of the plane that contains the two straight lines :
$\frac{x+1}{6}=\frac{y-2}{7}=z$, $\frac{x-3}{6}=\frac{y+4}{7}=z-1$ is ....

ⓐ $13x-2y-64z+17=0$ ⓑ $13x-2y-64z-17=0$

ⓒ $13x+2y-64z-17=0$ ⓓ $13x+2y-64z+17=0$

15 The equation of the plane contains the two points A (0 , 1 , 2) and B (– 1 , 0 , 3) and parallel to the straight line $x-1=\frac{y-1}{-2}=\frac{z-2}{3}$ is ....

ⓐ $x-4y-3z+10=0$ ⓑ $x-4y-3z=6$

ⓒ $3x+y+4z=9$ ⓓ $3x-2y+z=0$

16 The equation of the plane passing through the point (1 , 0 , – 1) and perpendicular to the straight line $6x+6=3y+9=-4z-28$ is ....

ⓐ $x+3y+7z=-6$ ⓑ $x+3y+7z=3$

ⓒ $2x+4y-3z=0$ ⓓ $2x+4y-3z=5$

17 If the projection of the point O (0 , 0 , 0) on the plane (P) is M (2 , 3 , 1) then the equation of this plane is ....

ⓐ $2x+3y+z-14=0$ ⓑ $x+3y+2z-14=0$

ⓒ $2x+3y+z=0$ ⓓ $x+3y+2z=0$

18 The plane contains the origin point and the vector (0 , 1 , – 2) is perpendicular to it is ....

ⓐ $x=2y$ ⓑ $y=2z$ ⓒ $z=2x$ ⓓ $y=1$

19 If the point $(12, -4, 3)$ is the projection of the origin on a plane then the equation of this plane is ....

(a) $x - y + z = 13$
(b) $12x - 4y + 3z = 13$
(c) $12x - 4y + 3z = 169$
(d) $\frac{x}{12} - \frac{y}{4} + \frac{z}{3} = 1$

20 Equation of the plane passing through the point $(3, 5, 7)$ and parallel to the cartesian plane $(x z)$ is ....

(a) $x + z = -1$
(b) $x = 3$
(c) $y = 5$
(d) $z = 7$

21 The equation of the plane which parallel to the plane y z and passes through the point $(1, 3, 5)$ is ....

(a) $x = 1$
(b) $y = 3$
(c) $z = 5$
(d) $y + z = 8$

22 The equation of the plane which parallel to the x-axis is ....

(a) $a x + c z + d = 0$
(b) $b y + c z + d = 0$
(c) $a x + b y + d = 0$
(d) $a x + b y + c z + d = 0$

23 The equation of the plane contains the z-axis and passes through $(1, -1, 3)$ is ....

(a) $3x + y = 6$
(b) $3x + y = 0$
(c) $x + y = 0$
(d) $x - 4y + 3z = 0$

24 The equation of the plane parallel to the y-axis and intercepts 3 parts and 4 parts from the x-axis and z-axis respectively is ....

(a) $3x + 4z = 1$
(b) $4x + 3z = 1$
(c) $3x + 4z = 12$
(d) $4x + 3z = 12$

25 The equation of the plane parallel to the x-axis and does not contains the straight line $y = 1$, $z = 2$ could be ....

(a) $3x - 5z + 7 = 0$
(b) $x + 4y - 3z - 5 = 0$
(c) $y + z - 3 = 0$
(d) $32 - 5y - 3 = 0$

26 The equation of the plane which passes through the point $(1, 2, 3)$ and parallel to the coordinate axes x, y is ....

(a) $x + y = 3$
(b) $z = 3$
(c) $x = 1$
(d) $y = 2$

27 The plane passes through the point $(\sqrt{2}, -1, 1)$ and its normal $\vec{n}$ makes an angle of measure 45° with the x-axis and an angle of measure 60° with y-axis and makes an acute angle with the z-axis , then its equation is ....

(a) $\sqrt{2}\,x + y + z = 2$ (b) $\sqrt{2}\,x - y - z = 2$

(c) $3\sqrt{2} - 4y - 3z = 7$ (d) $4\sqrt{2} + 7y + z = 2$

28 If the plane $2x - y + 2z = 4$ moves parallel to itself 3 units in $\overrightarrow{ox}$ direction , then its equation will be ....

(a) $2x - y + 2z = 1$ (b) $2x - y + 2z = 7$

(c) $2x - y + 2z = 10$ (d) $2x - y + 2z = -7$

29 If the plane $\vec{r} \cdot \left(\frac{1}{2}, \frac{-1}{2}, \frac{1}{\sqrt{2}}\right) = 3$ is translated paralleling itself a way from the origin point , then its equation may become ....

(a) $\vec{r} \cdot (1, -1, \sqrt{2}) = 2$ (b) $\vec{r} \cdot (3, -3, \sqrt{6}) = 12$

(c) $\vec{r} \cdot (1, -1, \sqrt{2}) = 6$ (d) $\vec{r} \cdot (1, -1, \sqrt{2}) = 8$

30 The value of a which makes the following position vectors :

$2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$, $3\hat{i} + a\hat{j} + 5\hat{k}$ lying in the same plane is ....

(a) 2 (b) 4 (c) -4 (d) 3

31 The system of equations $x - 2y + z = 5$, $2x + y - 3z = 1$ represents ....

(a) a plane. (b) a sphere. (c) a straight line. (d) a point.

32 The equation of the plane which passes through the points (1 , 2 , 3) , (–1 , 4 , 2) , (3 , 1 , 1) is ....

(a) $5x + y + 12z - 23 = 0$ (b) $5x + 6y + 2z - 23 = 0$

(c) $x + 6y + 2z - 13 = 0$ (d) $x + y + z - 13 = 0$

33 The equation of the plane passing through points (2 , 3 , 5) , (–1 , 3 , 1) , (4 , 3 , – 2) is ....

(a) $x + y - z = 0$ (b) $x = -1$ (c) $y = 3$ (d) $z = -2$

34 Equation of the plane intercepts from positive parts from x and z axes parts of length 2 , 4 units respectively and from negative part of y axis part of length 3 unit ....

(a) $2x - 3y + 4z = 1$
(b) $6x - 4y + 3z = 6$
(c) $6x + 4y + 3z = 12$
(d) $6x - 4y + 3z = 12$

---

35 The equation of the plane passing through the points (0 , 0 , – 4) , (0 , 5 , 0) , (–2 , 0 , 0) is ....

(a) $\vec{r} \cdot (-2, 0, 0) = [(0, 0, -4) \times (0, 5, 0)] \cdot (-2, 0, 0)$
(b) $-2x + 5y - 4z = 0$
(c) $a(x + 2) + b(y - 5) + c(z + 4) = 0$
(d) $-10x + 4y - 5z - 20 = 0$

---

36 Equation of the plane passing through the points (1 , 0 , 0) , (0 , 2 , 0) and (0 , 0 , 3) is ....

(a) $x + 2y + 3z = 6$
(b) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 6$
(c) $(6, 3, 2) \cdot \vec{r} = 6$
(d) $6x + 3y + 2z = 1$

---

37 The two non-parallel straight lines $\vec{r}_1 = \vec{A}_1 + t_1 \vec{d}_1$, $\vec{r}_2 = \vec{A}_2 + t_2 \vec{d}_2$ lie on the same plane if ....

(a) $\vec{A}_1 \times \vec{A}_2 = \vec{O}$
(b) $\vec{d}_1 \times \vec{d}_2 = \vec{O}$
(c) $(\vec{A}_1 - \vec{A}_2) \cdot (\vec{d}_1 \times \vec{d}_2) = 0$
(d) $\vec{d}_1 \cdot \vec{d}_2 = 0$

---

38 The straight line $\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ lies on the plane $a x + b y + c z + d = 0$ if ....

(a) $a\ell + bm + cn = 0$
(b) $a x_1 + b y_1 + c z_1 + d = 0$
(c) $a\ell + bm + cn = 0$, $a x_1 + b y_1 + c z_1 + d = 0$
(d) $(\ell, m, n) \times (a, b, c) = \vec{0}$

---

39 If the starighi line $\frac{x - 3}{b} = \frac{y - 4}{-7} = \frac{z + 3}{13}$ is contained in the plane $5x - y + z = a$, then a + b = ...

(a) – 3
(b) 2
(c) 4
(d) 9

40 The two straight lines $\ell_1 : \frac{X+1}{-1} = \frac{y-2}{-1} = \frac{z-1}{3}$, $\ell_2 : \frac{X+1}{1} = \frac{y-2}{-2} = \frac{z-1}{-1}$ lie on the plane ....

(a) $3X - 5y + z - 1 = 0$
(b) $5X - 4y + 2z - 7 = 0$
(c) $7X - 5y - z - 4 = 0$
(d) $7X + 2y + 3z = 0$

41 The equation $3X - 4z = 0$ in the space represents ....

(a) an equation of a straight line parallel to y-axis.
(b) an equation of a plane perpendicular to y-axis.
(c) an equation of a plane contains y-axis.
(d) an equation of a straight line its direction ratios are (3 , 0 , 4)

42 In space of three dimensions the equation $3y + 4z = 0$ represents ....

(a) a plane contains z-axis.
(b) a plane contains X-axis.
(c) a plane contains y-axis.
(d) a straight line its direction ratios (0 , 3 , 4)

43 $4X + 5y - 9 = 0$ is the equation of a plane ....

(a) parallel to X-axis.
(b) parallel to z-axis.
(c) contains the y-axis.
(d) contains z-axis.

44 $2y + 3z + 5 = 0$ equation of a plane ....

(a) passing through X-axis.
(b) parallels plane (y z)
(c) parallels X-axis.
(d) passing through the origin point.

45 $2y + 3 = 0$ is an equation of a plane ....

(a) parallels the plane (y z)
(b) parallels the plane (X z)
(c) parallels the plane (X y)
(d) otherwise.

46 The equation : $\|\vec{r}\|^2 - 2(\vec{r} \cdot \vec{A}) + k = 0$ represents ....

(a) a plane.
(b) a sphere.
(c) a straight line.
(d) vector.

47 The equation : $\|\vec{r}\|^2 - \vec{r} \cdot (2\hat{i} + 4\hat{j} + 2\hat{k}) - 10 = 0$ represents the equation of ....

(a) a circle of radius length 4
(b) a plane.
(c) a sphere of radius length 4
(d) a sphere of radius length $\sqrt{10}$

48 If the straight line $\frac{x-1}{4}=\frac{y-2}{b}=\frac{z}{6}$ never intersect the plane $2x-4y+3z=5$, then b = ...

(a) -10 (b) -8 (c) $\frac{13}{2}$ (d) $\frac{19}{2}$

49 The straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane ....

(a) $2x+y-2z=0$
(b) $x+y+z=0$
(c) $3x+4y+5z=7$
(d) $2x+3y+4z=0$

50 The straight line $\frac{x+1}{1}=\frac{y+3}{3}=\frac{z-2}{-2}$ is perpendicular to the plane ....

(a) $x-3y-2z+7=0$
(b) $2x+6y-4z+3=0$
(c) $x+3y-5z=0$
(d) $2x+6y+4z=5$

51 If the straight line L : $\frac{x-1}{3}=\frac{y+3}{2}=\frac{z-3}{-1}$ and the plane P : $x-2y-z=0$, then which of the following is correct ?

(a) $L \parallel P$ (b) $L \perp P$ (c) $L \subset P$ (d) L cuts P

52 If the straight line $x=3y=az$ is parallel to the plane $x+3y+2z+4=0$, then a = ...

(a) -3 (b) 3 (c) -1 (d) 1

53 The straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ with respect to the plane $x-2y+z-6=0$: ....

(a) The straight line is parallel to the plane.
(b) The straight line is perpendicular to the plane.
(c) The straight line lies on the plane.
(d) The straight line is incline to the plane.

54 The straight line : $\frac{x-1}{2}=\frac{y+2}{3}=1-z$ and the plane $3y-z+2x=3$ ....

(a) are parallel.
(b) the straight line lies on the plane.
(c) the straight line is perpendicular to the plane.
(d) the straight line intersects the plane but it is not perpendicular to it.

55 The plane which intersects the coordinate axes at the points A , B and C such that the point of intersection of the medians of the Δ ABC is (ℓ , m , n) , then the equation of the plane is ....

(a) $\ell x + my + nz = 1$
(b) $\ell x + my + nz = 3$
(c) $\frac{x}{\ell} + \frac{y}{m} + \frac{z}{n} = 1$
(d) $\frac{x}{\ell} + \frac{y}{m} + \frac{z}{n} = 3$

56 If a plane cuts the coordinate axes at A , B , C where the intersection point of the medians of Δ ABC is (3 , 3 , 3), then the equation of the plane is ....

(a) $3x + 3y + 3z = 1$
(b) $3x + 3y + 3z = 3$
(c) $\frac{x}{3} + \frac{y}{3} + \frac{z}{3} = 1$
(d) $\frac{x}{3} + \frac{y}{3} + \frac{z}{3} = 3$

57 The plane $3x - 2y + 4z = 12$ intercepts from x-axis a part of length ....

(a) 3
(b) -4
(c) 4
(d) 6

58 If the intercepted parts from the coordinate axes by the plane $x + 5y - 6z = 30$ are a , b , c , then a + b + c = ...

(a) 0
(b) 30
(c) 31
(d) 41

59 The equation of the plane is $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = a$ where $a > 0$ intercepts from the corrdinate axes total parts of length 12 units , then a = ...

(a) 1
(b) 2
(c) 3
(d) 6

60 If the plane $\frac{x}{4} + \frac{y}{2} + \frac{z}{2} = 1$ cuts the coordinate axes at points A, B, C , then the area of the triangle ABC = ...

(a) 12
(b) 10
(c) 6
(d) 4

61 The plane : $20x + 15y + 12z = 60$ intersects the coordinate axes x , y , z at the points A , B , C respectively , then the volume of the solid ABCO where O is the origin equals .... cubic unit.

(a) 30
(b) 60
(c) 90
(d) 10

62 If the plane $\frac{x}{2} + \frac{y}{4} + \frac{z}{3} = 1$ intersects the coordinate axes at the points A , B , C , then the volume of the pyramid OABC = ... cube units.

(a) 4
(b) 8
(c) 12
(d) 24

63 The volume of solid made by the plane $7x + 8y + 9z - 504 = 0$ with the cartesian planes = ... cube units.

ⓐ 14112 ⓑ 21168 ⓒ 42336 ⓓ 84632

64 The point that belongs to the plane : $\vec{r} = (-1, 0, 2) + t(0, 0, 1) + m(1, 0, -1)$ is ....

ⓐ $(0, 1, 2)$ ⓑ $(2, 1, 3)$ ⓒ $(3, 1, 2)$ ⓓ $(1, 0, 1)$

65 If $A(1, 0, 0)$ and $B(0, 1, 1)$ lie on the plane $kx + y + mz + 2 = 0$, then $k + m =$ ....

ⓐ -7 ⓑ -5 ⓒ -3 ⓓ -1

66 The point of intersection of the straight line $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z}{3}$ and the plane $x - 2y + 3z + 5 = 0$ is ....

ⓐ $(1, 2, 3)$ ⓑ $(-1, 2, 3)$ ⓒ $(-1, 2, 0)$ ⓓ $(0, 2, -1)$

67 If the straight line $\frac{x-2}{3} = \frac{y+1}{2} = 1 - z$ intersects the two planes $2x + 3y - z + 13 = 0$ and $3x + y + 4z = 16$ at the two points A and B , then AB = ... length unit.

ⓐ $\sqrt{14}$ ⓑ $\sqrt{28}$ ⓒ $2\sqrt{14}$ ⓓ 14

68 Which of the following points lying in the plane $3x + 2y - z = 5$?

ⓐ $(1, 1, 1)$ ⓑ $(2, 1, 0)$ ⓒ $(2, 0, 1)$ ⓓ $(2, 3, -1)$

69 The point $(2, -1, 3)$ lies on the plane ....

ⓐ $x + y - 2 = 6$ ⓑ $2x - 3y + z = -10$

ⓒ $3x - 2y + 4z = 20$ ⓓ $x - 2y + 5z = 4$

70 For any point (x, y, z) on the plane yz we find that ....

ⓐ $x = 0$ ⓑ $y = 0$ ⓒ $z = 0$ ⓓ $x = 0, z = 0$

71 If the plane $2ax - 3ay + 4az = 6$ is passing through the midpoint of the line segment drawn between centres of the two spheres : $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$, $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a = ...

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ -1

72 If $3x - 5y - 4z - 27 = 0$ is the general from of the equation of the plane, then the vector form of its equation is ....

(a) $(3, -5, -4) \cdot \vec{r} = 27$
(b) $(3, -5, -4) \cdot \vec{r} = -27$
(c) $(5, 4, 27) \cdot \vec{r} = 3$
(d) $(3, 4, 5) \cdot \vec{r} = 27$

---

73 If $(5, -1, 2) \cdot \vec{r} = 0$ is the vector form of the equation of the plane passes through the origin point, then the general form is ....

(a) $5x - y + 2z = 6$
(b) $5x - y + 2z = 0$
(c) $y - 5x + 2z = 8$
(d) $2x - y + 5z = 0$

---

74 If the point $(4, -1, 3)$ is the image of the point $(2, 1, 1)$ by reflection on a plane, then equation of that plane is ....

(a) $x - y + z = 5$
(b) $4x - y + 3z = 10$
(c) $2x + y + z = 5$
(d) $x - 2y + z = 5$

---

75 The equation of the plane which bisects the distants between the two points : $(-2, 3, 4)$, $(0, 1, 6)$ colud be ....

(a) $-x + 2y + 5z = 0$
(b) $3x - y + z = 0$
(c) $3x - y + z - 5 = 0$
(d) $6y + z = 7$

---

76 The equation of the plane which bisects $\overline{AB}$ and makes with it angle $\theta \in \left]0, \frac{\pi}{2}\right[$, $A(2, 3, 4)$, $B(6, 7, 8)$ may be ....

(a) $2x + y - 2z - 1 = 0$
(b) $x + z - y - 15 = 0$
(c) $x - y - z - 15 = 0$
(d) $x + y + z - 15 = 0$

---

77 The equation of the plane passing through the point $(1, 3, -5)$ and perpendicular to each of the two planes : $x + 2y - 3z + 5 = 0$ and $3x - y + 2z - 1 = 0$ is ....

(a) $x - 11y - 7z = 3$
(b) $x - 11y - 7z + 3 = 0$
(c) $x + 2y - 3z = 3$
(d) $3x - y + 2z = 3$

---

78 If the equation of the plane contains the straight line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z = 12$ is $ax + by + cz + 4 = 0$, then $a + b + c =$ ....

(a) 2
(b) 12
(c) 3
(d) -2

79 The equation of the plane that touches the sphere $x^2 + y^2 + z^2 = 9$ at the point A $(2, -1, 2)$ is ....

(a) $2x - y + 2z = 9$ (b) $2x - y + 2z = 3$ (c) $x + y + z = 3$ (d) $\frac{x}{2} - \frac{y}{1} + \frac{z}{2} = 3$

80 The equation of the plane that touches the sphere $x^2 + y^2 + z^2 = 4$ and parallels the plane xy can be ....

(a) $x + y + z = 4$ (b) $z = 4$ (c) $z = 2$ (d) $x + y = 4$

81 All the following points lie on the same side from the plane $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) = 5$ except ....

(a) $(1, 2, 1)$ (b) $(0, -1, 2)$ (c) $(2, -3, 1)$ (d) $(1, 2, 4)$

82 The two points A $(-2, 3, 5)$, B $(4, -7, -2)$, the plane P : $2x - 3y + z - 7 = 0$, then A, B ....

(a) belong to the plane P
(b) lies on the same side of the plane P
(c) lies on two different sides of the plane P
(d) one of the two points lies on the plane P

83 The projection of the point $(1, 2, 3)$ on the plane $x + 2y + 4z = 59$ is ....

(a) $(1, 2, 4)$ (b) $(9, 15, 5)$ (c) $(5, 5, 11)$ (d) $(3, 6, 11)$

84 If the point B $(2, 1, 3)$ is the projection of the point A $(x_1, y_1, 6)$ on the plane $2x + y + z = 8$, then $x_1 \times y_1 =$ ....

(a) 10 (b) 16 (c) 32 (d) 42

85 The image of the point $(-3, 4, 2)$ by reflection on y z-plane is ....

(a) $(3, -4, -2)$ (b) $(3, 4, 2)$ (c) $(-3, -4, 2)$ (d) $(-3, 4, -2)$

86 The point A is the image of the point N $(1, 2, 3)$ by reflection on the plane xy and B is the image of A by rotation about the origin with an angle of measure 180° and C is the image of B by translation 5 units in positive direction of y-axis, then C = ...

(a) $(1, 7, -3)$ (b) $(-1, 7, -3)$ (c) $(-1, -2, 8)$ (d) $(-1, 3, 3)$

87 The direction cosines of the normal to the plane : $3x + 4y + z = 1$ is ....

(a) $\pm\left(\frac{3}{5}, \frac{4}{5}, \frac{1}{5}\right)$
(b) $\pm(3, 4, 1)$
(c) $\pm\left(\frac{3}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right)$
(d) $\pm\left(\frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{3}{\sqrt{26}}\right)$

88 The normal direction vector to the plane : $-2x - 4y + z - 15 = 0$ is ....

(a) $(2, 4, 1)$ (b) $(2, 4, 5)$ (c) $(2, -4, -1)$ (d) $(-2, -4, 1)$

---

89 The direction cosines of the normal to the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ from the following is ....

(a) $(2, 3, 4)$ (b) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

(c) $(6, 4, 3)$ (d) $\left(\frac{6}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{3}{\sqrt{61}}\right)$

---

90 The plane whose equation $2x - y = 0$ is perpendicular to the plane ....

(a) xy (b) yz (c) xz (d) $y - 2x = 0$

---

91 The measure of the angle of inclination of the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z}{3}$ on the plane $3x + 2y + z = 11$ equals ....

(a) $60°$ (b) $30°$ (c) $45°$ (d) $90°$

---

92 The measure of the angle between the straight line : $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$ and the plane : $x + y + 4 = 0$ is ....

(a) zero (b) $30°$ (c) $45°$ (d) $90°$

---

93 The measure of the smaller angle between the straight line $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + t(-\hat{i} + \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 4$ is ....

(a) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (b) $\cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$ (c) $\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (d) $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

---

94 If θ is the measure of the angle that the plane $2x - 3y + 6z = 11$ makes with the positive direction of x-axis , then $\sin\theta =$ ....

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{7}$ (c) $\frac{\sqrt{2}}{3}$ (d) 1

---

95 If measure of the angle between the straight line : $\frac{x-5}{2} = \frac{y}{-1} = \frac{z+4}{a}$ and the plane $3x + 2y + z = 5$ equals $30°$, then $a =$ .... where $a \in \mathbb{Z}$

(a) -3 (b) 2 (c) 3 (d) -4

---

96 The measure of the angle included between the two planes :

$x + y - 1 = 0$, $y + z - 1 = 0$ equals ....

(a) $30°$ (b) $45°$ (c) $60°$ (d) $75°$

97 The measure of the angle between the two planes :
$3x - 6y + 6z - 4 = 0$, $x + z - 7 = 0$ is ....
(a) 90° (b) 60° (c) 45° (d) 30°

98 The measure of the angle between the two planes
$3x - 4y + 5z = 0$, $2x - y - 2z = 5$ is ....
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

99 The measure of the included angle between the two planes $\vec{r} \cdot (1, 1, 2) = 7$ and $2x - y + z = 6$ equals ....
(a) 30° (b) 45° (c) 60° (d) 90°

100 The sine of the angle between the two planes $\vec{r} \cdot (2, -1, 4) = 5$ and $3x - y + 2z = 4$ is ....
(a) $\frac{5\sqrt{6}}{14}$ (b) $\frac{23}{98}$ (c) $\frac{\sqrt{46}}{14}$ (d) $\frac{75}{98}$

101 The angle between the two planes :
$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$, $\vec{r} \cdot (-\hat{i} + \hat{j}) = 4$ of measure : ....
(a) $\frac{5}{\sqrt{58}}$ (b) $\cos^{-1}\left(\frac{5}{\sqrt{58}}\right)$ (c) $\cos^{-1}\left(\frac{\sqrt{58}}{5}\right)$ (d) $\cos\left(\frac{\sqrt{58}}{5}\right)$

102 The measure of the angle between the two planes $y = 0$ and $z = 0$ is ....
(a) 30° (b) 45° (c) 60° (d) 90°

103 The measure of the angle between the two planes $x = 2$, $y = -3$ is ....
(a) zero (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

104 *(2nd Session 2021)* The measure of the angle between the plane xy and the plane $x + \sqrt{3}z - 7 = 0$ equals ....°
(a) 60 (b) 90 (c) 30 (d) 45

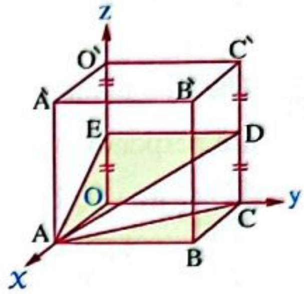
105 If measure of the angle between the two planes $(3, -4, 2) \cdot \vec{r} = 7$, $3x + 4y - mz = 12$ is 90° , then m = ...
(a) $\frac{-7}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{-25}{2}$ (d) $\frac{3}{2}$

106 *(1st Session 2021)* If the measure of the angle between the two planes :
$x + y - 1 = 0$, $ky + z - 1 = 0$ equals $60°$, then $k =$ ………… , where $k > 0$

(a) 4 (b) $\frac{1}{2}$ (c) 2 (d) 1

---

107 **In the opposite figure :**



A cube of side length 2 unit

D , E are midpoints of $\overline{CC`}$ and $\overline{OO`}$, then the measure of the angle between the plane ADE to the plane ABC equals ……………

(a) $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (b) $\cos^{-1}\left(\frac{2}{5}\right)$

(c) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (d) $\cos^{-1}\left(\frac{1}{5}\right)$

---

108 The two planes : $a_1 x_1 + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are parallel if …………

(a) $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ (b) $a_1 = m a_2$, $b_1 = m b_2$, $c_1 = m c_2$

(c) $a_1 = a_2$, $b_1 = b_2$ (d) $\frac{a_1}{a_2} = \frac{b_2}{b_1} = \frac{c_1}{c_2}$

---

109 If the plane $x - 3y + mz = 5$ and the plane $3x + ky + 6z = 10$ are parallel , then $k \times m =$ …………

(a) -6 (b) -12 (c) -18 (d) -24

---

110 If the two planes : $2x - y + kz = 5$ and $x + \ell y + 4z = 1$ are parallel , then $\ell + k =$ …………

(a) 8 (b) 9 (c) $7\frac{1}{2}$ (d) 6

---

111 *(1st Session 2021)* If the two planes $2x + cy + 4z = 1$ and $(a + 2)x + 6y + (b - 2)z = 5$ are parallel , then $2a - b =$ …………

(a) -6 (b) 6 (c) -12 (d) 12

---

112 The two planes $-2x + 3y - 5z = 3$, $-4x + 6y - 10z = 10$ are …………

(a) intersect. (b) parallel. (c) coincide. (d) perpendicular.

113 If the plane P_1 intercepted from positive parts of cartesian coordinates 2 , 3 , 5 units and the plane P_2 intercepted from positive parts of cartesian coordinates 4 , 6 , 10 units , then the two planes are ....

(a) coincident.
(b) parallel.
(c) perpendicular.
(d) intersecting and not perpendicular.

114 The two planes $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are perpendicular if ....

(a) $a_1 a_2 + b_1 b_2 + c_1 c_2 = 1$
(b) $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$
(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(d) $a_1 a_2 + b_1 b_2 + c_1 c_2 + d_1 d_2 = 0$

115 If the two planes : $3x - y + 2z + 4 = 0$, $x + 2y + kz = 2$ are perpendicular , then $k =$ ....

(a) -4
(b) $\frac{2}{3}$
(c) $\frac{1}{2}$
(d) $-\frac{1}{2}$

116 The two planes : $x + 2y + kz = 0$, $\frac{x}{2} + \frac{y}{4} + \frac{z}{-2} = 1$ are perpendicular to each other if $k =$ ....

(a) $-\frac{1}{2}$
(b) $\frac{1}{2}$
(c) -2
(d) 2

117 If the two planes $(\sin^2\theta)\, x + a\, y + z = 0$ and $2x - 4y + (\cos 2\theta)\, z = 7$ are perpendicular , then $a =$ .... for all values of θ

(a) 4
(b) 2
(c) $\frac{1}{2}$
(d) $\frac{1}{4}$

118 The two planes $2x - y + 2z = 8$, $3x + 4y - z - 7 = 0$ are ....

(a) perpendicular.
(b) parallel.
(c) coincide.
(d) the measure of their included angle $= \frac{\pi}{4}$

119 The two planes : $2x + 3y + 4z - 12 = 0$, $\vec{r} \cdot (3 , -6 , 0) = -1$ are ....

(a) coincident.
(b) parallel.
(c) intersecting and perpendicular.
(d) intersecting but not perpendicular.

120 If the plane $a x + b y + 3z = 1$ is perpendicular to both the two planes $3x - 4y + 2z = 0$ and $2x - 3y - 5 = 0$, then $a - b =$ ....

(a) -21
(b) -6
(c) 6
(d) 21

121 The line of intersection of the two planes $y = 5$, $x = 4$ ....

(a) parallel to the plane $z = 0$
(b) has direction vector $(4 , 5 , 0)$
(c) has direction vector $(0 , 0 , 9)$
(d) lies in the x y-plane.

---

122 The line of intersection of the two planes $\vec{r} \cdot (3 , -1 , 1) = 1$, $\vec{r} \cdot (1 , 1 , -2) = 2$ is parallel to the vector ....

(a) $(2 , 7 , 1)$
(b) $(-2 , 7 , 13)$
(c) $(1 , 7 , 4)$
(d) $(7 , 13 , 1)$

---

123 The equation of the line of intersection of the two planes $P_1 : 2x - y + z - 1 = 0$, $P_2 : x - 3y - z + 2 = 0$ is ....

(a) $\frac{x+1}{-1} = \frac{y}{2} = \frac{z}{3}$
(b) $\frac{x-1}{1} = -\frac{y}{3} = \frac{z-5}{1}$
(c) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z}{-1}$
(d) $\frac{x-1}{4} = \frac{y-1}{3} = \frac{z}{-5}$

---

124 Which of the following pairs of planes , their intersection line parallel x-axis ?

(a) $x + 2y - z = 5$, $2x - y + 3z = 3$
(b) $x - 4y = 1$, $x - y = 6$
(c) $y - 2z = 3$, $2y + z = -1$
(d) $x - 3z = 4$, $2x + z = 0$

---

125 The set of points equidistant from the two points $A(-4 , 2 , 1)$, $B(2 , -4 , 3)$ represents ....

(a) the straight line $\frac{x+4}{2} = \frac{y-2}{-4} = \frac{z-1}{3}$
(b) the straight line $\frac{x-2}{4} = \frac{y+4}{2} = \frac{z-3}{1}$
(c) the plane $3x - 3y + z = 2$
(d) the sphere $(x + 2)^2 + (y - 4)^2 + (z + 3)^2 = 2$

---

126 ***(Trial 2021)*** If the straight line : $\frac{x-2}{3} = \frac{y+1}{-4} = \frac{z+3}{5}$ makes angles of measures ℓ , m , n with the x y-plane , y z-plane and x z-plane respectively , then $\sin^2 \ell + \sin^2 m + \sin^2 n =$ ....

(a) 1
(b) 2
(c) $\frac{3}{2}$
(d) $\sqrt{3}$

127 *(Trial 2021)* A set of points in space satisfies the equations $x^2 + y^2 + z^2 = 25$, $z = -4$ represents ....

(a) a plane at a distance 4 length units from the plane $x y$

(b) a circle has centre $(0, 0, -4)$ and its radius length 3 length units.

(c) a circle has centre at the origin and its radius length 5 length units.

(d) a sphere has centre at the origin and its radius length 4 length units.

---

128 The distance between the point $(3, -1, 2)$ and the $x z$ plane equals .... length unit.

(a) 3 (b) -1 (c) 2 (d) 1

---

129 $x z$-plane divides the line segment $\overrightarrow{AB}$ where $A(2, 4, 5)$, $B(3, 5, -9)$ by ratio ....

(a) 4 : 5 internally. (b) 5 : 4 internally. (c) 4 : 5 externally. (d) 5 : 3 externally.

---

130 The plane y z divides $\overrightarrow{AB}$ where $A(2, 3, 1)$, $B(-7, 6, 1)$ by ratio ....

(a) 1 : 2 internally. (b) 2 : 7 externally. (c) 1 : 1 (d) 2 : 7 internally.

---

131 The length of the perpendicular from point $(2, 3, 1)$ to the plane $2x - 2y + z = 5$ is .... length unit.

(a) 1 (b) 2 (c) 3 (d) 4

---

132 The distance between the plane $2x - 3y + 6z + 14 = 0$ and the origin point is .... length unit.

(a) 11 (b) 2 (c) -2 (d) 14

---

133 If the length of the perpendicular drawn from the point $(2, -3, 1)$ to the plane $2x - 2y + z + a = 0$ equals 6 length unit , then $a =$ ....

(a) ± 6 (b) ± 7 (c) ± 29 (d) 7 or -29

---

134 If the direction angles of the normal to a plane are $\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}$ respectively and the distance between the plane and the origin equals $\sqrt{2}$, then the equation of the plane could be ....

(a) $x + y = 2$ (b) $\sqrt{2}\,x + y = 2$

(c) $\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 2$ (d) $x + y + 2\sqrt{2} = 0$

135 The distance between the two planes $y = 4$, $y = -2$ is ....

(a) 3 units. (b) two units. (c) 6 units. (d) 8 units.

136 If the distance between the two planes $y = 4$ and $y = a$ equals 6 length unit , then $a =$ ....

(a) 10 (b) -2 (c) 10 or -2 (d) 2

137 The length of the perpendicular drawn between the two planes

$3x + 12y - 4z = 9$, $3x + 12y - 4z = -17$ equals .... length unit.

(a) 2 (b) 3 (c) 4 (d) 5

138 The distance between the two planes : $x - 2y - 2z - 12 = 0$, $\vec{r} \cdot (-3, 6, 6) = 9$

equals .... length unit.

(a) 2 (b) 3 (c) 4 (d) 5

139 The distance between the two parallel planes : $ax + by + cz + d_1 = 0$

and $ax + by + cz + d_2 = 0$ equals ....

(a) $\frac{|ax + by + cz + d_1|}{\sqrt{a^2 + b^2 + c^2}}$ (b) $|d_2 - d_1|$

(c) $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$ (d) $\sqrt{a^2 + b^2 + c^2}$

140 The distance between the x-axis and the plane $\frac{y}{b} + \frac{z}{c} = 1$ is .... length unit

(where $bc > 0$)

(a) bc (b) $\sqrt{b^2 + c^2}$

(c) $\frac{bc}{\sqrt{b^2 + c^2}}$ (d) $\frac{b + c}{\sqrt{b^2 + c^2}}$

141 The distance between the origin point and the plane $\frac{x}{a} + \frac{y}{b} = 1$ is .... length unit

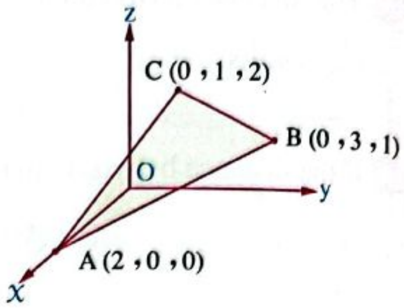
(where $ab > 0$)

(a) ab (b) $\sqrt{a^2 + b^2}$

(c) $\frac{ab}{\sqrt{a^2 + b^2}}$ (d) $\frac{a + b}{\sqrt{a^2 + b^2}}$

142 **In the opposite figure :**

The length of the perpendicular drawn from the origin to the plane ABC equals .... length units.



ⓐ $2\sqrt{5}$ ⓑ $\frac{2}{3}$

ⓒ $\frac{2}{\sqrt{5}}$ ⓓ $\frac{2\sqrt{5}}{3}$

---

143 If the distance between two parallel planes $2x + y + 2z - k = 0$ and $4x + 2y + 4z + 5 = 0$ equals 3.5 length units , then k = ...

ⓐ 8 or 13 ⓑ 8 or – 13 ⓒ – 8 or 13 ⓓ – 8 or – 13

---

144 If the intercepted parts of the coordinate axes by a plane are 8 , 4 , 4 , then the distance between the origin and this plane equals ....

ⓐ $\frac{4}{3}$ ⓑ $\frac{5}{3}$ ⓒ $\frac{8}{3}$ ⓓ $\frac{16}{3}$

---

145 If the distance between the point (1 , 1 , 1) from the origin equals half the distance between the same point and the plane $x + y + z + k = 0$, then $k \in$ ....

ⓐ $\{-3 , 3\}$ ⓑ $\{3 , -9\}$ ⓒ $\{-3 , 9\}$ ⓓ $\{-9 , 9\}$

---

146 If the two points A (1 , 1 , k) and B (– 3 , 0 , 1) are equidistant from the plane $3x + 4y - 12z + 13 = 0$, then k is one of the roots of the equation ....

ⓐ $3x^2 + 10x - 13 = 0$ ⓑ $3x^2 - 10x + 21 = 0$

ⓒ $3x^2 - 10x + 7 = 0$ ⓓ $3x^2 + 10x - 7 = 0$

---

147 The plane whose equation $2x - y + 3z - 5 = 0$ .... the sphere whose equation $(x - 1)^2 + (y + 2)^2 + (z - 3)^2 = 4$

ⓐ touches ⓑ intercests

ⓒ passes through the centre ⓓ outside

---

148 The centre of a sphere lies on the straight line : $\vec{r} = (3 , 5 , 4) + t\,(3 , -4 , 0)$ and touches the plane $4x + 3y - 12z = 5$, then its radius equals .... length unit.

ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 5

149 The volume of a sphere with centre $(-2, 1, -1)$ and touches the plane $2x + 2y + z = 6$ is .... volume units.

(a) 9π (b) 18π (c) 36π (d) 72π

150 If a sphere with equation $(x-2)^2 + (y+1)^2 + (z+2)^2 = 9$ is placed on the plane $2x + 6y - 3z + k = \text{zero}$ where $k > 0$, then $k =$ ....

(a) 12 (b) 13 (c) 15 (d) 17

151 The shortest distance between the plane $2x + y + 2z + 17 = 0$ and the sphere $x^2 + y^2 + z^2 - 2x - 4y = 4$ is .... length units.

(a) 2 (b) 4 (c) 7 (d) 9

152 *(Trial 2021)* If the plane $2x - y + 2z = 6$ touches the surface of the sphere whose equation $x^2 + y^2 + z^2 - 4x - 2y + 6z + 5 = \text{zero}$, then equation of the straight line which passing through the centre of the sphere and point of tangency is ....

(a) $\vec{r} = (2, 1, -3) + t(2, -1, 2)$ (b) $\vec{r} = (2, 1, -3) + t(4, 0, -1)$

(c) $\vec{r} = (4, 0, -1) + t(2, 1, -3)$ (d) $\vec{r} = (2, -1, 2) + t(2, 1, 3)$

153 The area of the resulted region from the intersection of the sphere $x^2 + y^2 + z^2 + 10x - 6y - 8z - 75 = 0$ and the y z-plane equals .... square unit.

(a) 100π (b) 125π (c) 20π (d) $10\sqrt{3}$

154 The radius length of the circular section resulting from intersecting the sphere : $x^2 + y^2 + z^2 - 4y + 2z = 15$ with the plane $2x + y - 2z = 10$ is .... length unit.

(a) $2\sqrt{5}$ (b) $\frac{4}{3}$ (c) 4 (d) 3

155 The radius length of the circular cross section of the sphere $x^2 + y^2 + z^2 - 2y - 4z = 20$ by the plane $x + 2y + 2z = 15$ is .... length unit.

(a) 7 (b) $\sqrt{7}$ (c) 4 (d) 3

156 If the length of the perpendicular drawn from the origin point to the plane P is 7 length units and the direction ratios of the straight line that carries it are $-3, 2, 6$ which of the following equations could be the equation of the plane P ?

(a) $-3x + 2y + 6z - 7 = 0$ (b) $-3x + 2y + 6z - 49 = 0$

(c) $3x - 2y + 6z + 7 = 0$ (d) $-3x + 2y - 6z - 49 = 0$

157 If the distance between point (A) and the plane (P) equals 16 length units and a straight line passing through (A) intersects the plane (P) at point (B) and makes with the plane (P) an angle of measure θ such that $\tan \theta = \frac{8}{15}$, then length of the projection of $\overline{AB}$ on the plane = ... length units.

(a) 30 (b) 15 (c) 16 (d) 8

---

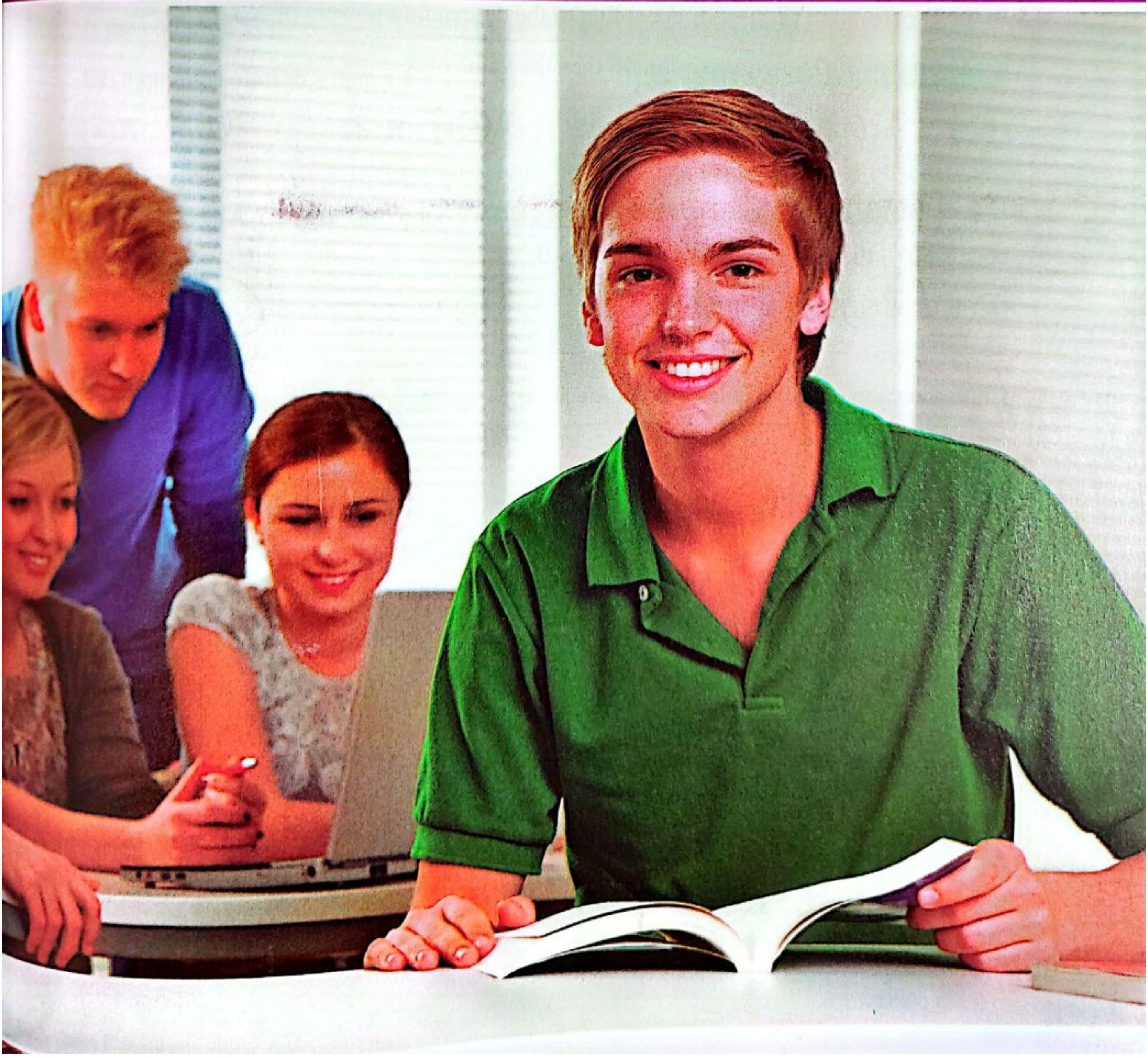
158 ***(Trial 2021)*** If the plane $b\,c\,x + a\,c\,y + a\,b\,z = a\,b\,c$ intercepts the coordinate axes at the point K , N , M respectively and the plane : $b\,c\,x + a\,c\,y - a\,b\,z = -\,a\,b\,c$ intercepts the coordinate axes at the points $\grave{K}$, $\grave{N}$, M respectively , then the pyramid M K N $\grave{K}$ $\grave{N}$ is .... where a , b , c are positive real numbers , $a \neq b$

(a) a right pyramid
(b) a regular quadrilateral pyramid
(c) a right triangular pyramid
(d) a regular triangular pyramid

Practice Exams

on

Algebra & Analytic Solid Geometry



Practice Exams

Exam 1

Choose the correct answer from the given ones :

1. The exponential form of the complex number $z = -2(1 - i)$ is ....

   (a) $2e^{\frac{3}{4}\pi i}$ (b) $2e^{-\frac{\pi}{4}i}$ (c) $2\sqrt{2}\,e^{\frac{2\pi}{3}i}$ (d) $2\sqrt{2}\,e^{\frac{3\pi}{4}i}$

2. $\left(\omega^2 + \frac{1}{\omega}\right)\left(1 + \frac{1}{\omega^2}\right)^2 =$ ....

   (a) 2 (b) zero (c) -3 (d) -5

3. In the expansion of $\left(\sqrt{x} + \frac{1}{x}\right)^8$ according to the descending powers of x , if $5T_4$, T_5 , T_7 , T_6 are proportional , then the value of $x =$ ....

   (a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) $\frac{5}{2}$ (d) $\frac{8}{5}$

4. If the measure of the angle between the two planes : $x + y - 1 = 0$, $k\,y + z - 1 = 0$ equals $60°$, then $k =$ .... , where $k > 0$

   (a) 4 (b) $\frac{1}{2}$ (c) 2 (d) 1

5. If the direction angles of the straight line are θ_x , θ_y , θ_z , then $\cos 2\theta_x + \cos 2\theta_y + \cos 2\theta_z =$ ....

   (a) -2 (b) -1 (c) 1 (d) 2

6. If z is a complex number and $z + \bar{z} = 2e^{\pi i}$, then z could be equal ....

   (a) $e^{\pi i}$ (b) $2e^{\frac{\pi}{2}i}$ (c) $e^{\frac{-\pi}{2}i}$ (d) $2e^{\pi i}$

7. In the triangle XYZ if $\begin{vmatrix} 0 & 0 & x \\ 0 & y & 0 \\ z & 0 & 0 \end{vmatrix} = -100\text{ cm}^3$ and the area of Δ XYZ $= 6.25\text{ cm}^2$, then the length of the diameter of circumcircle of triangle XYZ = ... cm.

   (a) 4 (b) 16 (c) 8 (d) 2

Scanned with CamScanner

8. The projection of the point A $(0, 9, 6)$ on the straight line passing through the two points B $(1, 2, 3)$, C $(7, -2, 5)$ is the point ....

(a) $(-2, 4, 2)$ (b) $(2, -4, 2)$ (c) $(1, 2, -1)$ (d) $(1, -2, -1)$

---

9. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, then $(A^2)^{-1} =$ ....

(a) $\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} -\cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$

---

10. If the equation system $x + y + z = 6$, $4x - ky - kz = 0$, $3x + 2y - 4z = -8$ has a unique solution, then $k \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{-4\}$ (d) $\mathbb{R}^*$

---

11. If the matrix (A_{xy}) is of the order 3×3 where $a_{xy} = 2x - y$, then the rank of the matrix A is ....

(a) 3 (b) 2 (c) 1 (d) zero

---

12. If $\vec{A} = \hat{i} + \hat{j} - \hat{k}$, $\vec{B} = -\hat{i} + \hat{j} + \hat{k}$, $\vec{C} = -\hat{i} + 2\hat{j} - \hat{k}$, then the unit vector perpendicular to each of $\vec{A} + \vec{B}$, $\vec{B} + \vec{C}$ from the following is ....

(a) $\hat{i}$ (b) $\hat{k}$ (c) $\hat{j}$ (d) $\dfrac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{3}}$

---

13. If the two planes $2x + cy + 4z = 1$ and $(a + 2)x + 6y + (b - 2)z = 5$ are parallel, then $2a - b =$ ....

(a) -6 (b) 6 (c) -12 (d) 12

---

14. The vector equation of the straight line passing through the point A $(2, -1, 4)$ and parallel to the bisector of the angle between $\overrightarrow{Oy}$, $\overrightarrow{Oz}$ in the plane y z is ....

(a) $\vec{r} = (2, -1, 4) + t(0, 1, 1)$ (b) $\vec{r} = (2, -1, 4) + t(0, -1, 1)$

(c) $\vec{r} = (2, -1, 4) + t(-1, 0, 1)$ (d) $\vec{r} = (2, -1, 4) + t(1, 0, -1)$

15 If $2\,{}^{n+1}C_r = {}^{n+1}P_r$, $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{5}{3}$, then ${}^nC_r + {}^nP_r = $ ....

(a) 63 (b) 33 (c) 60 (d) 36

16 In the expansion of $\left(x^2 + 2 + \frac{1}{x^2}\right)^6$ the coefficient of the term containing x^2 is ....

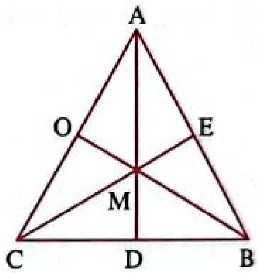
(a) ${}^{12}C_5$ (b) ${}^{12}C_6$ (c) ${}^{12}C_2$ (d) ${}^{6}C_5$

17 If $\overrightarrow{A} + \overrightarrow{BC} = 4\hat{i} + 12\hat{j} + 9\hat{k}$ where $\overrightarrow{A} = (0 , -1 , 3)$, $\overrightarrow{B} = (4 , -2 , 1)$, then $\overrightarrow{C} = $ ....

(a) $8\hat{i} + 13\hat{j} + 13\hat{k}$ (b) $8\hat{i} + 11\hat{j} + 7\hat{k}$

(c) $8\hat{i} + 9\hat{j} + 7\hat{k}$ (d) $8\hat{i} + 13\hat{j} - 7\hat{k}$

18 **In the opposite figure :**



ABC is an equilateral triangle of side length 4 cm.

, M is the point of intersection of its medians

, then $\overrightarrow{MB} \cdot \overrightarrow{CM} = $ ....

(a) $-\frac{8}{3}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{8}{3}$ (d) $\frac{16}{3}$

19 If $n = {}^mC_2$, then ${}^nC_2 = $ ....

(a) ${}^{m+1}C_4$ (b) ${}^{m-1}C_4$ (c) $3\;{}^{m+1}C_4$ (d) ${}^{m+2}C_4$

20 If the vector $\overrightarrow{n} = (a , 4 , c)$ is parallel to the cartesian plane y z and $\|\overrightarrow{n}\| = 5$

, then $c^2 = $ ....

(a) 3 (b) 9 (c) 12 (d) 20

21 In the triangle ABC , if $\begin{vmatrix} a^2 & c^2 & -b^2 \\ 1 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix} = ac$,

where a , b and c are the side lengths of triangle ABC , then m (∠ B) = ...°

(a) 45 (b) 90 (c) 60 (d) 120

22. If $z = k\left(\sin \frac{4}{3}\pi - i\cos \frac{4}{3}\pi\right)$, then $z^6 =$ .... , where $k >$ zero

(a) k^6 (b) $6k$ (c) $-k^6$ (d) $-6k$

23. If the shortest distance between the point A (3 , 5 , 1) and the surface of the sphere whose centre M (1 , 2 , – 5) is 2 length unit where A lies outside the sphere , then the radius of the sphere = ... length unit.

(a) 5 (b) 2 (c) 7 (d) 12

24. The middle term in the expansion of $\left(1 - \frac{1}{x}\right)^n (1 - x)^n$ equals ....

(a) ${}^{2n}C_n$ (b) $-{}^{2n}C_n$ (c) $-{}^{2n}C_{n-1}$ (d) ${}^{2n}C_{n-1}$

25. The number of even numbers formed from 3 different digits from the set $\{0 , 1 , 2 , 3\}$ equals ....

(a) 4 (b) 6 (c) 10 (d) 24

Choose the correct answer from the given ones :

1. If $\overrightarrow{A} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, $\overrightarrow{B} = -2\hat{i} + 2\hat{j} - \hat{k}$

, then : $\dfrac{\text{The projection of } \overrightarrow{A} \text{ in direction of } \overrightarrow{B}}{\text{The projection of } \overrightarrow{B} \text{ in direction of } \overrightarrow{A}} = \ldots\ldots\ldots\ldots$

(a) $\frac{3}{7}$ (b) 7 (c) 3 (d) $\frac{7}{3}$

2. An office has 9 men and 6 women. It is required to form a committee of 5 persons and the majority of them should be women and contains the two genders , then the number of committees equals ……………

(a) 11880 (b) 2871 (c) 3003 (d) 855

3. If 1 , ω , ω^2 are the cubic roots of unity , then the value of the determinant :

$$\begin{vmatrix} 1 & \omega & \omega - 1 \\ 1 & -1 & \omega + 1 \\ 1 & \omega & \omega \end{vmatrix} = \ldots\ldots\ldots\ldots$$

(a) $\omega - 1$ (b) ω^2 (c) ω (d) $\omega^2 + 1$

4. If the perpendicular distance between the point (– 1 , 2 , m) and the straight line : $\overrightarrow{r} = (-1, 3, 0) + t(0, -3, 0)$ is 8 length unit , then the value of m equals …………… , where $m \in \mathbb{R}^+$

(a) 4 (b) 16 (c) 8 (d) 2

5. If ${}^{n+1}P_r > {}^{n+1}P_{r-1}$, then $n >$ ……………

(a) $r - 1$ (b) $r - 3$ (c) $r + 1$ (d) $1 - r$

6. If $1 + 5x + \frac{5 \times 4}{2 \times 1} x^2 + \frac{5 \times 4 \times 3}{3 \times 2 \times 1} x^3 + \cdots + x^5 = 1024$, then $x =$ ……………

(a) 1 (b) 2 (c) 10 (d) 3

7 ABCD is a parallelogram and $\overrightarrow{AB} = (2, 2, -1)$, $\overrightarrow{AD} = (-1, 2, -3)$, then the area of the parallelogram = ... square unit.

(a) 6 (b) $7\sqrt{2}$ (c) $3\sqrt{11}$ (d) $\sqrt{101}$

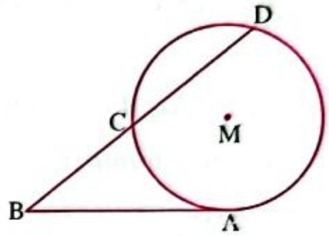
8 $\left(\frac{a}{\omega} - \frac{a}{\omega^2} + \frac{3a}{\omega^4} - \frac{3a}{\omega^5}\right)^2 =$ ....

(a) $-48a^2$ (b) $48a^2$ (c) $16a^2$ (d) $-16a^2$

9 **In the opposite figure :**

If $\overleftrightarrow{AB}$ is a tangent to the circle M at A , $\overline{DC}$ is a chord in the circle where

$\overleftrightarrow{DC} \cap \overleftrightarrow{AB} = \{B\}$, if $\begin{vmatrix} 1 & 0 & CD \\ -1 & AB & BC \\ 0 & -BC & AB \end{vmatrix} = 32$



, then AB = ... length unit.

(a) 8 (b) 4 (c) 16 (d) 6

10 If the greatest coefficient in the exponsion of $(a + x)^{20}$ is the coefficient of T_{11} , then $a \in$ .... where $a \in \mathbb{R}^+$

(a) $\left[\frac{10}{11}, \frac{11}{10}\right]$ (b) $[10, 11]$ (c) $\left[\frac{11}{10}, \frac{10}{11}\right]$ (d) $\left[\frac{-9}{11}, \frac{10}{11}\right]$

11 The plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line segment joining the two centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$, $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then the value of a = ...

(a) 4 (b) -2 (c) 6 (d) 1

12 If $\vec{n}$ is the perpendicular unit vector to the plane containing the two vectors $\vec{A}$, $\vec{B}$ where $\vec{n} = \left(\frac{3}{5}, 0, \frac{4}{5}\right)$ and $\|\vec{A} \times \vec{B}\| = 5$, then $(3\vec{A} + \vec{B}) \times (4\vec{A} + 2\vec{B})$ can be equal ....

(a) (3 , 0 , 4)
(b) (30 , 0 , 40)
(c) (– 30 , 0 , – 40)
(d) (6 , 0 , 8)

13 If the coefficient of the term containing x^4 in the expansion of $\left(x+\frac{a}{x^2}\right)^7$ equals 49 , then the value of the constant a = ...

(a) -7 (b) 49 (c) -49 (d) 7

14 If $\|\overrightarrow{A}\|=\sqrt{13}$, $\overrightarrow{A} \,//\, \overrightarrow{DC}$ and in its direction , such that D (1 , 3 , − 2) , C (1 , − 1 , 4) and $\overrightarrow{B}=(-2, 3, 5)$, then $\overrightarrow{A}\times\overrightarrow{B}=$ ....

(a) $-19\,\hat{i}+6\,\hat{j}+4\,\hat{k}$ (b) $-28\,\hat{i}-12\,\hat{j}-8\,\hat{k}$

(c) $-28\,\hat{i}+12\,\hat{j}-8\,\hat{k}$ (d) $-19\,\hat{i}-6\,\hat{j}-4\,\hat{k}$

15 If $z_1=\cos\theta+i\sin\theta$, $z_2=\cos 2\theta+i\sin 2\theta$, then the principle amplitude of the complex number $3\,z_1\,z_2=$ .... , where $\theta\in\,]0\,,\frac{\pi}{6}[$

(a) $9\,\theta$ (b) θ (c) $5\,\theta$ (d) $3\,\theta$

16 If ${}^{n}C_r : {}^{n-1}C_r = 3 : 1$, then $\underline{\left|\frac{4n}{r}\right.}=$ ....

(a) 24 (b) 120 (c) 720 (d) 5040

17 On Argand diagram , area of the triangle whose vertices are the points represent the cubic roots of one equals .... square units.

(a) $\frac{3\sqrt{3}}{4}$ (b) $\frac{3\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$

18 The plane equation passing through the points (2 , 3 , 5) , (− 1 , 3 , 1) , (4 , 3 , − 2) is ....

(a) $x+y-z=0$ (b) $x=-1$ (c) $y=3$ (d) $z=-2$

19 The measure of the angle between the plane xy and the plane $x+\sqrt{3}\,z-7=0$ equals ....°

(a) 60 (b) 90 (c) 30 (d) 45

20 If $z_1=3(\cos 300°+i\sin 300°)$, $z_2=2(\sin 240°+i\cos 240°)$, then which of the following is the exponential form of the number $(z_1\,z_2)$?

(a) $6\,e^{\frac{5}{6}\pi i}$ (b) $6\,e^{\pi i}$ (c) $\frac{3}{2}\,e^{\frac{5}{6}\pi i}$ (d) $\frac{3}{2}\,e^{\pi i}$

21 $\begin{vmatrix} x & y & y \\ y & x & y \\ y & y & x \end{vmatrix} = (x + 2y) \times$ ....

(a) $\begin{vmatrix} 1 & y & y \\ 0 & x-y & 0 \\ 0 & 0 & x-y \end{vmatrix}$

(b) $\begin{vmatrix} 1 & y & y \\ 0 & x+y & 0 \\ 0 & 0 & x+y \end{vmatrix}$

(c) $\begin{vmatrix} 1 & y & 0 \\ 0 & x+y & 0 \\ 0 & 0 & x-y \end{vmatrix}$

(d) $\begin{vmatrix} 1 & y & y \\ 0 & x-y & 2y \\ 0 & 0 & x+y \end{vmatrix}$

22 If A* is the augmented matrix for the system of the equation :

$3x + 2y - z = 4$, $x + y - z = 3$, $x = 2z$, then ....

(a) $2 < RK(A^*) < 4$

(b) $RK(A^*) < 3$

(c) $1 < RK(A^*) \le 2$

(d) $1 \le RK(A^*) < 3$

23 If the direction vector of the striaght line :

$\frac{2x - \ell}{4} = \frac{y-2}{2} = \frac{z-2}{5}$ is $\vec{d} = (m , n , 10)$ and the straight line passes through the point $(4 , 4 , 7)$, then $\ell + m + n =$ ....

(a) 10

(b) 12

(c) 14

(d) 16

24 A sphere touches the cartesian planes xy , xz , yz and passes through the point $(6 , 3 , -3)$, then the radius = ... length unit.

(a) 2 or 4

(b) 3 or 9

(c) 5 or 7

(d) 2 or 15

25 If A is a non-singular matrix , then Adj (A) = ...

(a) $|A| A^{-1}$

(b) $(A^t)^{-1}$

(c) $\frac{1}{|A|} A^{-1}$

(d) $|A^t|$

Choose the correct answer from the given ones :

1. If one of terms of the expansion $\left(x^2 + \frac{2}{x^3}\right)^{12}$ on the form $\frac{a}{x}$, then a = ...

   (a) $2^5 \times {}^{12}C_5$ (b) ${}^{12}C_4 \times {}^{7}C_2$ (c) $2^3 \times {}^{6}C_2$ (d) $3^5 \times {}^{5}C_2$

2. The measure of the angle between the two planes : $3x - 6y + 6z - 4 = 0$ and $x + z - 7 = 0$ is ....

   (a) 90° (b) 60° (c) 45° (d) 30°

3. If A is a square matrix of order 3 × 3 and $|A| = 5$, then $|2 \text{ Adj}(A)| =$ ....

   (a) 250 (b) 200 (c) 50 (d) 25

4. The perpendicular distance between the point (2 , 4 , 7) and the straight line $2x - 4 = \frac{2y - 8}{3} = \frac{2z - 14}{5}$ equals .... length unit.

   (a) zero (b) 1 (c) 2 (d) 5

5. If $x = \frac{4}{\sqrt{3} + i}$, $y = \frac{2}{\cos\frac{\pi}{6} - i \sin\frac{\pi}{6}}$, then ....

   (a) $x = y$ (b) x , y are conjugate.

   (c) $xy = 1$ (d) $x + y =$ zero

6. The direction cosines of the vector $\vec{A} = (-2k , 2k , k)$, where $k \in]0 , 1[$ are ....

   (a) $\left(\frac{-2}{3} , \frac{2}{3} , \frac{1}{3}\right)$ (b) $\left(\frac{-2k}{3} , \frac{2k}{3} , \frac{-K}{3}\right)$

   (c) $\left(\frac{-2}{3} , \frac{-2}{3} , \frac{-1}{3}\right)$ (d) $\left(\frac{2k}{3} , \frac{2k}{3} , \frac{K}{3}\right)$

7. The line of intersection of the two planes : $\vec{r} \cdot (3 , -1 , 1) = 1$, $\vec{r} \cdot (1 , 4 , -2) = 2$ is parallel to the vector ....

   (a) (−2 , 7 , 13) (b) (2 , 7 , −3) (c) (−2 , −7 , 13) (d) (2 , 7 , 13)

8 If $1, \omega, \omega^2$ are the cubic roots of unity, where a, b are two positive real numbers, then the conjugate of the number : $a\,\omega + b\,\omega^2$ is ....

(a) $a\,\omega^2 - b\,\omega$ (b) $a\,\omega - b\,\omega^2$ (c) $a\,\omega^2 + b\,\omega$ (d) $a\,\omega + b\,\omega^2$

9 If z_1, z_2 are roots of the equation $x + \frac{1}{x} = i$, then $z_1 z_2 =$ ....

(a) 1 (b) $e^{\frac{\pi}{2} i}$ (c) $\frac{1}{2} e^{\frac{\pi}{3} i}$ (d) ω

10 The straight line $\frac{x+3}{3} = \frac{y-2}{-2} = \frac{z+1}{1}$ and the plane $4x + 5y + 3z - 5 = 0$ are intersecting at the point ....

(a) $(3, 1, -2)$ (b) $(3, -2, 1)$

(c) $(2, -1, -3)$ (d) $(-1, -2, -3)$

11 If $4k \, \lfloor 2k-1 = \frac{32}{9} \times \frac{{}^{11}C_3 + {}^{11}C_4}{{}^{12}C_3} + 40$, then $k =$ ....

(a) 4 (b) 2 (c) 6 (d) $\lfloor 4$

12 If the point $A\left(\sqrt{k}, -\sqrt{k}\right)$ represents the complex number z on the Argand's plane, where $k > 1$, then the exponential form of the number z is ....

(a) $\sqrt{2k}\, e^{-\frac{\pi}{4} i}$ (b) $\sqrt{2}\, k\, e^{-\frac{\pi}{4} i}$ (c) $\sqrt{2k}\, e^{\frac{\pi}{4} i}$ (d) $\sqrt{2}\, k\, e^{\frac{\pi}{4} i}$

13 If $\overrightarrow{AB} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{CB} = -\hat{j} + \hat{i} - \hat{k}$, then $\overrightarrow{CA} =$ ....

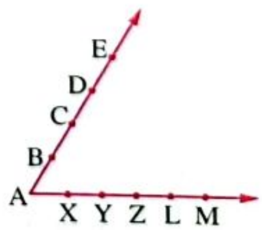
(a) $-\hat{i} + 2\hat{j} - \hat{k}$ (b) $3\hat{i} + 2\hat{j} - \hat{k}$ (c) $3\hat{i} + 4\hat{j} - \hat{k}$ (d) $-\hat{i} - 4\hat{j} + \hat{k}$

14 If ${}^7P_r = {}^5P_r$, then $r =$ ....

(a) 3 (b) 2 (c) 1 (d) zero

15 **In the opposite figure :**

The ten points lie on the two rays starting from the point A, then the number of different straight lines that can be drawn using these points equals ....



(a) 22 (b) 45 (c) 90 (d) 30

16 The area of the parallelogram in which : $\hat{i}$, $\hat{i} + \hat{j}$ are two adjacent vectors is .... square unit.

(a) 2
(b) $\frac{1}{2}$
(c) 1
(d) $\sqrt{2}$

---

17 If $A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & m & 6 \\ 5 & 7 & 9 \end{vmatrix}$ = , RK (A) = 3 , then $m \in$ ....

(a) $\{4\}$
(b) $\mathbb{R} - \{0\}$
(c) $\mathbb{R} - \{4\}$
(d) $\mathbb{R} - \{6\}$

---

18 If $\left(3\overrightarrow{A} \cdot 2\overrightarrow{B}\right) = 12$, then $\left(-\overrightarrow{B} \cdot 5\overrightarrow{A}\right) =$ ....

(a) -10
(b) -5
(c) 5
(d) 10

---

19 ABC is a triangle , where A (1 , 2 , 4) , B (– 2 , 0 , 5) , C (1 , 4 , 0) , if M is the intersection point of its medians , then the equation of straight line $\overleftrightarrow{AM}$ is ....

(a) $\vec{r} = (1 , 2 , 4) + t(-1 , 4 , 5)$
(b) $\vec{r} = (1 , 2 , 4) + t(-1 , 1 , 1)$
(c) $\vec{r} = (1 , 2 , 4) + t(-1 , 0 , -1)$
(d) $\vec{r} = (0 , 2 , 3) + t(1 , 2 , 4)$

---

20 In the expansion of $(1 + x)^{20}$ according to the ascending powers of x , if the coefficient of T_{r+2} = the coefficient of T_{r+4} , then the value of r = ...

(a) 9
(b) 8
(c) 10
(d) 11

---

21 In the expansion of $\left(x^5 - \frac{k}{x^2}\right)^{7n}$ according to the descending powers of x , the term free of x is .... , where k , $n \in \mathbb{Z}^+$

(a) T_{5n}
(b) T_{5n+1}
(c) T_{6n+1}
(d) T_{6n-1}

---

22 The system of equations $3x + y - z = 0$, $5x + 2y - 3z = 2$, $-x - 3y + 9z = 5$ has ....

(a) a unique solution.
(b) an infinite number of solutions.
(c) three solutions.
(d) no solution.

23 If $\begin{vmatrix} a & b & c \\ 1 & -2 & 3 \\ e & f & d \end{vmatrix} = 8$, then $\begin{vmatrix} 2 & -4 & 6 \\ 2a & 2b & 2c \\ e & f & d \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) 16　(b) 32　(c) -32　(d) -16

---

24 In the triangle ABC , if $\begin{vmatrix} a+2 & 3 & \sin C \\ 1 & b & 0 \\ 2 & 3 & \sin C \end{vmatrix} = 12$,

where a , b and c are the side lengths of the triangle ABC , then the surface area of the triangle ABC = ... unit area.

(a) 12　(b) 6　(c) 24　(d) 8

---

25 If M_1 , M_2 are two touching internally spheres and $M_1\left(-3 , 2 , -6\sqrt{2}\right)$, $r_1 = 8$ length unit $M_2\left(-2 , 1 , -5\sqrt{2}\right)$, then $r_2 =$ .... length unit , where $r_1 > r_2$

(a) 5　(b) 2　(c) 7　(d) 6

Choose the correct answer from the given ones :

1. By how many method to select at least one game from 20 games ?

(a) $2^{20} - 1$ (b) $\lfloor 20$ (c) 20 (d) $\lfloor 10$

2. If $z_1 = 15 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$ and $z_2 = 3 (\cos \theta + i \sin \theta)$, where $\theta \in]0, \frac{\pi}{2}[$,

then : $\frac{z_1}{z_2} = \ldots\ldots\ldots\ldots$

(a) $5 (\cos \theta + i \sin \theta)$ (b) $12 \left(\cos \left(\frac{\pi}{2} - \theta\right) + i \sin \left(\frac{\pi}{2} - \theta\right)\right)$

(c) $5 \left(\cos \left(\frac{\pi}{2} - \theta\right) + i \sin \left(\frac{\pi}{2} - \theta\right)\right)$ (d) $15 (\sin \theta + i \cos \theta)$

3. The sphere equation whose centre is the origin and passes through the point $(3, -1, 2)$ is $\ldots\ldots\ldots\ldots$

(a) $x^2 + y^2 + z^2 = 4$ (b) $(x - 3)^2 + (y + 1)^2 + (z - 2)^2 = 14$

(c) $(x - 3)^2 + (y + 1)^2 + (z - 2)^2 = \sqrt{14}$ (d) $x^2 + y^2 + z^2 = 14$

4. If $1, \omega, \omega^2$ are the cubic roots of unity, and $x = \frac{1}{1 + \omega i}$, $y = \frac{\omega + i}{1 + \omega^2 i}$, then $x - y = \ldots\ldots\ldots\ldots$

(a) $i + 1$ (b) $1 - i$ (c) 1 (d) i

5. If the straight line $\vec{r} = \hat{k} + t (2 \hat{i} + 3 \hat{j} + 4 \hat{k})$ is perpendicular to the plane $x + \ell y + m z = 5$, then $\ell \times m = \ldots\ldots\ldots\ldots$

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 12

6. If z_1, z_2 are two complex numbers, $z_1 = e^{5 + k \pi i}$, $z_2 = e^{(5 + k i) \pi}$,

where $-\frac{1}{2} < k < \frac{1}{2}$, then the principle amplitude of the complex number $z_1 + z_2$ could be equal $\ldots\ldots\ldots\ldots$

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{2}$

(c) π (d) $-\frac{\pi}{6}$

7. Length of perpendicular drawn between the two planes : $3x + 12y - 4z = 9$, $3x + 12y - 4z = -17$ equals .... length unit.

(a) 26 (b) 8 (c) 2 (d) 5

8. The straight line equation that passes through the two points A (1 , − 1 , 2) , B (− 1 , 0 , 1) could be ....

(a) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{1}$

(b) $\frac{x+1}{-2} = \frac{y}{1} = \frac{z+1}{-1}$

(c) $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-1}{2}$

(d) $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{1}$

9. If $(1 + a x)^n = 1 + 30x + 405x^2 + \cdots + a^n x^n$, then n : a = ...

(a) 5 : 1 (b) 10 : 3 (c) 3 : 10 (d) 5 : 3

10. Amplitude of the number : $z = (1 + \cos 40° + i \sin 40°)$ equals ....

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{18}$ (c) $\frac{\pi}{9}$ (d) $\frac{2\pi}{9}$

11. The general form of the equation of the plane which passing through the point (− 2 , 2 , − 1) and parallel to the plane whose equation : $(2, 3, -5) \cdot \vec{r} = 1$ is ....

(a) $2x + 3y - 5z = -7$

(b) $2x + 2y - z = 1$

(c) $2x - 3y + 5z = -7$

(d) $2x + 3y - 5z = 7$

12. If ${}^nP_r = {}^nP_{r+1}$, ${}^nC_r = {}^nC_{r-1}$, then n + r = ...

(a) 3 (b) 4 (c) 5 (d) 6

13. If $\vec{A}$, $\vec{B}$, $\vec{C}$ represent three adjacent edges in a parallelepiped . $\|\vec{A}\| = 2$ and the direction angles of vector $\vec{A}$ are (135° , 60° , 120°) , $\vec{B} = (1, \sqrt{2}, 0)$, $\vec{C} = (\sqrt{2}, 3, 5)$, then the volume of the parallelepiped = ... cubic unit.

(a) 16 (b) $6\sqrt{2}$ (c) 11 (d) $16\sqrt{2}$

14 If $\overrightarrow{A}$, $\overrightarrow{B}$ are two vectors where $\|\overrightarrow{A}\| = 5$, and the component of vector $\overrightarrow{B}$ in the direction of vector $\overrightarrow{A}$ is 3 , then $\overrightarrow{A} \cdot \overrightarrow{B} =$ ....

(a) 15
(b) $\frac{5}{3}$
(c) $\frac{3}{5}$
(d) 8

---

15 If : $(X - 3)$ is a factor of the determinant $\begin{vmatrix} 1 & m & 6 - X \\ X - 2 & X + 2 & X \\ X - 1 & 2 & 4 \end{vmatrix}$, then m = ...

(a) 3 (b) 4 (c) 5 (d) 6

---

16 If A is a non-singular matrix , then the false statment in each of following ....

(a) A has a multiplicative inverse.
(b) RK (A) = RK (A^{-1})
(c) $|A| = |A^t|$
(d) $A + A^{-1} = \square$

---

17 If the measure of the angle between the two planes : $(3 , -4 , 2) \cdot \vec{r} = 7$ and $3X + 4y - mz = 12$ is 90° , then m = ...

(a) $\frac{-7}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{-25}{2}$ (d) $\frac{3}{2}$

---

18 If the coefficient of T_6 in the expansion of $\left(aX + \frac{1}{bX}\right)^{10}$ according to the descending powers of X equals ${}^{10}C_5$, then $\frac{a}{b} =$ .... where $a \in \mathbb{R}^*$, $b \in \mathbb{R}^*$

(a) -1 (b) 1 (c) 10 (d) $\frac{1}{10}$

---

19 If $\frac{{}^nC_4 + {}^nC_3}{{}^{n+1}C_3} = 1$, then $\underline{|n - 6}$ = ...

(a) 6
(b) 1
(c) zero
(d) 24

---

20 If the cosine of the angle which the vector $\overrightarrow{A} = (k , 12 , 4)$ makes with the positive direction of X-axis equals $\frac{3}{13}$, then k = ... where $k \in \mathbb{R}$

(a) 4
(b) $\sqrt{3}$
(c) $-\sqrt{3}$
(d) 3

21 If $\begin{vmatrix} x-1 & a & b \\ 0 & x^2+x+1 & c \\ 0 & 0 & 1 \end{vmatrix} = 8$, then the value of $x^9 + 1 =$ ....

(a) 37 (b) 729 (c) 730 (d) 8

---

22 If the system of the equations : $x + 4y = 5$, $ay + z = 3$, $-x + az = 1$ has infinite number of solutions , then a = ...

(a) ± 2 (b) ± 1 (c) 2 (d) zero

---

23 The two vectors $\overrightarrow{AB} = 3\hat{i} - 3\hat{k}$, $\overrightarrow{AC} = \hat{i} - 2\hat{j} + \hat{k}$ are two sides of Δ ABC , then the median length drawn from the vertex A equals .... length unit.

(a) $\sqrt{3}$ (b) $\sqrt{6}$ (c) $2\sqrt{3}$ (d) $3\sqrt{2}$

---

24 If $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$, then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$ ....

(a) $\frac{11}{6}$ (b) $\frac{5}{6}$ (c) $\frac{7}{6}$ (d) $\frac{2}{3}$

---

25 In the expansion of $\left(x^3 + \frac{5}{x}\right)^n$ according to the descending powers of x , if the term free of x is T_7 , then the value of n = ...

(a) 9 (b) 7 (c) 10 (d) 8

Choose the correct answer from the given ones :

1. $\sum_{r=0}^{n} \frac{{}^nP_r}{\lfloor r} = \dots\dots$

ⓐ 2^n ⓑ $\lfloor n$ ⓒ $\frac{\lfloor n}{2}$ ⓓ $\lfloor {}^nP_r$

2. The equation of the sphere whose center is $(-1, 0, 5)$ and its volume 36π volume unit is $\dots\dots$

ⓐ $(x+1)^2 + y^2 + (z-5)^2 = 36$ ⓑ $(x-1)^2 + y^2 + (z+5)^2 = 6$

ⓒ $(x+1)^2 + y^2 + (z-5)^2 = 27$ ⓓ $(x+1)^2 + y^2 + (z-5)^2 = 9$

3. If the measure of the angle between the two planes : $2x - y + z = 5$, $a x - 2y - z = 7$ where $a > 0$ equals $60°$, then $a = \dots\dots$

ⓐ $\frac{1}{2}$ ⓑ 1 ⓒ 2 ⓓ $\frac{1}{3}$

4. If $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, then $A^{-2} = \dots\dots$

ⓐ I ⓑ $-2I$ ⓒ $-I$ ⓓ A

5. If $a = {}^nP_2$, then ${}^aC_2 = \dots\dots$

ⓐ ${}^nC_2$ ⓑ ${}^nC_2\,(n^2 - n - 1)$ ⓒ $\frac{{}^nC_2}{\lfloor 2}$ ⓓ ${}^nC_2 \times \lfloor n$

6. If the secret number of a lock consists of 3 different digits from the digits $\{1, 2, 3, \dots, 9\}$ in how many ways a secret number can be formed including 6 ?

ⓐ 168 ⓑ 126 ⓒ 336 ⓓ 224

7. If $32(\cos\theta + i\sin\theta)^2 = \left(1 + \sqrt{3}\,i\right)^5$, then $\theta = \dots\dots$

ⓐ $\frac{\pi}{3}$ ⓑ $\frac{\pi}{6}$ ⓒ $\frac{5\pi}{6}$ ⓓ $\frac{5\pi}{3}$

8. If the straight line : $\frac{x-2}{3} = \frac{y+1}{-4} = \frac{z+3}{5}$ makes angles of measures ℓ , m , n with the x y-plane , y z-plane and x z-plane respectively , then

$\sin^2 \ell + \sin^2 m + \sin^2 n =$ ....

ⓐ 1 ⓑ 2 ⓒ $\frac{3}{2}$ ⓓ $\sqrt{3}$

---

9. If the two straight lines $L_1 : \vec{r}_1 = (1 , 2 , 3) + t_1 (-1 , 3 , 4)$ and $L_2 : \vec{r}_2 = (-2 , 5 , -1) + t_2 (m , n , 1)$ are perpendicular , then : $3n - m =$ ....

ⓐ -4 ⓑ $\frac{3}{4}$ ⓒ $\frac{-4}{3}$ ⓓ 4

---

10. The equation of the plane bisects one of the two angles between the two planes :

$2x - y + 2z + 3 = 0$, $3x - 2y + 6z + 8 = 0$ is ....

ⓐ $5x - y - 4z + 3 = 0$ ⓑ $5x - y + 4z - 3 = 0$

ⓒ $5x - y - 4z - 3 = 0$ ⓓ $5x + y + 4z - 3 = 0$

---

11. If the plane $2x - y + 2z = 6$ touches surface of the sphere $x^2 + y^2 + z^2 - 4x - 2y + 6z + 5 =$ zero , then equation of the straight line passing through the centre of the sphere and point of tangency is ....

ⓐ $\vec{r} = (2 , 1 , -3) + t (2 , -1 , 2)$ ⓑ $\vec{r} = (2 , 1 , -3) + t (4 , 0 , -1)$

ⓒ $\vec{r} = (4 , 0 , -1) + t (2 , 1 , -3)$ ⓓ $\vec{r} = (2 , -1 , 2) + t (2 , 1 , 3)$

---

12. If $f(x) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & x & 5 \\ 1 & 2 & x+3 \end{vmatrix}$, then $f(1) + f(2) + f(3) +$ .... $+ f(10) =$ ....

ⓐ 55 ⓑ 385 ⓒ 25 ⓓ 165

---

13. In the expansion of $\left(a x^2 - \frac{b}{x}\right)^{12}$ according to the descending powers of x , T_7 is ....

ⓐ the term containing x^6 ⓑ the term free of x

ⓒ the term before the last ⓓ the term containing x^7

14 $\sqrt{2(\omega+i)(\omega^2+i)} =$ ....

(a) $1+i$
(b) $1-i$
(c) $\pm(1+i)$
(d) $\pm(1-i)$

---

15 If $x + y = 2$, then the greatest value of the determinant $\begin{vmatrix} 1 & x & y \\ x & 1 & y \\ y & x & 1 \end{vmatrix}$ is ....

(a) 3
(b) zero
(c) -3
(d) 6

---

16 The possible values of k which make the distance between the two points A (2 , k , 3) , B (− 4 , 4 , 2) equals $\sqrt{62}$ are ....

(a) − 1 or 9
(b) − 5 or − 9
(c) 1 or 5
(d) 1 or − 9

---

17 Number of possible solutions to the system of equations :

$x - y + z = 2$, $2x + y - z = 2$, $4x + y + z = 0$ is ....

(a) unique solution.
(b) zero
(c) infinite number of solutions.
(d) three solutions.

---

18 If the coefficient of the ninth term in the expansion of $\left(a\sqrt{x} - \frac{1}{a\sqrt{x}}\right)^{12}$ according to the descending powers of x equals 7920 , then a = ...

(a) $\pm \frac{1}{2}$
(b) ± 2
(c) $\pm \frac{1}{4}$
(d) ± 4

---

19 In the expansion of $\left(x^2 - \frac{1}{x}\right)^{15}$ according to descending powers of x , the value of the term free of x equals ....

(a) ${}^{15}C_5$
(b) $-{}^{15}C_5$
(c) ${}^{15}C_9$
(d) $-{}^{15}C_9$

---

20 If ABC is a triangle in which D is the midpoint of $\overline{BC}$, A (3 , 1 , 5) , B (2 , 3 , 7) , C (0 , 3 , 1) , then the length of $\overline{AD}$ = ... length unit.

(a) 9
(b) 2
(c) 7
(d) 3

21 If $a = \cos\theta + i\sin\theta$, then $\frac{1+a}{1-a} = \dots\dots$

ⓐ $\cot\frac{\theta}{2}$ ⓑ $\cot\theta$

ⓒ $i\cot\frac{\theta}{2}$ ⓓ $i\tan\frac{\theta}{2}$

22 If A (3 , −4 , 0) , B (15 , 0 , 2) , C (0 , −8 , 4) are three points in the space and they form triangle ABC , then the distance between its centroid and the x z-plane is ....

ⓐ greater than the distance from the x y-plane.

ⓑ smaller than or equal to the distance from the x y-plane.

ⓒ greater than the distance from the y z-plane.

ⓓ greater than or equal to the distance from the y z-plane.

23 The value of determinant $\begin{vmatrix} 7^{2n} & 7^{3n} & 7^{4n} \\ 7^{3n} & 7^{4n} & 7^{5n} \\ 7^{4n} & 7^{5n} & 7^{6n} \end{vmatrix} = \dots\dots$, where $n \in \mathbb{Z}^+$

ⓐ zero ⓑ 7^{2n} ⓒ 7^n ⓓ 7^{3n}

24 If $a e^{2\theta i} + b e^{-2\theta i} = 5\cos 2\theta - i\sin 2\theta$, where a , b are positive real roots ,

$\theta \in \left]0, \frac{\pi}{2}\right[$, $i^2 = -1$, then a b = ...

ⓐ 6 ⓑ 2 ⓒ 5 ⓓ 3

25 If the plane $b c x + a c y + a b z = a b c$

intercepts the coordinate axes at the point K , N , M respectively and the plane :

$b c x + a c y - a b z = -a b c$ intercepts the coordinate axes at the points $\grave{K}$, $\grave{N}$, M

respectively , then the pyramid M K N $\grave{K}$ $\grave{N}$ is ....

where a , b , c are positive real numbers , $a \neq b$

ⓐ a right pyramid ⓑ a regular quadrilateral pyramid

ⓒ a right triangular pyramid ⓓ a regular triangular pyramid

Choose the correct answer from the given ones :

1 If 30° , 70° , θ are the direction angles of a vector , then θ = ...

(a) 100° (b) 80° (c) 260° (d) 68.6°

2 $e^{\pi i} - e^{-\pi i} =$ ....

(a) – 2 (b) zero (c) 1 (d) 2

3 The value of the expression $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 =$ ....

(a) $^{52}C_5$ (b) $^{47}C_5$ (c) $^{52}C_2$ (d) $^{52}C_4$

4 The unit vector perpendicular to each of the two vectors $\overrightarrow{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\overrightarrow{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ is ....

(a) $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$ (b) $\frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$

(c) $\frac{1}{3}(2\hat{i} + \hat{j} + 2\hat{k})$ (d) $\frac{1}{3}(2\hat{i} - 2\hat{j} + 2\hat{k})$

5 **The equation of the plane that contains the straight line**

$L_1 : \overrightarrow{r} = (0 , 3 , -5) + t_1 (6 , -2 , -1)$

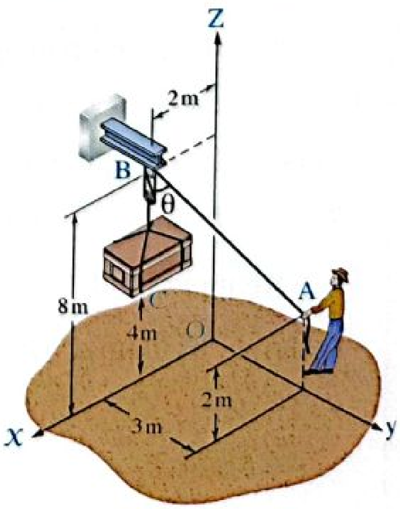
and parallel to the straight line : $L_2 : \overrightarrow{r} = (1 , 7 , -4) + t_2 (1 , -3 , 3)$

(a) $9x + 19y + 16z + 23 = 0$ (b) $9x - 19y - 16z - 23 = 0$

(c) $9x - 19y - 16z + 23 = 0$ (d) $9x + 19y + 16z - 23 = 0$

6 **In the opposite figure :**

C is the intersection point of the base diagonals of the box , then the measure of the angle included between $\overrightarrow{BA}$ and $\overrightarrow{BC} \simeq$ ....



(a) 59° (b) 41°

(c) 31° (d) 49°

7. If the middle term in the expansion of $\left(\frac{2a}{3}+\frac{b}{a^2}\right)^{8n}$ is the ninth term , then n = ...

(a) 1 (b) 2 (c) 3 (d) 4

8. The student should answer 10 questions out of 13 questions on a condition that he should answer 4 questions at least from the first 5 questions. How many ways can he answer ?

(a) 140 (b) 196 (c) 280 (d) 346

9. The system $\begin{pmatrix} 2 & 2 & 3 \\ 1 & -2 & -3 \\ 4 & -2 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\square$ has ....

(a) the trivial solution only.

(b) an infinite number of solutions other than the zero solution.

(c) a finite number of solutions except the zero solution.

(d) no solution at all.

10. The measure of the angle between the straight line :

$\vec{r}=(1 , 2 , -1)+t(1 , -1 , 1)$ and the plane $\vec{r}\cdot(2 , -1 , 1)=4$ approximatly equals ....

(a) $19^\circ\ 28'$ (b) $70^\circ\ 32'$ (c) $43^\circ\ 19'$ (d) $46^\circ\ 41'$

11. If the two straight lines $L_1 : \frac{x}{2}=\frac{y-1}{-1}=\frac{z-2}{m}$, $L_2 : \frac{x-1}{m}=\frac{y-2}{1}=\frac{z}{-1}$ are perpendicular , then m = ...

(a) -1 (b) 2 (c) 1 (d) -3

12. The radius length of the circular sector of the sphere $x^2+y^2+z^2-2y-4z=20$ with the plane $x+2y+2z=15$ is .... length unit.

(a) 7 (b) $\sqrt{7}$ (c) 4 (d) 3

13. In the expansion $\left(x^5+\frac{1}{x}\right)^6$ the ratio between the term free of x and the coefficient of the middle term equals ....

(a) $\frac{\lfloor 6}{\lfloor 5}$ (b) $\frac{\lfloor 6}{\lfloor 5 \lfloor 3}$ (c) $\frac{3}{10}$ (d) $\frac{10}{3}$

14 $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = \dots\dots$

(a) $2\begin{vmatrix} y & 0 & x \\ 0 & y & z \\ z & x & 0 \end{vmatrix}$ (b) $2\begin{vmatrix} y & 0 & x \\ 0 & z & y \\ z & 0 & x \end{vmatrix}$

(c) $2\begin{vmatrix} y & x & 0 \\ 0 & z & y \\ z & 0 & x \end{vmatrix}$ (d) $2\begin{vmatrix} y & 0 & z \\ x & z & 0 \\ y & 0 & x \end{vmatrix}$

---

15 If the system of equation : $2x - y - 3z = 2$, $x + 2y + z = 1$, $3x - 5y + 2z = 13$ has a unique solution , then $x + y + z = \dots\dots$

(a) 4 (b) 2 (c) -2 (d) zero

---

16 If $z_1 = 1 - \sqrt{3}\, i$, $z_2 = 1 + i$ where $i^2 = -1$ the number : $z = \frac{z_1}{z_2}$ in the exponential form is $\dots\dots$

(a) $2\, e^{-\frac{\pi}{3} i}$ (b) $\sqrt{2}\, e^{\frac{\pi}{4} i}$ (c) $\sqrt{2}\, e^{\frac{7}{12}\pi i}$ (d) $\sqrt{2}\, e^{-\frac{7}{12}\pi i}$

---

17 The number $2\sqrt{2}\,(1 + i)$ in trigonometric form is $\dots\dots$

(a) $16\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$ (b) $4\left(\cos\left(\frac{-\pi}{4}\right) + i \sin\left(\frac{-\pi}{4}\right)\right)$

(c) $4\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$ (d) $16\left(\cos\left(\frac{-\pi}{4}\right) + i \sin\left(\frac{-\pi}{4}\right)\right)$

---

18 If $\vec{a}$, $\vec{b}$ are two unit vectors , measure of the angle between them θ , then $2\,\|\vec{a} \times \vec{b}\|\,(\vec{a} \cdot \vec{b}) = \dots\dots$

(a) $\sin\theta \cos\theta$ (b) $2\sin\theta$

(c) $\cos 2\theta$ (d) $\sin 2\theta$

---

19 ${}^{n}C_{r} = {}^{n}P_{r}$ if $\dots\dots$

(a) $n = r$ (b) $r = \frac{n}{2}$

(c) $r = 1$ or n (d) $r = 0$ or 1

20 All the following points lying on the same side of the plane : $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) = 5$ except ....

(a) $(1, 2, 1)$ (b) $(0, -1, 2)$ (c) $(2, -3, 1)$ (d) $(1, 2, 4)$

---

21 The number of the integer terms in the expansion $\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)^7$ is ....

(a) zero (b) 2 (c) 3 (d) 4

---

22 $\left(\frac{\omega}{1+2\omega}\right)^2 + \left(\frac{\omega^2}{1+2\omega}\right)^2 =$ ....

(a) 1 (b) zero (c) $\frac{-1}{3}$ (d) $\frac{1}{3}$

---

23 The volume of the parallelepiped in which $\overline{AB}$, $\overline{AC}$ and $\overline{AD}$ are three sides where $A(1, 1, 1)$, $B(2, 1, 3)$, $C(3, 2, 2)$, $D(3, 3, 4)$ equals .... cubic unit.

(a) 4 (b) 5 (c) 6 (d) 8

---

24 If $A = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$, then $|A||A^{adi}| =$ ....

(a) a^3 (b) a^6 (c) a^9 (d) a^{27}

---

25 If $\begin{vmatrix} a^3-1 & a^2 & a \\ b^3-1 & b^2 & b \\ c^3-1 & c^2 & c \end{vmatrix} = \text{zero}$ where $a \neq b \neq c$, then $abc =$ ....

(a) zero (b) 1 (c) -1 (d) 3

Choose the correct answer from the given ones :

1 The number of ways to choose 2 different letters together or 3 different letters together from the elements of the set $\{a , b , c , d , e , h\}$ is ....

(a) ${}^6C_2 \times {}^6C_3$ (b) ${}^6P_2 \times {}^6P_3$

(c) ${}^6C_2 + {}^6C_3$ (d) ${}^6P_2 + {}^6P_3$

---

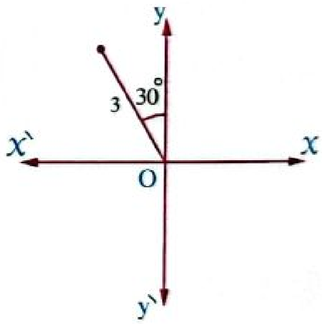
2 The sum of coefficients of binomial expansion of $\left(x^2 - \frac{1}{x}\right)^7$ equals ....

(a) 2^7 (b) 2^5

(c) 2^6 (d) zero

---

3 The opposite figure represents the complex number ....



(a) 3 (cos 30° + i sin 30°)

(b) 3 (cos 60° + i sin 60°)

(c) 3 (cos 120° + i sin 120°)

(d) 3 (cos 150° + i sin 150°)

---

4 The plane equation passes through (– 1 , 2 , 1) perpendicular to the straight line passes through the two points (– 3 , 1 , 2) , (2 , 3 , 4) is ....

(a) $-5x + 2y - 2z + 1 = 0$ (b) $-5x - 2y - 2z + 1 = 0$

(c) $-5x + 2y + 2z - 1 = 0$ (d) $5x - 2y - 2z - 1 = 0$

---

5 The two straight lines :

$\vec{r_1} = (3 , -1 , 2) + t_1 (4 , 1 , 3)$, $\vec{r_2} = (0 , 4 , -1) + t_2 (1 , -1 , 2)$ are ....

(a) parallel. (b) intersected and perpendicular.

(c) skew. (d) intersected and not perpendicular.

6 The direction cosines of the vector which makes equal angles with the coordinate axes are ....

(a) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (b) $\pm\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

(c) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (d) $\pm\left(\frac{1}{\sqrt{7}}, \sqrt{\frac{3}{14}}, \sqrt{\frac{1}{14}}\right)$

---

7 If A^{-1} is the multiplicative inverse of the coefficient matrix for the system :

$x + 3y + 2z = 0$, $x + z = -1$, $x + 2y = 3$, then $A^{-1} =$ ....

(a) $\begin{pmatrix} -2 & 4 & 3 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & -4 & 3 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}$

(c) $\frac{1}{5}\begin{pmatrix} -2 & 4 & 3 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ (d) $\frac{1}{5}\begin{pmatrix} -2 & 4 & 3 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}$

---

8 In the expansion of $\left(\frac{2x}{3} + \frac{3}{2x^2}\right)^{12}$ if the ratio between the middle term and the term contains x^{-3} equals $\frac{7}{9}$, then $x =$ ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{27}{8}$ (d) $\frac{8}{27}$

---

9 In Δ ABC if a , b , c are the side lengths , then $\overrightarrow{CA} \cdot \overrightarrow{CB} =$ ....

(a) $\frac{1}{2}(a^2 + b^2 - c^2)$ (b) $\frac{1}{2}(a^2 + c^2 - b^2)$

(c) $a^2 + c^2 - b^2$ (d) $\frac{1}{2}(c^2 - a^2 - b^2)$

---

10 Which of the following is false where $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ are non-zeroes non coplaner vectors ?

(a) $\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C} = -\overrightarrow{B} \times \overrightarrow{C} \cdot \overrightarrow{A}$ (b) $\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C} = \overrightarrow{B} \cdot \overrightarrow{C} \times \overrightarrow{A}$

(c) $\overrightarrow{A} \cdot (\overrightarrow{A} \times \overrightarrow{B}) = \overrightarrow{C} \cdot (\overrightarrow{B} \times \overrightarrow{C})$ (d) $\overrightarrow{A} \times (\overrightarrow{B} + \overrightarrow{C}) = \overrightarrow{A} \times \overrightarrow{B} + \overrightarrow{A} \times \overrightarrow{C}$

---

11 If $x^2 + y^2 + z^2 - 2x + 4y - 4z + k = 0$ is an equation of a sphere , then k could be ....

(a) 9 (b) 18 (c) 5 (d) 10

12. The direction cosines of the straight line passing through the two points $(4 , 3 , -5)$, $(-2 , 1 , -8)$ could be equal ....

(a) $(2 , 4 , -13)$
(b) $(6 , 2 , 3)$
(c) $\left(\frac{6}{7} , \frac{2}{7} , \frac{3}{7}\right)$
(d) $\left(\frac{-6}{7} , \frac{2}{7} , \frac{-3}{7}\right)$

13. If $z_1 = \cos\frac{7\pi}{4} - i\sin\frac{7\pi}{4}$, $z_2 = -1 + i$, then the number : $z = z_1^2 z_2^6$ in the exponential form $(i^2 = -1)$ is ....

(a) $e^{\frac{\pi}{2} i}$
(b) $\sqrt{2}\, e^{\frac{3}{4}\pi i}$
(c) $8\, e^{\pi i}$
(d) $8\, e^{\frac{\pi}{2} i}$

14. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} =$ ....

(a) $(a - b)(b - c)(a - c)$
(b) $(a - b)(b - c)(c - a)$
(c) $(b - a)(b - c)(c - a)$
(d) $(b - a)(c - b)(a - c)$

15. Values of x which satisfy the following equation in C : $(3x - 2)^3 = 8$ is ....

(a) $1 , \omega , \omega^2$
(b) $2 , 2\omega , 2\omega^2$
(c) $\frac{4}{3} , \frac{-2}{3}\omega , \frac{-2}{3}\omega^2$
(d) $1 , \frac{2}{3}\omega , \frac{2}{3}\omega^2$

16. The perpendicular distance between the point $(2 , 4 , -1)$ and the straight line $\frac{x + 5}{1} = \frac{y + 3}{4} = \frac{z - 6}{-9}$ equals .... length unit.

(a) 3
(b) 5
(c) 7
(d) 9

17. The value of x which makes the matrix $\begin{pmatrix} x - 1 & 2 \\ 4 & x + 1 \end{pmatrix}$ singular is ....

(a) -3
(b) 3
(c) ± 3
(d) 9

18. If $1 , \omega , \omega^2$ are the cubic roots of unity , then $\left(\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \cdots\right) + \omega + \omega^2$ $=$ ....

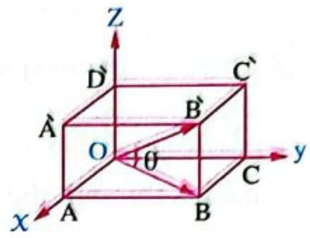
(a) -1
(b) 1
(c) $-i$
(d) i

19 If A is a matrix in order 2×2 and $|A| = 5$, then : $|A \times A^{adj}| =$ ....

(a) 5 (b) 25 (c) 125 (d) 1

---

20 **In the opposite figure :**



A B C O A` B` C` D` is a cuboid , B` (3 , 5 , 4) , then θ ≅ ....

(a) 27° 38`  (b) 34° 27`

(c) 45° (d) 35° 43`

---

21 The equation of the straight line parallel to the straight line :

$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and passing through the point (0 , 7 , − 7) is ....

(a) $x + 3 = y - 4 = z + 10$ (b) $x + y + z = 2$

(c) $x + y + z = 1$ (d) $\frac{x}{-3} = \frac{y-7}{2} = \frac{z+7}{1}$

---

22 If ${}^{2n}C_3 : {}^{n}C_2 \leq 11 : 1$, then n could be equals ....

(a) 5 (b) 4

(c) 6 (d) 7

---

23 The term free of x in the expansion $\left(\frac{3}{2} x^2 - \frac{1}{3x}\right)^9$ equals ....

(a) $\frac{7}{18}$ (b) $\frac{5}{18}$ (c) $\frac{11}{18}$ (d) $\frac{13}{18}$

---

24 $\begin{vmatrix} a & a & a \\ a & b & c \\ c & c & c \end{vmatrix} =$ ....

(a) zero (b) a c (c) b c (d) a b c

---

25 $\sqrt{5 + 12\,i} =$ ....

(a) ± (2 + 3 i) (b) ± (3 + 2 i) (c) ± (2 − 3 i) (d) ± (3 − 2 i)

Choose the correct answer from the given ones :

1. Three points in space , have position vectors :

$\overrightarrow{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{B} = \hat{i} - 3\hat{j} - 5\hat{k}$, $\overrightarrow{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, then the plane equation passes through the three points A , B , C is ....

(a) $8x - 11y - 5z = 0$
(b) $-8x - 11y + 5z = 0$
(c) $-8x + 11y - 5z = 0$
(d) $-8x - 11y + 5z - 1 = 0$

2. The exponential form of the number : $z_1 = -1 + \sqrt{3}\, i$ where $i^2 = -1$ is ....

(a) $2e^{\frac{2\pi}{3}i}$
(b) $2e^{\frac{\pi}{3}i}$
(c) $2e^{-\frac{2}{3}\pi i}$
(d) $2e^{\pi i}$

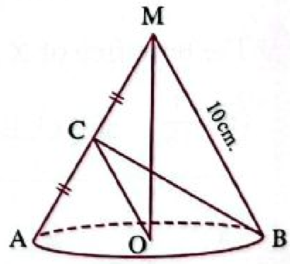
3. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ according to the ascending powers of x are 35 , 21 , 7 , then n = ...

(a) 7
(b) 2
(c) 6
(d) 4

4. **In the opposite figure :**

A right circular cone , its base circumference = 12π cm.

, then $\overrightarrow{BC} \cdot \overrightarrow{CO}$ = ...



(a) – 43
(b) – 40
(c) – 37
(d) – 33

5. The two straight lines : $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{7}$, $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{7}$ are ....

(a) perpendicular.
(b) intersecting.
(c) skew.
(d) parallel.

6. Which of the following could be direction angles of a vector ?

(a) 15° , 45° , 45°
(b) 45° , 135° , zero°
(c) 45° , 60° , 120°
(d) 60° , 60° , 60°

7. If $(x-1)$ is a factor of the determinant $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & 1 & x+1 \\ -1 & 1 & x+k \end{vmatrix}$, then k = ...

(a) 5 (b) − 5 (c) 1 (d) − 1

8. The coefficient of the fifth term according to the ascending powers of x in the expansion of $(1+2x)^{10}$ equals ....

(a) $16\ {}^{10}C_5$ (b) $\frac{1}{16}\ {}^{10}C_5$ (c) $16\ {}^{10}C_4$ (d) $\frac{1}{16}\ {}^{10}C_4$

9. If $A = \begin{pmatrix} 1 & -2 & 3 \\ x & 0 & 1 \\ 3 & 2 & -1 \end{pmatrix}$ and RK (A) = 2 , then x = ...

(a) − 2 (b) zero (c) 2 (d) 6

10. For any matrix A if $A^2 - A + I = \boxed{}$, then A^{-1} = ...

(a) A^{-2} (b) A + 1 (c) I − A (d) A − I

11. ${}^{n-1}C_6 + {}^{n-1}C_7 > {}^{n}C_6$ if ....

(a) n > 4 (b) n > 12 (c) n > 13 (d) n ≥ 13

12. If $\overrightarrow{A} \perp \overrightarrow{B}$, $\overrightarrow{A} \perp \overrightarrow{C}$ and $\overrightarrow{B} = (2, 3, 2)$, $\overrightarrow{C} = (1, 2, 1)$ and $\|\overrightarrow{A}\| = 4\sqrt{2}$, then $\overrightarrow{A}$ = ...

(a) ± (− 4 , − 4 , 0) (b) ± (− 4 , 0 , 4) (c) ± (4 , 4 , 0) (d) ± (0 , − 4 , 4)

13. The trigonometric form of the number $(i^{21})^3$ where $i^2 = -1$ is ....

(a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (b) $\cos\pi + i\sin\pi$

(c) $\cos(-\pi) + i\sin(-\pi)$ (d) $\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)$

14. The radius of the circular section produced by intersection of the sphere $x^2 + y^2 + z^2 - 2y - 4z = 11$ and the plane $x + 2y + 2z = 15$ equals ....

(a) 4 (b) 3 (c) $\sqrt{7}$ (d) 7

15 If $\begin{vmatrix} 2 & y \\ 1 & x \end{vmatrix} = 1$, $\begin{vmatrix} 3 & z \\ 2 & y \end{vmatrix} = 1$, $\begin{vmatrix} 3 & x \\ 1 & z \end{vmatrix} = 2$, then $x + y + z =$ ....

(a) 2 (b) -1 (c) 3 (d) -2

16 If the plane $\frac{x}{4} + \frac{y}{2} + \frac{z}{2} = 1$ intersects the coordinate axes at the points A , B , C , then the area of Δ ABC = ... area unit.

(a) 12 (b) 10 (c) 6 (d) 4

17 **In the opposite figure :**

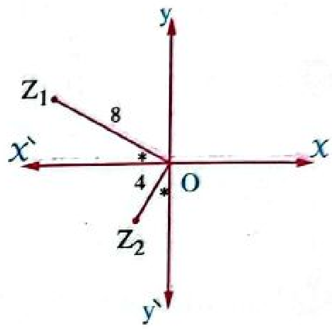
z_1 , z_2 are two complex numbers , then $\frac{z_1}{z_2} =$ ....

(a) 2

(b) -2

(c) 2 i

(d) -2 i



18 If A (2 , 4 , 5) , B (3 , 5 , -9) , then the yz-plane divides $\overline{AB}$ by ratio = ...

(a) 2 : 3 (b) 3 : 2 (c) $-2 : 3$ (d) $-4 : 3$

19 In the expansion of $(x + y)^n$ according to the descending power of x , if the seventh term is the unique term which has the greatest coefficient , then n = ...

(a) 12 (b) 13 (c) 14 (d) 15

20 The sphere which its equation is $(x - 2)^2 + (y + 4)^2 + (z + 3)^2 = 4$ touches ....

(a) x-axis. (b) yz-plane. (c) x y-plane. (d) y-axis.

21 The value of the determinent : $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$ ....

(a) zero (b) 1 (c) ω (d) ω^2

22 If $(1 + a\mathcal{X})^n = 1 + 8\mathcal{X} + 24\mathcal{X}^2 + \cdots + a^n \mathcal{X}^n$, then $\frac{a-n}{a+n} =$ ....

(a) 3 (b) -3 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

---

23 The number of ways to form a 2-digit number from the digits set $\{1 , 2 , 3 , 4 , 5\}$ equals ....

(a) ${}^5C_2$ (b) ${}^5P_2$ (c) 2^5 (d) 5^2

---

24 The measure of the included angle between the two straight lines

$L_1 : \frac{\mathcal{X}-3}{2} = \frac{z+1}{-2}$, $y = 1$, $L_2 : \frac{\mathcal{X}+1}{1} = \frac{y-2}{2} = \frac{z+1}{-2}$ equals ....

(a) $15°$ (b) $30°$ (c) $45°$ (d) $60°$

---

25 If $(1 + \omega)^7 = a + b\,\omega$ where a and b are two real numbers , then $(a , b) =$ ....

(a) $(0 , -1)$ (b) $(1 , 1)$ (c) $(0 , 1)$ (d) $(1 , -1)$

Choose the correct answer from the given ones :

1. The term free of x in the expansion $\left(2x - \frac{1}{2x^2}\right)^{12}$ is ....

(a) $-{}^{12}C_3 \times 2^6$ (b) $-{}^{12}C_5 \times 2^2$ (c) ${}^{12}C_6$ (d) ${}^{12}C_4 \times 2^4$

2. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a & b & c \end{vmatrix} = k$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a+1 & b+1 & c+1 \end{vmatrix} =$ ....

(a) k (b) $k-1$ (c) $k-6$ (d) $k-3$

3. If $A^{-1} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, then $y =$ ....

(a) 5 (b) 6 (c) 7 (d) 8

4. How many ways can a 7 member team be selected out of 9 girls and 5 boys if the team has only 3 boys ?

(a) ${}^{14}C_9$ (b) ${}^{5}C_3 \times {}^{9}C_7$ (c) ${}^{5}C_3 + {}^{9}C_4$ (d) ${}^{5}C_3 \times {}^{9}C_4$

5. If $z = \omega^{x}$, then $|z| =$ .... where x is a positive real number.

(a) 1 (b) ω (c) x (d) ω^2

6. The distance between the point $A(x , -3 , 4)$ and x-axis equals .... length unit.

(a) $\sqrt{x^2+25}$ (b) 5 (c) x (d) -5

7. If the sum of the coefficients of the terms of expansion $(a^2 x^2 - 2ax + 1)^{51}$ equals zero , then $a =$ ....

(a) 2 (b) -2 (c) 1 (d) -1

8. The distance between the two parallel planes :

$2x - y + 3z - 4 = 0$, $6x - 3y + 9z + 13 = 0$ equals .... length unit.

(a) zero (b) $\frac{25}{\sqrt{126}}$ (c) $\frac{25}{\sqrt{162}}$ (d) $\frac{17}{\sqrt{126}}$

9 The measure of the angle between the two straight lines :

$x - 1 = \frac{y+2}{\sqrt{2}} = -z + 1$, $-x = z + 3$, $y = 4$ equals ....

(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$

10 If ABCD is a square , then ....

(a) $(\overrightarrow{B} - \overrightarrow{A}) = (\overrightarrow{C} - \overrightarrow{B})$

(b) $\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C} = \overrightarrow{O}$

(c) $(\overrightarrow{C} - \overrightarrow{A}) \cdot (\overrightarrow{D} - \overrightarrow{B}) =$ zero

(d) $(\overrightarrow{C} - \overrightarrow{A}) \times (\overrightarrow{D} - \overrightarrow{B}) = \overrightarrow{O}$

11 The length of the perpendicular drawn from the origin point to the plane P is 7 length unit and the direction ratios of the straight line contains it are -3 , 2 , 6 , which of the following equations could be representing the plane equation of P ?

(a) $-3x + 2y + 6z - 7 = 0$

(b) $-3x + 2y + 6z - 49 = 0$

(c) $3x - 2y + 6z + 7 = 0$

(d) $-3x + 2y - 6z - 49 = 0$

12 If $z = \left(1 + \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^6$, then the number z in the exponential form = ...

(a) $27\, e^{\frac{\pi}{6} i}$

(b) $3\sqrt{3}\, e^{\pi i}$

(c) $27\, e^{\pi i}$

(d) $3\sqrt{3}\, e^{\frac{\pi}{6} i}$

13 In the expansion of $\left(\frac{x^3}{2} - \frac{4}{x}\right)^{11}$ according to the descending power of x the value of x which makes the sum of two middle terms equals to zero is equal to ....

(a) $\pm\sqrt{8}$

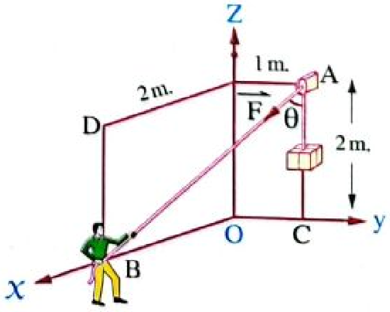
(b) $\pm\sqrt[4]{8}$

(c) ± 8

(d) $2\sqrt{2}$

14 **In the opposite figure :**

If $\|\overrightarrow{F}\| = 21$ newton , then the algebraic components of the force $\overrightarrow{F}$ in directions of coordinate axes is ....



(a) $(14 , -7 , 14)$ (b) $(14 , 7 , -14)$

(c) $(-14 , -7 , 14)$ (d) $(14 , -7 , -14)$

15 The cosine directions of the vector $\overrightarrow{A} = (-2, 1, 2)$ are ....

(a) $(-2, 1, 2)$ (b) $\left(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ (c) $\left(\frac{5}{-2}, 5, \frac{5}{3}\right)$ (d) $(-1, 1, 1)$

16 If $1, \omega, \omega^2$ are the cubic roots of one, $n \in \mathbb{Z}^+$

, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ equals ....

(a) 1 (b) ω (c) ω^2 (d) zero

17 The coordinates of the point $A \in$ the plane $x + y - 2z = 3$ such that $\overrightarrow{BA} \perp$ the plane , $B(2, 1, 6)$ is ....

(a) $(4, 3, 2)$ (b) $(2, 2, 0)$ (c) $(1, 4, 1)$ (d) $(-2, 5, 0)$

18 Given that : ${}^nC_r : {}^nC_{r+2} : {}^nC_{r+4} = 3 : 14 : 14$, then ${}^nP_r =$ ....

(a) 10 (b) 90 (c) 720 (d) 9

19 The sphere equation whose centre is $(-2, 1, -4)$ and its radius length is 25 is ....

(a) $(x+2)^2 + (y-1)^2 + (z+4)^2 = 5$

(b) $(x-2)^2 + (y+1)^2 + (z-4)^2 = 625$

(c) $x^2 + y^2 + z^2 + 4x - 2y + 8z - 625 = 0$

(d) $x^2 + y^2 + z^2 + 4x - 2y + 8z - 604 = 0$

20 The measure of the angle between the straight line $\overrightarrow{r} = (2\hat{i} - \hat{j} + \hat{k}) + t(-\hat{i} + \hat{j} + \hat{k})$ and the plane $\overrightarrow{r}.\ (3\hat{i} + 2\hat{j} - \hat{k}) = 4$ is ....

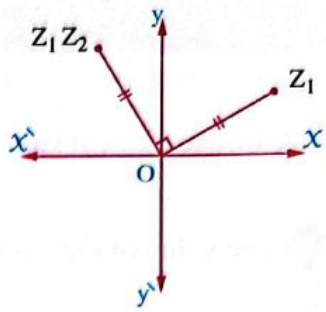
(a) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (b) $\cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$ (c) $\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (d) $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

21 The rank of the augmented matrix of the system $2x - 3y = 5$, $6x - 9y = 15$ is ....

(a) zero (b) 1 (c) 2 (d) 3

22 **In the opposite figure :**

z_1 , z_2 are two complex numbers and $(z_1\ z_2)$ is a complex number , then $z_2 =$ ....



(a) $-2i$

(b) $-i$

(c) i

(d) $2i$

---

23 If ${}^{14}C_{r^2} = {}^{14}C_{r+2}$, then $r =$ ....

(a) 2 or -1 (b) 4 (c) 3 (d) 2 or 3 or -1

---

24 $\left(\frac{-1+\sqrt{-3}}{2}\right)^5 + \left(\frac{-1-\sqrt{-3}}{2}\right)^8 =$ ....

(a) -1 (b) 1 (c) ω (d) ω^2

---

25 The equation system $3x + y - z = 0$, $5x + 2y - 3z = 2$, $15x + 6y - 9z = 5$ ....

(a) has a unique solution. (b) has an infinite number of solutions.

(c) has 3 solutions. (d) has no solution.

Choose the correct answer from the given ones :

1. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} = \dots\dots$

(a) $a+b+c$ (b) zero (c) 1 (d) $a\,b\,c$

2. If $z_1 = 2 + 2\sqrt{3}\,i$, $z_2 = -3 - 3\sqrt{3}\,i$, then the amplitude of the number : $z_1 + z_2 = \dots\dots$

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $-\frac{2\pi}{3}$ (d) $-\frac{\pi}{3}$

3. If $(1+\omega)^7 = a + b\,\omega$, where a , b are real numbers , then $(a , b) = \dots\dots$

(a) $(0 , -1)$ (b) $(1 , 1)$ (c) $(0 , 1)$ (d) $(1 , -1)$

4. The equation of the straight line passing through the point $A(-1 , 0 , 2)$ and the vector $\vec{d} = (1 , -1 , 3)$ its direction vector is $\dots\dots$

(a) $\frac{x-1}{2} = \frac{y}{1} = \frac{z-1}{-1}$ (b) $\frac{x+1}{1} = \frac{y}{-1} = \frac{z-2}{3}$

(c) $\frac{x-1}{3} = \frac{y}{-1} = \frac{z}{1}$ (d) $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z}{2}$

5. If $(x-2)^2 + (y+4)^2 + (z-2)^2 = 1$, $(x+4)^2 + (y-4)^2 + (z-2)^2 = 4$ are the equations of two spheres , then the two spheres are $\dots\dots$

(a) distant. (b) touching internally. (c) touching externally. (d) intersecting.

6. If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = 2\hat{i} - \hat{j} - \hat{k}$, then $\vec{A} \times (\vec{A} - \vec{B}) = \dots\dots$

(a) $\hat{i} + \hat{k}$ (b) $-3\hat{j} + 3\hat{k}$ (c) $-3\hat{i} - 3\hat{j}$ (d) $3\hat{i} - 2\hat{j}$

7. If $A = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$, then $|A| \times |Adj(A)| = \dots\dots$

(a) k^3 (b) k^6 (c) k^9 (d) k^{27}

8) If $(x-2)$ is one of the factors of the determinant $\begin{vmatrix} x-1 & x+3 & 2 \\ -3 & x+5 & -6 \\ x+3 & 2 & x+k \end{vmatrix}$, then k = ...

(a) -8 (b) 8 (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

9) If $\frac{\lfloor n+2}{\lfloor n} + 3n = 74$, $\lfloor r = n$, then ${}^nC_r$ = ...

(a) 720 (b) 20 (c) 12 (d) 6

10) The plane equation which bisects the line segment joining the two points B (2 , 3 , 4) , C (6 , 7 , 8) of the following equations is ....

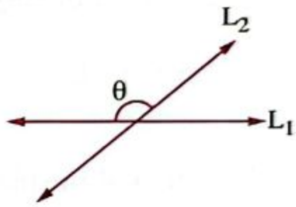
(a) $x - y - z - 15 = 0$ (b) $x - y + z - 15 = 0$

(c) $x + y + z - 15 = 0$ (d) $x + y + z + 15 = 0$

11) **In the opposite figure :**

If $L_1 : x = 0$, $y = z$, $L_2 : y = 0$, $x = z$

, then θ = ...



(a) 120° (b) 135°

(c) 150° (d) 165°

12) In the expansion $\left(x^2 + \frac{1}{x}\right)^{18}$, the ratio between the term free of x to the coefficient of the middle term = ...

(a) $\frac{21}{55}$ (b) $\frac{55}{21}$ (c) $\frac{13}{10}$ (d) $\frac{10}{13}$

13) If $x + y - z = 0$, $x - y - 5z + 7 = 0$ are equations of two planes in the space.

the straight line whose equation is $\frac{x+3}{3} = \frac{y-1}{-2} = \frac{z-5}{1}$ ....

(a) is parallel to the line of intersection of the two planes.

(b) is skew with the line of intersection of the two planes.

(c) perpindicular to the line of intersection of the two planes.

(d) intersects the line of intersection of the two planes at a point.

14 The solution set of system of the equations : $x + 2z = 5$, $y - 3z - 1 = 0$, $y = 7 - x$ is ....

(a) $\{(3, -4, 1)\}$ (b) $\{(3, 4, -1)\}$ (c) $\{(-3, 4, 1)\}$ (d) $\{(3, 4, 1)\}$

---

15 **In the opposite figure :**

A man lifts a box using a string passing over a smooth pully and inclines to the vertical with an angle of measure 30°

If the tension force in the string is 120 newten to rise the box 3 metre above the ground , then the work done by the tension force = ... Joule.

(a) 180 (b) $180\sqrt{3}$ (c) $-180\sqrt{3}$ (d) -180

---

16 The number of ways to distribute 3 similar balls in 4 boxes equals ....

(a) ${}^{4}C_{3}$ (b) ${}^{4}P_{3}$ (c) ${}^{6}C_{3}$ (d) ${}^{6}P_{3}$

---

17 Equation of the plane which contains the origin point and the vector $(0, 1, -2)$ is perpendicular to it is ....

(a) $x = 2y$ (b) $y = 2z$ (c) $z = 2x$ (d) $y = 1$

---

18 If $A(5, 3, 2)$, $B(-1, 0, -4)$, $C(1, 1, -2)$ lie on the same straight line , then the ratio by which the point B divides the segment $\overrightarrow{CA}$ is ....

(a) $-1 : 3$ (b) $2 : 3$ (c) $3 : -1$ (d) $1 : 2$

---

19 The rank of the matrix $\begin{pmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix}$ equals ....

(a) zero if $a = 6$ (b) 1 if $a = -1$ (c) 3 if $a = 2$ (d) 1 if $a = -6$

---

20 ${}^{n}C_{0} + 2\,{}^{n}C_{1} + 2^{2}\,{}^{n}C_{2} + \dots + 2^{r}\,{}^{n}C_{r} + \dots + 2^{n}\,{}^{n}C_{n} =$ ....

(a) 5^{n} (b) 6^{n} (c) 4^{n} (d) 3^{n}

21 $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j}) = \dots\dots$

(a) zero (b) -3 (c) -1 (d) 3

22 The number : $z = 2(\omega + i)(\omega^2 + i)$ in the exponential form is $\dots\dots$

(a) $2e^{\frac{\pi}{2}i}$ (b) $2e^{-\frac{\pi}{2}i}$ (c) $2e^{\pi i}$ (d) $2e^{-\pi i}$

23 If $z = 1 + \cos 40° + i \sin 40°$, then the amplitude of $(z) = \dots\dots$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{9}$ (c) $\frac{\pi}{18}$ (d) $\frac{2\pi}{9}$

24 In the expansion $\left(\frac{1}{x} - x^2\right)^9$ according ascending power of x , if $T_5 = 12\, T_4$, then $x = \dots\dots$

(a) 2 (b) -2 (c) 8 (d) -8

25 If ${}^7C_r > 1$, ${}^rC_5 > 1$, then the value of $\lfloor 6 - r = \dots\dots$

(a) zero (b) 1 (c) 720 (d) 6

Choose the correct answer from the given ones :

1. If $z_1 = r_1 (\cos\theta_1 + i\sin\theta_1)$, $z_2 = r_2 (\cos\theta_2 + i\sin\theta_2)$ and $\theta_1 + \theta_2 = \pi$, then $z_1 z_2 =$ ....

   (a) $r_1 r_2$ (b) $-r_1 r_2$ (c) $r_1 r_2 i$ (d) $-r_1 r_2 i$

2. $\sum_{r=1}^{6} (1 + \omega^r) =$ ....

   (a) zero (b) 6 (c) 1 (d) $1 + \omega$

3. If $A = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 2 & -2 & 1 \end{pmatrix}$, then RK $(A^t) =$ ....

   (a) zero (b) 1 (c) 2 (d) 3

4. In the expansion of $x^3 (1 + x)^7$, the coefficient of the term containing x^4 is ....

   (a) ${}^7C_4$ (b) ${}^7C_3$ (c) ${}^7C_1$ (d) 21

5. The sphere equation which passes through the points (4 , 0 , 0) , (0 , 4 , 0) , (0 , 0 , 4) and its radius length is as small as possible is ....

   (a) $(x-4)^2 + (y-4)^2 + (z-4)^2 = \frac{32}{3}$ (b) $\left(x - \frac{4}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 + \left(z - \frac{4}{3}\right)^2 = 16$

   (c) $(x-4)^2 + (y-4)^2 + (z-4)^2 = 16$ (d) $\left(x - \frac{4}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 + \left(z - \frac{4}{3}\right)^2 = \frac{32}{3}$

6. The number $z = \frac{5 - 3\sqrt{3}\,i}{1 + 2\sqrt{3}\,i}$ where $i^2 = -1$ in the trigonometric form is ....

   (a) $4\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right)$ (b) $2\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right)$

   (c) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ (d) $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

7. The equation of the line of intersection of the two planes : $x + y + z - 6 = 0$, $2x + 3y + 4z + 5 = 0$ is ....

   (a) $x = \frac{y - 29}{2} = z + 32$ (b) $x = \frac{y + 29}{2} = z - 23$

   (c) $x = \frac{-y + 29}{2} = z + 23$ (d) $x = \frac{-y + 29}{2} = 23 - z$

8. $\begin{vmatrix} a+b & 5 & c \\ b+c & 5 & a \\ a+c & 5 & b \end{vmatrix} = \text{..............}$

(a) 5 (b) 4 (c) 3 (d) zero

9. In the expansion of $\left(x^3 + \frac{5}{x}\right)^n$ according to the descending powers of x, if the seventh term is the term free of x, then the value of n equals ....

(a) 11 (b) 10 (c) 8 (d) 9

10. If the two vectors : $(2, k, -3)$, $(4, 6, -6)$ are parallel, then $k =$ ....

(a) 6 (b) 3 (c) -3 (d) 1

11. The solution set of the matrix equation : $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix}$ is ....

(a) $\{(7, 0, 1)\}$ (b) $\{(7, 0, -1)\}$ (c) $\{(-7, 0, 1)\}$ (d) $\{(7, 1, 0)\}$

12. The measure of the angle between the two planes $\vec{r} \cdot (3, 1, -1) = 1$, $\vec{r} \cdot (1, 4, -2) = 2$ is ....

(a) $\cos^{-1}\left(\frac{9}{\sqrt{231}}\right)$ (b) $\cos^{-1}\left(\frac{4}{\sqrt{231}}\right)$ (c) $-\cos^{-1}\left(\frac{11}{\sqrt{231}}\right)$ (d) $\cos^{-1}\left(\frac{11}{\sqrt{231}}\right)$

13. The plane equation passing through the point $(1, 2, 3)$ and parallel to x and y axes is ....

(a) $x + y = 3$ (b) $z = 3$ (c) $x = 1$ (d) $y = 2$

14. If $\vec{A} \times \vec{B} = \vec{C} \times \vec{B}$ which of the following is always true ?

(a) $\vec{A} = \vec{C}$

(b) $(\vec{A} \times \vec{B}) \cdot (\vec{B} \times \vec{C}) = \text{zero}$

(c) $\vec{B} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{B} \times \vec{C}) = \text{zero}$

(d) $\vec{A} \times \vec{B} = \vec{B} \times \vec{C}$

15. ${}^nC_1 + (2 \times {}^nC_2) + (3 \times {}^nC_3) + (4 \times {}^nC_4) + \cdots + (n \times {}^nC_n) =$ ....

(a) $n \times 2^n$ (b) $n \times 2^{n+1}$ (c) $n \times 2^{n-1}$ (d) $n \times 2^{2n}$

16 The image of the point A $(1 , 3 , 4)$ by reflection in the plane $2x - y + z + 3 = 0$ is ....

(a) $(2 , 5 , -3)$ (b) $(-3 , 5 , -2)$ (c) $(-3 , 5 , 2)$ (d) $(-3 , -5 , 2)$

17 The position vector of the point A is $6\hat{i} + \hat{j} - 3\hat{k}$ and the position vector of the point B is $4\hat{i} - 4\hat{j} - 2\hat{k}$, then the work done from the force $\vec{F} = \hat{i} - 3\hat{j} + 5\hat{k}$ to move the body from A to B is ....

(a) 15 (b) 18 (c) -15 (d) -18

18 The number of ways to select 3 persons in the same time from a group consisting of 5 men and 4 women where 2 persons from the selected are of the same gender equals ....

(a) ${}^9C_3$ (b) ${}^5C_2 \times {}^4C_1$

(c) ${}^4C_2 \times {}^5C_1$ (d) ${}^5C_2 \times {}^4C_1 + {}^5C_1 \times {}^4C_2$

19 If the straight line $L_1 : \frac{x+2}{-1} = \frac{y+3}{3} = \frac{z+5}{2}$ is perpendicular to the straight line $L_2 : \frac{x}{2} = \frac{y-5}{k} = \frac{z-6}{m}$, then $3k + 2m =$ ....

(a) -1 (b) 2 (c) 3 (d) 4

20 If the middle term in the expansion : $\left(x^2 + \frac{1}{2x}\right)^{10}$ equals $\frac{28}{27}$, then $x =$ ....

(a) $\frac{3}{2}$ (b) 5 (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

21 If $a = 2\omega - 3\omega^2$, $b = 3 + 5\omega^2$, then $a^2 + b^2 =$ ....

(a) -37 (b) -19 (c) 1 (d) 38

22 If $z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$, then $z + 1 =$ .... (on the exponential form)

(a) $e^{\frac{\pi}{6}i}$ (b) $\sqrt{3}\,e^{\frac{\pi}{6}i}$ (c) $\sqrt{3}\,e^{\frac{\pi}{3}i}$ (d) $(\sqrt{2}+1)\,e^{\frac{\pi}{3}i}$

23 The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$ equals ....

(a) $3\sqrt{3}\,i$ (b) $\pm 3\sqrt{3}\,i$ (c) $-\sqrt{3}\,i$ (d) $2\sqrt{3}\,i$

24 If (30° , 70° , θ) are the direction angles of a vector where θ is an acute angle , then $\theta \simeq$ ....

(a) 42.6° (b) 80° (c) 60.8° (d) 68.6°

25 The term which has the greatest coefficient in the expansion $(1 + x)^{10}$ a ccording ascending power of x is ....

(a) T_{11} (b) T_5 (c) T_6 (d) T_{10}

Choose the correct answer from the given ones :

1. The length of perpendicular drawn between the two planes $3x + 12y - 4z = 9$ and $3x + 12y - 4z = -17$ equals .... length unit.

   (a) 2 (b) 3 (c) 4 (d) 5

2. If ABC is a triangle where A (1 , 2 , 3) , B (0 , 1 , 2) , C (2 , 1 , 0) , then the length of the median drawn from A equals .... length unit.

   (a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) 5 (d) 10

3. The point (x , y , z) moves in the path parallel to the x-axis so which of the variables x , y , z remains constant ?

   (a) z , x (b) x , y (c) y , z (d) x

4. The coefficient of x^7 in the expansion of $(1 - x)^4 (1 + x)^9$ is ....

   (a) 27 (b) -24 (c) 36 (d) -36

5. $\left(1 + 2\omega^5 + \frac{1}{\omega^2}\right)\left(1 + 2\omega + \frac{1}{\omega^4}\right) =$ ....

   (a) zero (b) 1 (c) -1 (d) 2

6. If z_1 , z_2 are two non-zero complex numbers , $|z_1| = |z_2|$ and amplitude of z_1 + amplitude of $z_2 = \pi$, then $z_1 =$ ....

   (a) $\overline{z_2}$ (b) $-\overline{z_2}$ (c) z_2 (d) $-z_2$

7. Equation of the straight line passing through the point (2 , – 1 , 1) parallel to the straight line passing through the two points (– 1 , 4 , 1) , (1 , 2 , 2) could be equal to ....

   (a) $\vec{r} = (2 , -1 , 1) + t(-2 , 2 , 1)$

   (b) $x = 2 - 2t , y = -1 + 2t , z = 1 + t$

   (c) $\frac{x - 2}{-2} = \frac{y + 1}{2} = z - 1$

   (d) $\frac{x - 2}{-2} = \frac{y + 1}{2} = \frac{z - 1}{-1}$

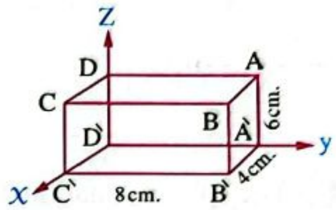
8. The value of k which makes $(x-2)$ one of the factors of

the determinant : $\begin{vmatrix} x+1 & 1 & -3 \\ 2 & 5 & x-1 \\ 1 & -4 & x+k \end{vmatrix}$ is ....

(a) 6 (b) -6 (c) -2 (d) 78

---

9. **In the opposite figure :**



ABCD À B̀ C̀ D̀ is a cuboid

$\overrightarrow{AC} \times \overrightarrow{AD} =$ ....

(a) $-48\,\hat{i} - \hat{j} - 32\,\hat{k}$ (b) $-48\,\hat{i} - 32\,\hat{j}$

(c) $48\,\hat{i} - 32\,\hat{k}$ (d) $-48\,\hat{i} - 32\,\hat{k}$

---

10. If $(a-x)^{14} = C + C_1\,x + C_2\,x^2 + C_3\,x^3 + \cdots + C_{14}\,x^{14}$ and $4\,C_4 + 11\,(C_3 + C_2) =$ zero ,

then $a =$ ....

(a) 2 (b) -2 (c) $\frac{7}{4}$ (d) $\frac{-11}{4}$

---

11. If $\overrightarrow{A} = \hat{i} - 2\,\hat{j} + \hat{k}$, $\overrightarrow{B} = m\,\hat{i} - 5\,\hat{j} + 3\,\hat{k}$, $\overrightarrow{C} = 5\,\hat{i} - 9\,\hat{j} + 4\,\hat{k}$

lie in one plane , then $m =$ ....

(a) 2 (b) -2 (c) 3 (d) -3

---

12. If $A = \begin{pmatrix} 2 & 5 \\ 5 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 3 \\ -46 & 17 \end{pmatrix}$ and $A \times C = B$, then $C =$ ....

(a) $\begin{pmatrix} -8 & 1 \\ 3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -11 & 4 \\ 3 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} -17 & 4 \\ 6 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} -9 & 6 \\ 3 & -1 \end{pmatrix}$

---

13. In ΔABC : $\begin{vmatrix} a & b & c \\ 5 & 7 & 8 \\ \sin A & \sin B & \sin C \end{vmatrix} =$ ....

(a) 5 a (b) 7 b (c) 8 c (d) zero

14 The number of solutions of the system : $2x + 5y = 0$, $3x - z = 0$, $2y - 3z = 0$ is ....

(a) zero
(b) 1
(c) 2
(d) an infinite number of solutions.

15 If $\| \overrightarrow{A} \| = \sqrt{75}$ and the vector $\overrightarrow{A}$ is perpendicular to each of the two vectors $2\hat{i} + \hat{j} + \hat{k}$, $3\hat{i} - 2\hat{j} - \hat{k}$, then $\overrightarrow{A} =$ ....

(a) $-\hat{i} + 5\hat{j} + 7\hat{k}$
(b) $7\hat{i} + 5\hat{j} + \hat{k}$
(c) $\hat{i} + 5\hat{j} - 7\hat{k}$
(d) $-7\hat{i} - 5\hat{j} - \hat{k}$

16 The sphere equation which touches the plane $z = 5$ and whose centre is $(3, -2, -1) =$ ....

(a) $(x - 3)^2 + (y + 2)^2 + (z + 1)^2 = 36$
(b) $(x - 3)^2 + (y + 2)^2 + (z + 1)^2 + 36 = 0$
(c) $x^2 + y^2 + z^2 - 6x + 4y + 2z = 36$
(d) $x^2 + y^2 + z^2 - 6x + 4y + 2z + 22 = 0$

17 If $x = \sqrt{3} - i$, $y = \sqrt{3} + i$ and $z = x^2 - 2xy + y^2$, then one of the cubic roots of the complex number z is ....

(a) $4 e^{\pi i}$
(b) $\sqrt[3]{4}\left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3}\right)$
(c) $4 e^{-\frac{\pi}{3} i}$
(d) $\sqrt[3]{4}\, e^{\frac{\pi}{3} i}$

18 The distance between the two parallel straight lines :

$L_1 : \frac{x - 2}{2} = \frac{y - 1}{3} = \frac{z + 4}{-2}$, $L_2 : \vec{r} = (1, -1, 2) + t(2, 3, -2)$

approximately equals .... length units.

(a) 5.13
(b) 2.26
(c) 1.17
(d) 4.18

19 The equations : $2x + 3y - z = 0$, $x - 3y + 2z = 2$, $y + z = -2$ are represented by ....

(a) three planes are intersected at one point.
(b) three planes are intersected at a straight line.
(c) three parallel planes.
(d) a plane intersects both of the two other planes.

20 Solution set of the equation : $2 \times {}^nC_4 - 3 \times {}^nC_3 + 2 \times {}^nC_2 = 0$ is ....

(a) $\{5, 6\}$ (b) $\{3, 4\}$ (c) $\{2, 4\}$ (d) $\{2, 3\}$

21 The two spheres $(x-1)^2 + y^2 + (z-3)^2 = 16$, $(x+1)^2 + (y-2)^2 + (z-k)^2 = 25$ are touching externally , then $k =$ ....

(a) $\sqrt{11}$ (b) $3 \pm \sqrt{73}$ (c) $2 \pm \sqrt{19}$ (d) $\sqrt{7}$

22 The number of ways can a 6 member team selected out of 8 girls and 6 boys such that the team contains only three boys equals ....

(a) 2110 (b) 1120 (c) 1008 (d) 810

23 If number of the terms of the expansion $(a+b)^{2n} + (a-b)^{2n}$ is 11 terms , then $n =$ ....

(a) 11 (b) 10 (c) 9 (d) 8

24 If ${}^nP_2$, ${}^nP_3$, ${}^{n+1}P_3$ form an arithmetic sequence , then $n =$ ....

(a) 6 (b) 7 (c) 8 (d) 9

25 The exponential form of the complex number $z = 2\cos\frac{\pi}{3} + i\sin\frac{\pi}{2}$ is ....

(a) $2e^{\frac{\pi}{3}i}$ (b) $e^{\frac{\pi}{2}i}$ (c) $e^{\frac{5\pi}{6}i}$ (d) $\sqrt{2}\,e^{\frac{\pi}{4}i}$

Choose the correct answer from the given ones :

1. $\overline{AB}$ is a diameter in the sphere $(x-5)^2+(y+2)^2+(z-1)^2=11$ where $A=(8,-1,2)$, then the point $B=$ ....

(a) $(5,-2,1)$ (b) $(10,-4,5)$ (c) $(2,-3,0)$ (d) $(10,3,6)$

2. If $\Delta = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ 0 & 4 & 2 \end{vmatrix}$, then $\begin{vmatrix} 4 & 12 & 4 \\ 8 & -4 & 4 \\ 0 & 16 & 8 \end{vmatrix} =$ ....

(a) $12\,\Delta$ (b) $64\,\Delta$ (c) $4\,\Delta$ (d) $16\,\Delta$

3. If ${}^{n+2}C_4 = n^2-1$, then $n=$ ....

(a) 2 (b) 4 (c) 6 (d) 10

4. If the middle term in the expansion of $\left(3x^2+\frac{2}{3x}\right)^8$ according to ascending powers of x equals 17920, then $x=$ ....

(a) ± 2 (b) 3 (c) ± 4 (d) 5

5. The plane equation which contains the two straight lines

$\frac{x+1}{-1}=\frac{y-2}{-1}=\frac{z-1}{3}$, $\frac{x+1}{1}=\frac{y-2}{-2}=\frac{z-1}{-1}$ is ....

(a) $3x-5y+z-1=0$ (b) $5x-4y+2z-7=0$

(c) $7x-5y-z-4=0$ (d) $7x+2y+3z=0$

6. If $\vec{A}$, $\vec{B}$ are two non-zero vectors and θ is their included angle, then $\|\vec{A}\times\vec{B}\|^2+|\vec{A}\cdot\vec{B}|^2=$ ....

(a) $\|\vec{A}\|^2\,\|\vec{B}\|^2$ (b) 1

(c) $\|\vec{A}\|^2\,\|\vec{B}\|^2\sin\theta$ (d) zero

7 The equation of the straight line which passes through the point (2 , 3 , – 5) and makes equal angles with the coordinate axes is ....

(a) $\frac{x}{2} = \frac{y}{3} = \frac{z}{-5}$
(b) $x - 2 = y - 3 = z + 5$
(c) $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{-5}$
(d) $\frac{x-2}{\sqrt{3}} = \frac{y-3}{\sqrt{3}} = \frac{z+5}{3}$

---

8 If $z = \left(1 + \sqrt{3}\, i\right)^n$ and $|z| = 8$, then the principle amplitude of the number z equals ....

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$
(d) π

---

9 If $z_1 = 4\left(\cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}\right)$, $z_2 = e^{\frac{\pi}{2}i}$ where $i^2 = -1$

, then the number : $z = \frac{z_2}{z_1}$ in the trigonometric form is ....

(a) $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
(b) $\frac{1}{4}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
(c) $\frac{1}{4}\left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)$
(d) $4\left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)$

---

10 The area of the triangle whose vertices are A (1 , 2 , 3) , B (2 , 5 , – 1) , C (– 1 , 1 , 2) is .... square unit.

(a) 150
(b) 145
(c) $\frac{\sqrt{155}}{2}$
(d) $\frac{155}{2}$

---

11 If the expansion $\left(x^2 + \frac{1}{x}\right)^n$ has term free of x , then n must be multiple of ....

(a) 2
(b) 3
(c) 5
(d) all of pervious.

---

12 For the system : $3x + y + 4z = 0$, $2x + 3y + 5z = 0$, $-x + 2y + z = 0$

, there is ....

(a) unique solution other than the trivial solution.
(b) the trivial solution only.
(c) infinite number of solution one of them trivial solution.
(d) has no solution.

13. $1 + \omega + \omega^2 + \omega^3 + \cdots + \omega^{100} = \ldots\ldots\ldots\ldots$

(a) zero (b) 1 (c) ω (d) $-\omega^2$

---

14. The number of ways can a 4 member team of the same gender be selected out of 9 boys and 6 girls equals ....

(a) ${}^{15}C_4$ (b) ${}^{9}C_4$ (c) ${}^{9}C_4 \times {}^{6}C_4$ (d) ${}^{9}C_4 + {}^{6}C_4$

---

15. If the two planes : $P_1 : a\,x + b\,y + c\,z + d_1 = 0$, $P_2 : a\,x + b\,y + c\,z + d_2 = 0$ are parallel , then the distance between them is ....

(a) $\frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$ (b) $\frac{|d_2 + d_1|}{\sqrt{a^2 + b^2 + c^2}}$ (c) $\frac{|d_2 - d_1|}{a^2 + b^2 + c^2}$ (d) $\frac{|d_2 + d_1|}{a^2 + b^2 + c^2}$

---

16. If the vectors $\vec{A} = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $\vec{B} = -5\hat{i} + 7\hat{j} - 3\hat{k}$, $\vec{C} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ are the position vectors of 3 points in space , then the measure of the angle between $(\vec{A} \times \vec{B})$, $\vec{C} \simeq$ ....

(a) $64° 22'$ (b) $115° 38'$ (c) $154° 22'$ (d) $25° 38'$

---

17. The plane equation passing through the two points A (0 , 0 , 0) , B (3 , – 1 , 2) parallel to the straight line $\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}$ is ....

(a) $x - 19\,y - 11\,z = 0$ (b) $x + 19\,y - 11\,z = 0$

(c) $x + 19\,y + 11\,z = 0$ (d) $x - 19\,y + 11\,z = 0$

---

18. The determinant $\begin{vmatrix} 1 & -2c + 2c^2 & -3c + 3c^2 \\ 1 & 1 + 2c^2 & 3c^2 \\ c & 2c^3 & 1 + 3c^3 \end{vmatrix} = \ldots\ldots\ldots\ldots$

(a) $6\,c^5$ (b) $1 + 2\,c$ (c) 1 (d) $1 + 2\,c + c^2$

---

19. If the midpoint of $\overline{AB}$ lies on the cartesian plane $x\,z$ and if A (– 3 , 12 + t , 5) , B (1 , 3 t , – 2) , then t = ...

(a) 5 (b) – 3 (c) – 2 (d) 1

20. The vector component of the force $\vec{F} = \hat{i} + 2\hat{j} - 4\hat{k}$ in direction of the vector $\vec{A} = 2\hat{i} + 4\hat{j} - 4\hat{k}$ is ....

(a) $\frac{13}{9}(-\hat{i} + 2\hat{j} - 2\hat{k})$
(b) $\frac{13}{9}(\hat{i} + 2\hat{j} - 2\hat{k})$
(c) $\frac{13}{9}(-\hat{i} - 2\hat{j} - 2\hat{k})$
(d) $\frac{13}{9}(\hat{i} - 2\hat{j} + 2\hat{k})$

---

21. If the ratio between the seventh term from the starting to the seventh term from the end of the expansion : $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}\right)^n$ is 1 : 4 , then n = ...

(a) 7
(b) 8
(c) 9
(d) 10

---

22. If the number : $z = \frac{-1+\sqrt{3}\,i}{2}$ where $i^2 = -1$ and $z_1 = \frac{1-z}{1+z}$, the two square roots of the number z_1 in the exponential form are ....

(a) $\sqrt{3}\,e^{-\frac{\pi}{4}i}, \sqrt{3}\,e^{\frac{3\pi}{4}i}$
(b) $\sqrt{3}\,e^{-\frac{\pi}{6}i}, \sqrt{3}\,e^{\frac{5\pi}{6}i}$
(c) $\sqrt{3}\,e^{\frac{\pi}{2}i}, \sqrt{3}\,e^{-\frac{\pi}{2}i}$
(d) $\sqrt[4]{3}\,e^{-\frac{\pi}{4}i}, \sqrt[4]{3}\,e^{\frac{3\pi}{4}i}$

---

23. If A is not singular matrix , then (A^{adj}) = ...

(a) $|A|A^{-1}$
(b) $(A^t)^{-1}$
(c) $\frac{A^{-1}}{|A|}$
(d) $|A^t|$

---

24. In the expansion : $(2 + X)^9$ according ascending powers of X. If $T_6 + \frac{1}{4}T_7 = 7T_8$, then X = ...

(a) $\frac{3}{2}$ or $\frac{4}{3}$
(b) $\frac{-3}{2}$ or $\frac{-4}{3}$
(c) $\frac{3}{2}$ or $\frac{-4}{3}$
(d) $\frac{-3}{2}$ or $\frac{4}{3}$

---

25. The S.S. of the equation : $\begin{vmatrix} |X-1| & 2 \\ 4 & 3 \end{vmatrix} = 7$ is ....

(a) {6 , 4}
(b) {4 , −6}
(c) {6 , −4}
(d) ∅

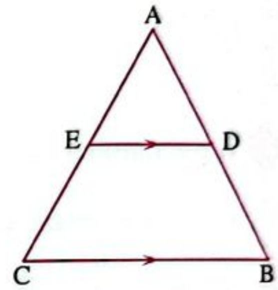
Choose the correct answer from the given ones :

1. The fifth term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^{12}$ according to the descending powers of x equals ....

ⓐ $\frac{7920}{x^4}$ ⓑ $\frac{-7920}{x^4}$ ⓒ $7220\, x^{-4}$ ⓓ $-7520\, x^4$

2. **In the opposite figure :**

If $\overline{DE} \,//\, \overline{BC}$, then $\begin{vmatrix} 5 & 6 & 7 \\ DE & AD & AE \\ BC & AB & AC \end{vmatrix} =$ ....



ⓐ 7 ⓑ 6
ⓒ 5 ⓓ zero

3. The number of ways of selecting an even number and two odd numbers from 4 even numbers , 5 odd numbers is ....

ⓐ $4\ {}^5C_2$ ⓑ $4 + {}^5C_2$ ⓒ $4\ {}^5P_2$ ⓓ $4 + {}^5P_2$

4. The plane equation which contains the two straight lines :
$\vec{r_1} = (\hat{i} + \hat{j}) + t_1\,(\hat{i} + 2\hat{j} - \hat{k})$, $\vec{r_2} = (\hat{i} + \hat{j}) + t_2\,(-\hat{i} + \hat{j} - 2\,\hat{k})$ is ....

ⓐ $-3x + 3y + 3z = 0$ ⓑ $x - y + z = 0$
ⓒ $-3x + 2y + 3z = 0$ ⓓ $-3x + y + z = 0$

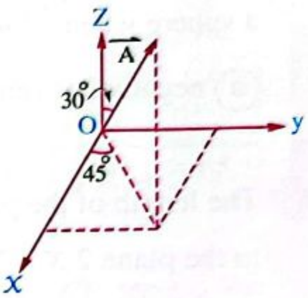
5. The equation system : $x - 2y + 2z = 0$, $3x + 4z = 0$, $6z - y = 0$ has ....

ⓐ infinite number of solutions.
ⓑ unique solution other than the trivial solution.
ⓒ only the trivial solution.
ⓓ no solution.

6 The determinant $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} =$ ....

(a) c^2 (b) abc (c) zero (d) $-$ abc

7 The opposite figure represents the vector $\overrightarrow{A}$ whose magnitude is 10 units , then the direction angle measures of the vector $\overrightarrow{A}$ is ....



(a) 45° , 45° , 30

(b) 69° 18̀ , 69° 18̀ , 30°

(c) 110° 32̀ , 110° 32̀ , 150°

(d) 20° 32̀ , 20° 32̀ , 60°

8 If $\left(\frac{\omega^2}{1+2\omega^2}\right)^4 + \left(\frac{1+2\omega}{\omega}\right)^4 = a\,\omega^2$ where ω are the cubic root of one , then a = ...

(a) $\frac{-82}{9}$ (b) 9 (c) $\frac{80}{9}$ (d) $\frac{82}{9}$

9 The coefficient of x^5 in the expansion of $(1 + x + x^2)^6$ equals ....

(a) 126 (b) 180 (c) 120 (d) 960

10 If $\overrightarrow{A} = 4\hat{i} - 3\hat{j} + 5\hat{k}$, then the component of $\overrightarrow{A}$ in direction of z-axis equals ....

(a) 5 (b) 4 (c) -3 (d) 3

11 If $\overrightarrow{A}$, $\overrightarrow{B}$ are two unit vectors , the measure of their included angle is θ

, then $2\,\|\overrightarrow{A} \times \overrightarrow{B}\|\,(\overrightarrow{A} \cdot \overrightarrow{B}) =$ ....

(a) sin θ cos θ (b) 2 sin θ (c) cos 2 θ (d) sin 2 θ

12 The straight line equation passing through the two points (2 , − 1 , 3) , (0 , 3 , 1) is ....

(a) $\overrightarrow{r} = (2 , -1 , 3) + t\,(2 , -4 , 2)$

(b) $\overrightarrow{r} = (2 , -1 , 3) + t\,(2 , 2 , 4)$

(c) $\overrightarrow{r} = (2 , -4 , 2) + t\,(2 , -1 , 3)$

(d) $\overrightarrow{r} \cdot (2 , -4 , 2) =$ zero

13 The value of ℓ which makes the equation system :

$x + y + z = 6$, $4x + \ell y - \ell z = 0$, $3x + 2y - 4z = -8$ has a unique solution $\in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\{3\}$ (d) $\{3, -1\}$

14 The equation $x^2 + y^2 + z^2 + 2\ell x + 2my + 2nz + d = 0$ represents an equation of a sphere when $\ell^2 + m^2 + n^2 - d =$ ....

(a) negative or zero. (b) negative. (c) zero. (d) positive.

15 The length of the perpendicular drawn from the point $(2, 3, 1)$ to the plane $2x - 2y + z = 5$ is .... length unit.

(a) 1 (b) 2 (c) 3 (d) 4

16 If $z_1 = \cos 75° + i \sin 75°$, $z_2 = \cos 15° + i \sin 15°$, then the algebraic form of the number $(z_1 + z_2)$ is ....

(a) $\frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2} i$ (b) $\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} i$ (c) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$ (d) $\sqrt{3} + \sqrt{3} i$

17 The radius length of the surface produced from the intersection of the sphere $x^2 + y^2 + z^2 = 49$ and the plane whose equation is $2x + 3y - z - 5\sqrt{14} = 0$ is .... length unit.

(a) $4\sqrt{6}$ (b) $2\sqrt{6}$ (c) $\sqrt{6}$ (d) 6

18 If $\vec{A} = (k, 3, -4)$, $\vec{B} = (-2, 9, m)$ and $\vec{A} \parallel \vec{B}$, then $k \times m =$ ....

(a) -8 (b) 8 (c) -18 (d) 18

19 If $z = (\sin \theta - i \cos \theta)^n$, then the principle amplitude of the number (z) is ....

(a) $-\theta n$ (b) $\left(\frac{\pi}{2} - \theta\right) n$ (c) $\left(\frac{-\pi}{2} + \theta\right) n$ (d) $\left(\frac{-\pi}{2} - \theta\right) n$

20 If A , B are two matricies in order (3×3) and $A = 2B$, $|B| = 5$, then $|A| =$ ....

(a) 8 (b) 16 (c) 32 (d) 40

21 $^nC_0 - {^nC_1} + {^nC_2} - {^nC_3} + \cdots (-1)^n\, {^nC_n} = \ldots\ldots\ldots\ldots$

(a) 2^n (b) $(-1)^n$ (c) zero (d) $(-2)^n$

---

22 The two non parallel straight lines $\overrightarrow{r_1} = \overrightarrow{A_1} + t_1\, \overrightarrow{d_1}$, $\overrightarrow{r_2} = \overrightarrow{A_2} + t_2\, \overrightarrow{d_2}$ lie on the same plane if $\ldots\ldots\ldots\ldots$

(a) $\overrightarrow{A_1} \times \overrightarrow{A_2} = \overrightarrow{O}$ (b) $\overrightarrow{d_1} \times \overrightarrow{d_2} = \overrightarrow{O}$

(c) $(\overrightarrow{A_2} - \overrightarrow{A_1}) \cdot (\overrightarrow{d_1} \times \overrightarrow{d_2}) = \text{zero}$ (d) $\overrightarrow{A_1A_2} \times \overrightarrow{d_1} = \overrightarrow{O}$

---

23 The coefficient of the middle term of the expansion $(16 + 32\,x + 24\,x^2 + 8\,x^3 + x^4)^5$ equals $\ldots\ldots\ldots\ldots$

(a) $^{20}C_{11}$ (b) $^{22}C_{11}$ (c) $64 \times {^{15}C_8}$ (d) $1024 \times {^{20}C_{10}}$

---

24 $e^{\theta i} + e^{-\theta i} = \ldots\ldots\ldots\ldots$

(a) $e^{2\theta i}$ (b) $2\cos\theta$ (c) $2\sin\theta$ (d) $e^{-2\theta i}$

---

25 If $^nP_x + {^xP_n} = 1440$, then the value of $^{n+4}P_{x-5} = \ldots\ldots\ldots\ldots$

(a) 4 (b) 5 (c) 9 (d) 10

Choose the correct answer from the given ones :

1. $x^2 + y^2 + z^2 + 4x - 6y + 8z + 4 = 0$ is a sphere equation whose diameter length equals .... length unit.

   (a) 5 (b) 10 (c) 15 (d) 20

2. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+1}C_1 & {}^{n+2}C_1 \\ {}^{n}C_2 & {}^{n+1}C_2 & {}^{n+2}C_2 \end{vmatrix} =$ ....

   (a) 1 (b) -1 (c) zero (d) n

3. In the expansion $\left(a x^2 + \frac{1}{a x}\right)^{11}$ according to the descending powers of x if the coefficients of x^4 and x^7 are equal , then a = ...

   (a) 1 (b) -1 (c) ± 1 (d) ± 2

4. If $\vec{A} + \vec{B} = \vec{C}$, $\|\vec{A}\| = 4$, $\|\vec{B}\| = 6$, $\|\vec{C}\| = 8$, then the measure of the angle between the two vectors $\vec{A}$, $\vec{B}$ is ....

   (a) $\frac{\pi}{3}$ (b) $\cos^{-1}\frac{1}{3}$ (c) $\cos^{-1}\frac{1}{4}$ (d) $\frac{\pi}{4}$

5. The equation of the straight line of intersection of the two planes $L_1 : 2x - y + z - 1 = 0$, $L_2 : x - 3y - z + 2 = 0$ is ....

   (a) $\frac{x+1}{-1} = \frac{y}{2} = \frac{z}{3}$ (b) $\frac{x-1}{1} = \frac{y}{3} = \frac{z-5}{1}$

   (c) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z}{-1}$ (d) $\frac{x-1}{4} = \frac{y-1}{3} = \frac{z}{-5}$

6. If z , $\bar{z}$ are two conjugate numbers , then $z + \bar{z}$ could be ....

   (a) $9 - 4i$ (b) $5i$

   (c) 13 (d) $1 + i$

7. If $\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, then $\begin{pmatrix} x \\ y \end{pmatrix} =$ ....

(a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

---

8. The number of ways of parking 4 cars, side by side in a parking area containing 10 places in a row equals ....

(a) $7 \underline{|3}$ (b) $7 \underline{|4}$ (c) $9 \underline{|3}$ (d) $9 \underline{|4}$

---

9. For any point (x, y, z) on x-axis, then ....

(a) $y = 0$, $z = 0$ (b) $x = 0$, $z = 0$

(c) $x = 0$, $y = 0$ (d) $x = 0$

---

10. If ${}^{n+2}P_r : {}^{n+2}C_r = 2 : 1$, ${}^nC_{r+1} : {}^nC_r = 5 : 3$, then the value of : ${}^{2n}C_{n-r} =$ ....

(a) ${}^6P_3$ (b) ${}^{14}C_5$

(c) ${}^{11}C_3$ (d) ${}^7P_2$

---

11. In the expansion of $\left(a x + \frac{1}{b x}\right)^{10}$ according to the descending power of x, if the term free of x equals the coefficient of the seventh term, then ab = ...

(a) 1 (b) ${}^{10}C_2$ (c) $\frac{5}{6}$ (d) ${}^6P_3$

---

12. The volume of the parallelepiped in which three non parallel edges are represented by the vectors $\overrightarrow{A} = (3, -4, 1)$, $\overrightarrow{B} = (0, 2, -3)$, $\overrightarrow{C} = (3, 2, 2)$ equals .... cubic unit.

(a) $\frac{39}{2}$ (b) 60 (c) 25 (d) $61 \frac{1}{2}$

---

13. If $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = a^2 + b^2 + c^2 + \ell$, then $\ell =$ ....

(a) 2 (b) 11 (c) -3 (d) 1

14 The two straight lines :

$\overrightarrow{r_1} = (1 , 2 , 4) + t_1 (2 , -1 , 1)$, $\overrightarrow{r_2} = (1 , 1 , 1) + t_2 (-2 , 7 , 11)$ are ....

(a) perpendicular and skew.
(b) perpendicular intersected.
(c) parallel.
(d) intersected and not perpendicular.

---

15 If $z = \frac{2\left(5 - 3\sqrt{3}\, i\right)}{1 + 2\sqrt{3}\, i}$ where $i^2 = -1$, then $\sqrt{z}$ equals ....

(a) $e^{\frac{3\pi}{4} i}$, $e^{\frac{-\pi}{4} i}$
(b) $\sqrt{2}\, e^{\frac{-\pi}{6} i}$, $\sqrt{2}\, e^{\frac{2\pi}{3} i}$
(c) $2\, e^{\frac{-\pi}{3} i}$, $2\, e^{\frac{2\pi}{3} i}$
(d) $4\, e^{\frac{\pi}{6} i}$, $4\, e^{\frac{-5\pi}{6} i}$

---

16 If the plane $3x + 2y + 4z = 12$ intersects the coordinate axes x , y , z at the points A , B , C respectively , then the area of the triangle ABC = ... square unit.

(a) $\sqrt{173}$
(b) $3\sqrt{29}$
(c) 13.5
(d) $6\sqrt{29}$

---

17 If the two middle terms in the expansion $(a + 2b)^{2n+1}$ are equal , then ....

(a) $\frac{a}{b} = \frac{1}{2}$
(b) $a = 4b$
(c) $a = 8b$
(d) $a = 2b$

---

18 The system $2x + 5y = 0$, $3x - z = 0$, $2y - 3z = 0$ has ....

(a) unique solution other than the trivial solution.
(b) the trivial solution only.
(c) infinite number of solutions except the trivial solution.
(d) infinite number of solution contains the trivial solution.

---

19 If z is a complex number where $z^2 = 1 + \omega^2$, then z = ...

(a) $\pm (1 + i)$
(b) $\pm \omega^2 i$
(c) $\pm \omega i$
(d) $\pm \omega$

---

20 If L and M are two complex numbers where : $L = a\omega + b\omega^2$, $M = a\omega^2 + b\omega$, $a , b \in \mathbb{R} - \{0\}$, then the incorrect statement of the following is ....

(a) L and M are one of them multiplicative inverse of each other.
(b) L and M are conjugate.
(c) $L^2 - M^2 = \pm\sqrt{3}\, i\, (a^2 - b^2)$
(d) $(\omega L + \omega^2 M) \in \mathbb{R}$

21 If A , B are two non singular matrices , then $(AB)^{-1} =$ ....

(a) $-AB$ (b) $A^{-1} B^{-1}$ (c) $B^{-1} A^{-1}$ (d) $(BA)^{-1}$

---

22 The length of perpendicular from the point A (3 , 0 , – 5) on the plane $2x + \sqrt{5}\, y + 4z - 6 = 0$ equals .... length unit.

(a) $\frac{14}{5}$ (b) $\sqrt{17}$ (c) 1.3 (d) 4

---

23 The equation of the plane which passes through the point (3 , 2 , 0) and contains the straight line $x - 3 = \frac{y-6}{5} = \frac{z-4}{4}$ is ....

(a) $x + y - z + 2 = 0$
(b) $x - y + z - 1 = 0$
(c) $2x + y - z - 3 = 0$
(d) $2x - y + z - 2 = 0$

---

24 The plane y z divides the line between A (2 , 4 , 5) and B (3 , 5 , – 4) by the ratio ....

(a) 2 : 3 internally
(b) 3 : 2 internally
(c) 2 : 3 externally
(d) 4 : 3 externally

---

25 The S.S. of the equation : $\underline{|1 + \log x} = 1$ is ....

(a) $\{\frac{1}{10}\}$ (b) $\{1\}$ (c) $\{\frac{1}{10} , 1\}$ (d) $\{0 , -1\}$

Choose the correct answer from the given ones :

1. If ${}^{10}C_x + 2 \times {}^{10}C_{x+1} + {}^{10}C_{x+2} = {}^{12}C_7$, then the sum of possible values of x equals ....

(a) 6 (b) 11 (c) 7 (d) 8

2. The area of the parallelogram whose diagonals $\overrightarrow{A} = 3\,\hat{i} + \hat{j} - 2\,\hat{k}$, $\overrightarrow{B} = \hat{i} - 3\,\hat{j} + 4\,\hat{k}$ equals .... square unit.

(a) $3\sqrt{21}$ (b) $5\sqrt{3}$ (c) $4\sqrt{11}$ (d) $4\sqrt{13}$

3. The value of k which makes x is a factor of the determinant : $\begin{vmatrix} x+1 & 3 & 1 \\ k & 2 & 3 \\ x & 2 & 1 \end{vmatrix}$ equals ....

(a) 2 (b) 3 (c) zero (d) -4

4. The equation system $3y - z = 5$, $2x + y + z = 9$, $x + 2y + 2z = 3$ has ....

(a) unique solution.

(b) infinite number of solutions one of them the trivial solution.

(c) no solution at all.

(d) infinite number of solutions other than the trivial solution.

5. The measure of the angle between the straight line $\vec{r} = (1\,, 2\,, -1) + t\,(1\,, -1\,, 1)$ and the plane $\vec{r} \cdot (2\,, -1\,, 1) = 4$ is ....

(a) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (b) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (c) $\sin^{-1}\left(\frac{1}{3}\right)$ (d) $\frac{\pi}{6}$

6. If the equations $3x - 2y + z = 0$, $6x - 5y + 2z = 0$, $9x - 6y + kz = 0$ have solutions other than the zero solution , then k = ...

(a) zero (b) 1 (c) 3 (d) 4

7. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are represented by the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ of ΔABC respectively in one way around , then ....

(a) $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$
(b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
(c) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$
(d) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

---

8. A force of magnitude $10\sqrt{3}$ force unit acts in vector direction $\hat{i} - \hat{j} - \hat{k}$ to move a particle from the point A (4 , 2 , – 1) to the point B (2 , 1 , – 3) , then the work done equals .... work unit.

(a) 20 (b) 10 (c) $\sqrt{20}$ (d) $\sqrt{10}$

---

9. The value of the term free of x in the expansion : $\left(\frac{1}{x^2}\right)\left(x + \frac{1}{x^3}\right)^{10}$ equals ....

(a) 54 (b) 90 (c) 45 (d) 20

---

10. The area of the circle produced by the intersection of the plane : $2x + 3y - z - 5\sqrt{14} = 0$ with the sphere $x^2 + y^2 + z^2 = 49$ equals .... square unit.

(a) 16π (b) 24π (c) 18π (d) 12π

---

11. If the point A (2 , 3 , 1) and the straight L : $\vec{r} = (3\hat{i} + \hat{j} + 3\hat{k}) + t(\hat{i} - 2\hat{j} + 2\hat{k})$, then the perpendicular length drawn from the point A to the straight line equals .... length unit.

(a) 3 (b) 2 (c) 1 (d) zero

---

12. If $z = \sin\frac{\pi}{9} + i\cos\frac{\pi}{9}$, then the sum of amplitudes of the cubic roots of the number $(\bar{z})^9$ equals ....

(a) $\frac{\pi}{2}$ (b) $\frac{7\pi}{6}$ (c) $\frac{-\pi}{6}$ (d) $\frac{-\pi}{2}$

---

13. If $\vec{A}$, $\vec{B}$ are unit vectors and their acute included angle is θ , then $\|\vec{A} - \vec{B}\| =$ ....

(a) $2\cos\theta$ (b) $2\sin\theta$ (c) $2\cos\frac{\theta}{2}$ (d) $2\sin\frac{\theta}{2}$

14 If $A^{-1} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, then y = ...

(a) 5 (b) 6 (c) 7 (d) 8

15 The number of terms in the expansion $(x + y)^{100} - (x - y)^{100}$ after simplifying equals ....

(a) 50 (b) 51 (c) 202 (d) 101

16 $(1 + \omega)^4 + (1 + \omega^2)^4 + (\omega + \omega^2)^4 =$ ....

(a) 1 (b) zero (c) ω (d) ω^2

17 If $N = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & 5 \\ 1 & 4 & 2 \end{vmatrix}$, $M = \begin{vmatrix} 3 & 0 & 9 \\ 4 & 6 & 10 \\ 5 & 20 & 10 \end{vmatrix}$, then M = ...

(a) N (b) 10 N (c) 20 N (d) 30 N

18 The number of ways which we can put 3 identical balls in 5 places where each place can take one place = ...

(a) 8 (b) 10 (c) 15 (d) 60

19 If the angle between two straight lines $\frac{x}{a} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$ equals $\frac{\pi}{3}$, then a may be equal ....

(a) 1 (b) 2 (c) 3 (d) $\frac{12}{5}$

20 If $z = x + \sqrt{3}\, i$ where $i^2 = -1$ and $|z| = |z + \bar{z}|$, then $x =$ ....

(a) zero (b) ± 1 (c) ± 2 (d) 1

21 If $z = 1 + \omega$, $\bar{z}$ is the conjugate of z, then $\left(\frac{\bar{z}}{z}\right)^3 =$ ....

(a) $3 - 2\omega$ (b) $1 - \omega^2$ (c) 1 (d) ω

22 If the straight line $x = 3y = kz$ is parallel to the plane $x + 3y + 2z + 4 = 0$, then $k =$ ....

(a) 5 (b) -1 (c) 3 (d) -4

---

23 The ratio between the coefficient of two consecutive terms in the expansion $(1 + x)^{24}$ according ascending power of x is $4 : 1$, then the two terms are ....

(a) T_4, T_5 (b) T_{20}, T_{21} (c) T_3, T_4 (d) T_{21}, T_{22}

---

24 If ${}^{n+2}C_3$, ${}^{n}P_2$, ${}^{n}C_2$ are in geometrical sequent, then the value of $n =$ ....

(a) 2 (b) 7 (c) 2 or 7 (d) 2 or 5

---

25 The intersection of the straight line $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z}{3}$ and the plane $x - 2y + 3z + 5 = 0$ is ....

(a) $(1, 2, 3)$ (b) $(-1, 2, 3)$ (c) $(-1, 2, 0)$ (d) $(0, 2, -1)$

Choose the correct answer from the given ones :

1. If $1, \omega, \omega^2$ are the cubic roots of unity , $i^2 = -1$, then $\left(1 + \frac{1}{\omega} + i\right)\left(1 + \frac{1}{\omega^2} + i\right) =$ ....

(a) ω^2 (b) ωi (c) i (d) $-i$

2. If the point $A(2, -1, 3)$, the straight line $L : \vec{r} = (1, -1, 2) + t(2, 2, -1)$, then the plane equation which contains each the point A and the straight line L is ....

(a) $3x - 2y - 2Z - 1 = 0$ (b) $2x - 3y + 2Z - 1 = 0$

(c) $2x - 3y - 2Z - 1 = 0$ (d) $2x - 3y - 2Z + 1 = 0$

3. If $N = \begin{vmatrix} x & y & z \\ \ell & m & n \\ k & p & q \end{vmatrix} = 2$, then the value of : $\begin{vmatrix} 2x & 2y & 2z \\ 5\ell + x & 5m + y & 5n + z \\ 7k - 3\ell & 7p - 3m & 7q - 3n \end{vmatrix}$

= ....

(a) 700 (b) 140 (c) 250 (d) 100

4. The sphere equation whose centre is $(1, -1, 1)$ and its radius length equals the radius length of the sphere : $2x^2 + 2y^2 + 2z^2 - 2x + 4y - 6z = 1$ is ....

(a) $x^2 + y^2 + z^2 + 2x - 2y + 2z + 1 = 0$

(b) $x^2 + y^2 + z^2 - 2x + 2y - 2z - 1 = 0$

(c) $x^2 + y^2 + z^2 - 2x + 2y - 2z + 1 = 0$

(d) $x^2 + y^2 + z^2 + 2x + 2y + 2z - 1 = 0$

5. If the two straight lines $L_1 : x = 2t_1 - 1$, $y = t_1 + 1$, $z = t_1 - 1$

, $L_2 : x = a t_2 - 1$, $y = 2t_2 + 1$, $z = b t_2 - 2$ are parallel

, then $a + b =$ ....

(a) 4 (b) 2 (c) 6 (d) -2

6 The system of the equations : $x + 2y + z = 3$, $4x - y - z = 6$, $x + y + 3z = 10$ has ....

(a) unique solution.
(b) infinite number of solution.
(c) only the trivial solution.
(d) no solutions at all.

---

7 The multiplicative inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$ is ....

(a) $-A^+$
(b) $\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$
(c) $\frac{1}{\Delta} A^T$
(d) I

---

8 The vector which lies in the coordinate plane $x z$ where $x \geq 0$, $z \geq 0$ and makes an angle of measure 30° with the positive direction of x-axis , then its direction cosines is ....

(a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$
(b) $\left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$
(c) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$
(d) $\left(\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}\right)$

---

9 If $\overrightarrow{A} = (4, -k, 6)$, $\overrightarrow{B} = (2, 2, m)$ and $\overrightarrow{A} \parallel \overrightarrow{B}$, then $k + m =$ ....

(a) -3
(b) -2
(c) -1
(d) zero

---

10 If A is a square matrix , $|A| = 4$, then $A \times \text{Adj}(A) =$ ....

(a) 4
(b) 4 I
(c) I
(d) $\frac{1}{4}$ I

---

11 The number of ways can 8 awards be distributed on 4 students equals ....

(a) ${}^8C_4$
(b) ${}^8C_2$
(c) ${}^8C_2 + {}^6C_2 + {}^4C_2 + 1$
(d) ${}^8C_2 \times {}^6C_2 \times {}^4C_2$

---

12 The term which has the greatest coefficient in the expansion $(3 + 2x)^6$ according to the ascending powers of x is ....

(a) T_1
(b) T_3
(c) T_4
(d) T_7

13) The two planes : $x + 2y + kz = 0$, $2x + y - 2z = 0$ are perpendicular if k = ...

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) -2 (d) 2

14) The amplitude of the complex number $z = \frac{1 + i\tan 18^\circ}{1 - i\tan 18^\circ}$ is ....

(a) $\frac{1}{2}\pi$ (b) $\frac{1}{4}\pi$ (c) $\frac{1}{5}\pi$ (d) $\frac{1}{12}\pi$

15) If ${}^{n}C_3 : {}^{n-1}C_4 = 8 : 5$, then n = ...

(a) 5 (b) 7 (c) 8 (d) 9

16) The length of the intercepted part of the straight line : $\frac{x - 5}{1} = \frac{y - 3}{3} = \frac{z - 1}{-2}$ by the sphere $x^2 + y^2 + z^2 - 2x - 4y - 2z - 39 = 0$ equals .... length unit.

(a) $\sqrt{101}$ (b) $\sqrt{111}$ (c) $\sqrt{126}$ (d) $\sqrt{139}$

17) The measure of the angle between the straight line : $\frac{x + 8}{9} = \frac{y - 10}{4} = \frac{z - 9}{-2}$ and the plane $3x + 4y + 5z = 76$ equals ....

(a) 62° 20` (b) 27° 40` (c) 13° 25` (d) 88° 16`

18) In the expansion $\left(2x + \frac{3}{x^2}\right)^{20}$ according to the descending powers of x, if the ninth and tenth terms are equal , then the value of x = ...

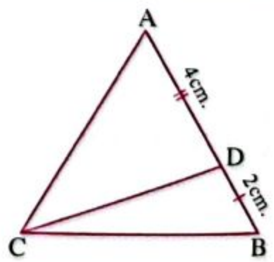
(a) 8 (b) 2 (c) $\sqrt[3]{2}$ (d) 7

19) **In the opposite figure :**

ΔABC is an equilateral triangle

, $D \in \overline{AB}$ when $AD = 2\,DB = 4$ cm

, then $\overrightarrow{CD} . \overrightarrow{CB}$ = ...



(a) $6\sqrt{3}$ (b) 30 (c) $2\sqrt{21}$ (d) 52

20) The vectors $\overrightarrow{A} = (3 , -4 , 5)$, $\overrightarrow{B} = (0 , -4 , 3)$, $\overrightarrow{C} = (0 , 0 , 5)$ represented by three adjacent edges in parallelpiped , its volume = ... cubic units.

(a) 12 (b) 125 (c) 50 (d) 60

21 If $\|\overrightarrow{A}\| = 6$, $\|\overrightarrow{B}\| = 4$ and component of $\overrightarrow{A}$ in direction $\overrightarrow{B} = 3$ then component of $\overrightarrow{B}$ in direction $\overrightarrow{A}$ equals ....

(a) -2 (b) 2 (c) -8 (d) 8

---

22 The rank of the Augment matrix of the system of equations : $2x - 3y = 5$, $6x + y = 15$ is ....

(a) zero (b) 1 (c) 2 (d) 3

---

23 In the expansion $\left(\frac{x^3}{4} - \frac{4}{x}\right)^{11}$ according descending power of x , the value of x to make the sum of the two middle terms = zero is ....

(a) 1 (b) 2 (c) $\pm\sqrt{2}$ (d) ± 2

---

24 In the expansion : $(1 - m x)^n$ according ascending power of x , if $T_2 = \frac{-1}{4} x$, $T_3 = \frac{3}{100} x^2$, then m = ...

(a) $\frac{1}{4}$ (b) 4 (c) $\frac{1}{100}$ (d) $\frac{3}{400}$

---

25 The exponential form of the complex Number $Z = \left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ is ....

(a) $2e^{\frac{-\pi}{12}i}$ (b) $e^{\frac{-\pi}{3}i}$ (c) $e^{\frac{\pi}{6}i}$ (d) $e^{\frac{\pi}{12}i}$

Choose the correct answer from the given ones :

1. All the following matrices haven't multiplicative inverse except ....

(a) $\begin{pmatrix} 3 & -6 \\ -2 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & -8 \\ 2 & -4 \end{pmatrix}$ (d) $\begin{pmatrix} 4 & -2 \\ 10 & 5 \end{pmatrix}$

2. If ${}^{n}C_{r} = {}^{n}P_{r}$, then ....

(a) $n = r$ (b) $r = \frac{n}{2}$ (c) $r = 1$ or n (d) $r = 0$ or 1

3. The coefficient of the middle term in the expansion $(1 + x)^{2n}$ is ....

(a) $\frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{\underline{|n}} \times 2^{n}$ (b) ${}^{2n}C_{n+1}$

(c) ${}^{2n}C_{n-1}$ (d) $\frac{\underline{|2n}}{\underline{|n}}$

4. The two straight lines : $\vec{r}_1 = (1, 2, 4) + t_1 (2, -1, 1)$, $\vec{r}_2 = (1, 1, 1) + t_2 (-2, 7, 11)$ are ....

(a) parallel. (b) perpendicular and intersected.

(c) perpendicular and skew. (d) coincident.

5. The value of the determinant : $\begin{vmatrix} 3 & 5 & 6 \\ 8 & -4 & 12 \\ 4 & 7 & 5 \end{vmatrix} - \begin{vmatrix} -2 & 1 & -3 \\ 12 & 20 & 24 \\ 4 & 7 & 5 \end{vmatrix} =$ ....

(a) 4 (b) 1 (c) zero (d) 2

6. The equation : $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$ has ....

(a) unique solution. (b) infinite number of solution.

(c) no solution at all. (d) trivial solution.

7. If $\overrightarrow{A}+\overrightarrow{B}+\overrightarrow{C}=\overrightarrow{0}$, $\|\overrightarrow{A}\|=6$, $\|\overrightarrow{B}\|=8$, $\|\overrightarrow{C}\|=10$

, then $\overrightarrow{A}\cdot\overrightarrow{B}+\overrightarrow{B}\cdot\overrightarrow{C}+\overrightarrow{C}\cdot\overrightarrow{A}=$ ....

(a) -40 (b) -50 (c) -100 (d) -200

---

8. The equation : $\|\overrightarrow{r}\|^2-\overrightarrow{r}\cdot(2,4,-2)-10=0$ represents an equation of ....

(a) a circle of radius length 4 (b) a plane.

(c) a sphere of radius length 4 (d) a sphere of radius length 3

---

9. If $|z_1|=|z_2|=1$ and the amplitude of $(z_1 z_2^3)=81°$ and the amplitude of $\left(\frac{z_1}{z_2}\right)=33°$

, then the number $(z_1^{15}\times z_2^{15})$ on the form $x+y\,i$ is ....

(a) $1+\sqrt{2}\,i$ (b) $\frac{-1}{2}+\frac{\sqrt{3}}{2}i$ (c) $\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$ (d) $\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$

---

10. The measure of the angle between the two planes :

$\overrightarrow{r}\cdot\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$, $\overrightarrow{r}\cdot\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$ equals .... (to nearest minute)

(a) $75°\,14'$ (b) $61°\,13'$ (c) $32°\,11'$ (d) $78°\,37'$

---

11. If $B(-2,1,2)$, $C(2,4,-3)$, then the equation of the straight line $\overleftrightarrow{BC}$ is ....

(a) $\overrightarrow{r}=(4,3,-5)+t(2,4,-3)$ (b) $\overrightarrow{r}=(2,4,-3)+t(4,3,-5)$

(c) $\overrightarrow{r}=(-2,4,3)+t(4,3,-5)$ (d) $\overrightarrow{r}=(-4,3,-5)+t(2,4,-3)$

---

12. In the expansion $\left(2x+\frac{3}{x^2}\right)^n$ according to the descending power of x , the ninth term and the tenth term are equal and the ratio between the sixth term and the seventh term as $8:15$, then the value of n equals ....

(a) 16 (b) 20 (c) 15 (d) 18

---

13. If ${}^{n+2}P_r=2\,{}^{n+2}C_r$, $\frac{{}^{n}C_{r+1}}{{}^{n}C_r}=\frac{5}{3}$, then the value of : $2\,{}^{n}C_{n-r}+{}^{n+3}P_{r-1}$ is ....

(a) 1692 (b) 8101 (c) 4032 (d) 2012

14 The measure of the angle between the two straight lines : $\frac{x-3}{2}=\frac{z-1}{-2}$, $y=1$

, $\frac{x+1}{1}=\frac{y-2}{2}=\frac{z+1}{-2}$ equals ....

(a) 15° (b) 30° (c) 45° (d) 60°

---

15 $(a+b\,\omega+a\,\omega^2)(a+b\,\omega^2+a\,\omega^4)=$ ....

(a) 1 (b) $a-b$ (c) $(a-b)^2$ (d) b^2-a^2

---

16 The radius length of the sphere whose equation is :

$x^2+y^2+z^2-6x-8y-10z+1=0$ is ....

(a) 7 (b) 5 (c) 2 (d) 15

---

17 $\begin{vmatrix} ab & a & \frac{1}{c} \\ ac & c & \frac{1}{b} \\ bc & b & \frac{1}{a} \end{vmatrix}=$ ....

(a) zero (b) 1 (c) 2 (d) 5

---

18 The direction cosines of the vector $\overrightarrow{A}=(-2,1,2)$ are ....

(a) $(-2,1,2)$ (b) $\left(\frac{-2}{3},\frac{1}{3},\frac{2}{3}\right)$ (c) $\left(\frac{5}{-2},5,\frac{5}{3}\right)$ (d) $(-1,1,1)$

---

19 If the position vectors of the points A , B , C are $\hat{i}+\hat{j}+\hat{k}$, $4\hat{i}+5\hat{j}+\hat{k}$, $5\hat{i}-2\hat{j}+\hat{k}$

, then the area of $\Delta ABC=$ .... square unit.

(a) 5 (b) $\frac{25}{2}$ (c) 25 (d) 50

---

20 If A (1 , 1 , 1) , B (3 , 2 , – 1) , then the plane $2x+3y-z=5$ divides $\overline{AB}$

by ratio ....

(a) 2 : 3 externally. (b) 4 : 1 internally.

(c) 1 : 8 internally. (d) 3 : 5 externally.

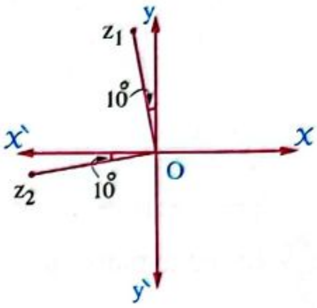
21 In the expansion of $\left(x^2 + \frac{1}{a x}\right)^8$ according to the descending powers of x , if the coefficient of the middle term equals the coefficient of x^7 then a = ...

(a) $\frac{2}{5}$ (b) $\frac{-14}{5}$ (c) $\frac{-15}{4}$ (d) $\frac{5}{4}$

---

22 **In the opposite figure :**

the principle amplitude of the complex number $\left(\frac{z_1}{z_2}\right)$ is ....



(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$

---

23 If $x + y\,i = e^{\theta i}$, then $x\,y \in$ ....

(a) $[\frac{-1}{2} , \frac{1}{2}]$ (b) $[-1 , 1]$ (c) $[\frac{-1}{2} , 0]$ (d) $[0 , \frac{1}{2}]$

---

24 If L , M are the two roots of the equation $x^2 - 11\,x + 27 = 0$

, then $\begin{vmatrix} \log_3 L & \log_3 M \\ -1 & 1 \end{vmatrix}$ = ...

(a) 3 (b) 1 (c) -1 (d) -3

---

25 If $1 + 20\,x + 190\,x^2 + \cdots + x^{20} = x^{18} + 18\,x^{17} + 153\,x^{16} + \cdots + 1$, then the set of real values of x are ....

(a) $\{-2 , 0\}$ (b) $\{0 , -1 , -2\}$ (c) $\{0 , 1 , 2\}$ (d) $\{0 , 2\}$

Choose the correct answer from the given ones :

1. The general form of the straight line passes through the point (2 , – 1 , 4) and its direction $\hat{i} + \hat{j} - 2\hat{k}$ is ....

(a) $\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-4}{-2}$ (b) $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z+2}{4}$

(c) $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ (d) $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-2}{4}$

2. In the expansion $\left(x^2 + \frac{1}{2x}\right)^{3n}$ according to the descending power of x , then the ratio between the term free of x and the middle term at n = 4 , $x = 1$ is ....

(a) $\frac{15}{112}$ (b) $\frac{3}{16}$ (c) $\frac{4}{21}$ (d) $\frac{26}{15}$

3. If $x + 2y + 3z = 5$, $3x + 3y + z = 9$ are two plane equations , then the plane equation passes through the point (– 1 , 3 , 2) perpendicular to each of the two planes is ....

(a) $3x + 6y + 3z = 45$ (b) $\vec{r} \cdot (3, 6, 3) = 14$

(c) $-7x + 8y - 3z - 25 = 0$ (d) $\vec{r} \cdot (3, 3, 1) = 4$

4. The two straight lines $\overleftrightarrow{xx'}$, $\overleftrightarrow{zz'}$ form the coordinate plane whose equation is ....

(a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $y = 2$

5. $\left(\omega^2 + \frac{1}{\omega}\right)\left(1 + \frac{1}{\omega^2}\right)^2 =$ ....

(a) 2 (b) zero (c) – 3 (d) – 5

6. The polygon which contains 44 diagonals , then the number of its sides equals ....

(a) 11 (b) 10 (c) 12 (d) 13

7. Each of the following equals ${}^nC_r$ except ....

(a) $\frac{n}{r} \times {}^{n-1}C_{r-1}$ (b) ${}^nC_{n-r}$ (c) $\frac{{}^nP_r}{\underline{|r}}$ (d) $\frac{\underline{|n}}{\underline{|n-r}}$

8 A straight line makes an angle of measure 60° with y-axis and 60° with z-axis , then it makes an angle with x-axis of measure may be equal ....

(a) 60° (b) 30° (c) 45° (d) 75°

9 If $z = 1 + \sqrt{3}\, i$, then its two square roots in the exponential form are ....

(a) $\pm\sqrt{2}\, e^{\frac{\pi}{3} i}$

(b) $\sqrt{2}\, e^{\frac{\pi}{3} i}$, $\sqrt{2}\, e^{\frac{-2\pi}{3} i}$

(c) $\sqrt{2}\, e^{\frac{\pi}{6} i}$, $\sqrt{2}\, e^{\frac{-5\pi}{6} i}$

(d) $\pm\sqrt{2}\, e^{\frac{2\pi}{3} i}$

10 The system of equations : $x + y + z = 6$, $x + 2y + 3z = 14$, $3x + 4y + kz = 14$, then the value of k which makes the system has a unique solution is ....

(a) 4 (b) 5 (c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{5\}$

11 If $\lfloor n = 720$, ${}^{n+1}P_{r-2} = 210$, then the value of : ${}^{n+1}C_r + {}^{n+1}C_{r-1} =$ ....

(a) 24 (b) 56 (c) 16 (d) 12

12 The plane equation which passes through the point A (3 , 2 , 0) and contains the straight line :

$\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is ....

(a) $x - y + z = 1$

(b) $x + y + z = 5$

(c) $x + 2y + z = 1$

(d) $2x - y + z = 5$

13 The centre of the sphere whose equation is :

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0$ is ....

(a) (x_2 , y_2 , z_2)

(b) $\left(\frac{x_1 - x_2}{2} , \frac{y_1 - y_2}{2} , \frac{z_1 - z_2}{2}\right)$

(c) $\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} , \frac{z_1 + z_2}{2}\right)$

(d) (x_1 , y_1 , z_1)

14 The projection of $\overrightarrow{AB}$ in direction of $\overrightarrow{CD}$ where A = (4 , – 3 , 2) , B = (1 , – 1 , – 1) , C = (2 , 2 , 2) , D = (3 , 3 , 3) is ....

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $-\frac{4}{\sqrt{3}}$ (d) $\frac{-2}{\sqrt{3}}$

15 The solution set of the equation : $\begin{vmatrix} x & 2x & 3x \\ 3x & 2x & x \\ x & -x & 0 \end{vmatrix} = 96$ in $\mathbb{R}$ is ....

(a) $\{4\}$ (b) $\{3\}$ (c) $\{2\}$ (d) $\{-2\}$

---

16 The intersection line of the two planes $\vec{r} \cdot (3, -1, 1) = 1$, $\vec{r} \cdot (1, 1, -2) = 2$ is a parallel to the vector ....

(a) $(2, 7, 1)$ (b) $(-2, 7, 13)$ (c) $(1, 7, 4)$ (d) $(7, 13, 1)$

---

17 If $\hat{e}$ is unit vector which perpendicular to each $\overrightarrow{A}$, $\overrightarrow{B}$, $\|\overrightarrow{A}\| = 9$, $\|\overrightarrow{B}\| = 16$, $\overrightarrow{A} \cdot \overrightarrow{B} = -72\sqrt{3}$, then $\overrightarrow{A} \times \overrightarrow{B}$ could be ....

(a) $-36\,\hat{e}$ (b) $36\,\hat{e}$ (c) $72\,\hat{e}$ (d) 72

---

18 If $\|\overrightarrow{A} + \overrightarrow{B}\| = \|\overrightarrow{A} - \overrightarrow{B}\|$, then the measure of the angle between $\overrightarrow{A}$, $\overrightarrow{B}$ where $\overrightarrow{A} \neq \overrightarrow{O}$, $\overrightarrow{B} \neq \overrightarrow{O}$ is ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

---

19 The term which has greatest coefficient in the expansion : $(2x + 7y)^3$ according descending powers of x is ....

(a) T_4 (b) T_3 (c) T_2 (d) T_1

---

20 In the expansion : $\left(x + \frac{1}{2x}\right)^n$ according descending powers of x

If $2 \times$ coefficient of T_2 = coefficient of T_1 + coefficient of T_3 , then $n =$ ....

(a) 8 (b) 122 (c) 15 (d) 18

---

21 If $A = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 2 & -2 & 1 \end{pmatrix}$, then $R\,k\,(A^T) =$ ....

(a) zero (b) 1 (c) 2 (d) 3

22 If $f(x) = \begin{vmatrix} \sin x & 0 & 0 \\ 1 & \cos\left(\frac{\pi}{2} - x\right) & 0 \\ 3 & -2 & \tan 3x \end{vmatrix}$, then $\lim\limits_{x \to 0} \frac{f(x)}{x^3} =$ ....

(a) 3 (b) $\frac{3}{2}\pi$ (c) 1 (d) $\frac{\pi}{2} - 3$

23 If $\begin{vmatrix} a & b & c \\ \ell & m & n \\ x & y & z \end{vmatrix} = 24$, then $\begin{vmatrix} 3a-4\ell & 3b-4m & 3c-4n \\ \ell & m & n \\ x & y & z \end{vmatrix} =$ ....

(a) 24 (b) 36 (c) 72 (d) 84

24 If $x(1-i) + y(1+i) + 2i =$ zero, then values of $\sqrt{3x + 4yi}$ are ....

(a) $(3+i), (1-i)$ (b) $\pm(2-i)$ (c) $\pm(5-i)$ (d) $6, -i$

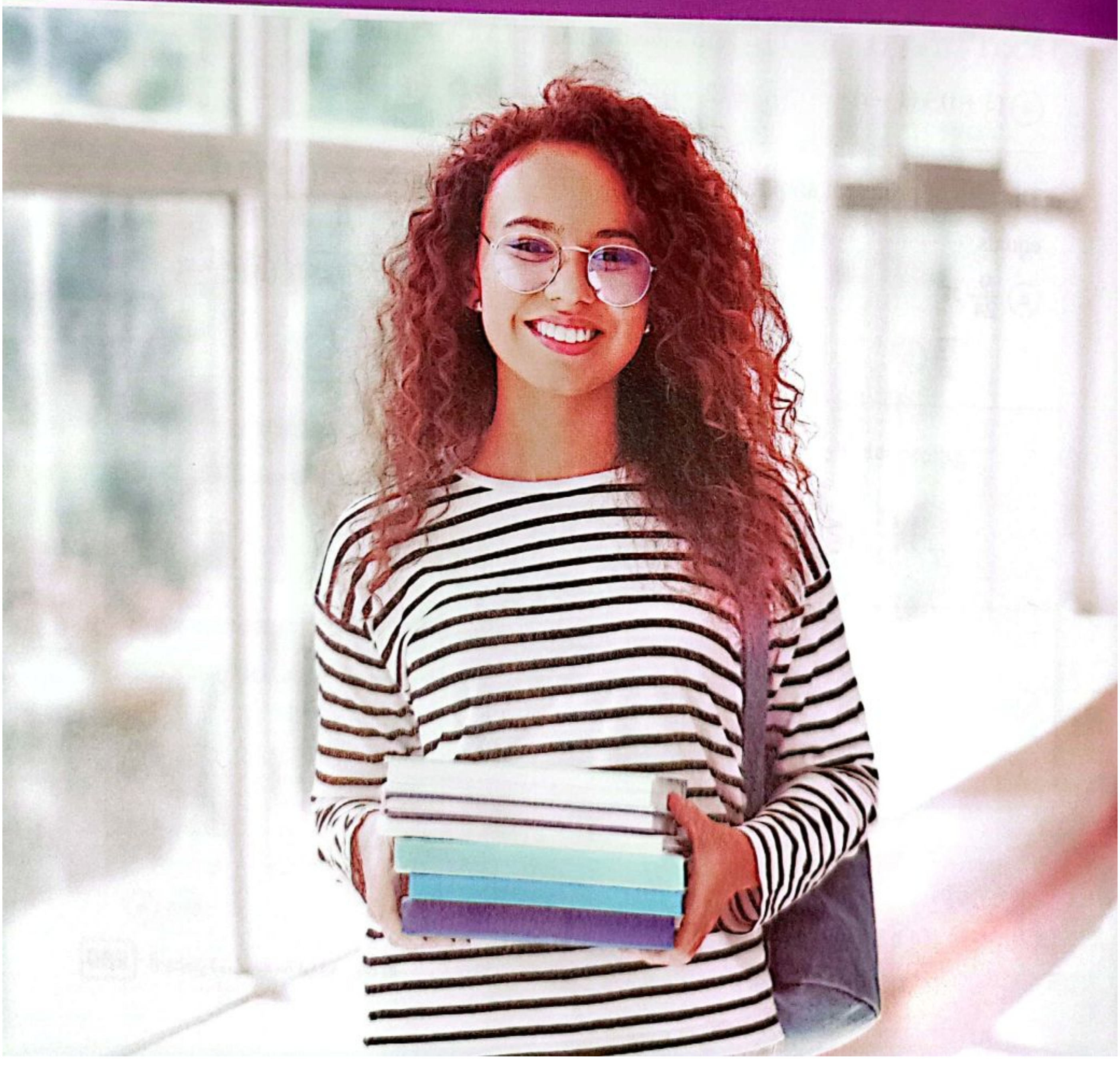
25 The sum of principal amplitudes of the cubic roots of the number $z = \left(1 + \sqrt{3}\, i\right)^2$ equals ....

(a) $\frac{9}{24}\pi$ (b) $\frac{2}{3}\pi$ (c) $\frac{-1}{9}\pi$ (d) $\frac{21}{9}\pi$

School Book Examinations

in

Algebra & Analytic Solid Geometry



School Book Examinations

Model 1

First Answer one of the following questions

1 Choose the correct answer :

(1) If ${}^{n}C_{n-3} = 20$, then n = ...

(a) 3 (b) 4 (c) 5 (d) 6

(2) $i + i^2 + i^3 + \cdots + i^{100} =$ ....

(a) 0 (b) 1 (c) 2 (d) 100

(3) If A (7 , – 1 , 8) , B (11 , 2 , – 4) , then the length of $\overline{AB}$ = ... length unit.

(a) 10 (b) 11 (c) 12 (d) 13

(4) $x^2 + y^2 + z^2 + 4x - 6y + 8z + 4 = 0$ is an equation of a sphere, whose diameter length = ... length unit.

(a) 5 (b) 10 (c) 15 (d) 20

(5) If $L_1 : \frac{x-3}{1} = \frac{y+2}{-2} = \frac{z+1}{-4}$ is parallel to $L_2 : \frac{x+5}{-2} = \frac{y}{k+1} = \frac{z-1}{8}$, then k = ...

(a) 3 (b) 4 (c) 5 (d) 6

(6) If θ is the measure of the angle included between the two vectors :

$\overrightarrow{A} = (-2 , -6 , 1)$, $\overrightarrow{B} = (2 , 6 , -1)$, then θ = ...

(a) 30° (b) 60° (c) 120° (d) 180°

2 Complete each of the following :

(1) The coefficient of x^5 in the expansion of $(3 - 2x)^7$ equals ....

(2) The solution set of $\begin{vmatrix} x & 1 & 2 \\ 0 & x & 3 \\ 0 & 0 & x \end{vmatrix} - 8 = 0$ in $\mathbb{R}$ is ....

(3) If $\overrightarrow{A} = 2\hat{i} + 3\hat{j} + m\hat{k}$, $\overrightarrow{B} = -6\hat{i} - 4\hat{j} + 4\hat{k}$ and $\overrightarrow{A} \perp \overrightarrow{B}$, then m = ...

(4) If $\overrightarrow{A} = (3 , 0 , 4)$, $\overrightarrow{B} = \hat{i} - 2\hat{j} + 3\hat{k}$, then $\overrightarrow{A} \times \overrightarrow{B}$ = ...

(5) The equation of the sphere whose centre is (2 , – 3 , 1) and its radius length equals $2\sqrt{5}$ length unit is ....

(6) The equation of the straight line passing through the two points :

A (2 , – 1 , 4) , B (– 1 , 0 , 2) is ....

Second Answer the following questions

3 [a] In the expansion of $\left(2x+\frac{1}{x^2}\right)^{15}$, find the value of the term free of x and prove that this expansion does not contain a term includes x^5 « 3075072 »

[b] Find all the different forms of the equation of the straight line :

$\frac{x+3}{2}=\frac{2y-1}{5}=\frac{3z+2}{4}$

---

4 **[a] Find the multiplicative inverse of the matrix :** $A=\begin{pmatrix} 1 & -1 & 2 \\ 2 & -3 & 1 \\ 1 & 5 & 21 \end{pmatrix}$

[b] Find the two square roots of the complex number :

$z=2-2\sqrt{3}\,i$ in the trigonometric form. « $2\left(\cos\frac{-\pi}{6}+i\sin\frac{-\pi}{6}\right)$, $2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ »

---

5 **[a] Solve the following equations :**

$x+3y+2z=13$, $2x-y+z=3$, $3x+y-z=2$

using the multiplicative inverse of the matrix. « 1 , 2 , 3 »

[b] Find the point of intersection of the planes :

$2x+y-z=-1$, $x+y+z-2=0$, $3x-y-z=6$ « $\left(2,\frac{-5}{2},\frac{5}{2}\right)$ »

School Book Examinations

Model 2

First Answer one of the following questions

1 Choose the correct answer :

(1) If the two equations : $2x + y = 1$, $4x + 2y = k$ have an infinite number of solutions , then $k = \dots\dots$

(a) zero (b) 1 (c) 2 (d) 3

(2) If ${}^{n+1}C_3 : {}^{n}C_4 = 2 : 3$, then $n = \dots\dots$

(a) 2 (b) 3 (c) 5 (d) 11

(3) If $x^2 + y^2 + z^2 + 6x - 4y + 10z - 8 = 0$ is the equation of a sphere whose centre is M , then $M = \dots\dots$

(a) $(-3, 2, -5)$ (b) $(4, -2, -5)$ (c) $(-3, -2, -5)$ (d) $(3, 2, 5)$

(4) If $\overrightarrow{A} = (-2, 4, 6)$, $\overrightarrow{B} = (0, k, 3)$ where $k \in \mathbb{Z}^+$ and $\|\overrightarrow{AB}\| = 7$, then the value of $k = \dots\dots$

(a) 10 (b) 8 (c) 6 (d) 4

(5) If θ is the measure of the angle included between $\overrightarrow{A} = (2, 0, 2)$, $\overrightarrow{B} = (0, 0, 4)$, then $\theta = \dots\dots$

(a) 30° (b) 45° (c) 60° (d) 90°

(6) If $L_1 : \frac{x-3}{2} = \frac{-y-1}{6} = \frac{z}{k}$ is parallel to $L_2 : \frac{x+2}{6} = \frac{y-4}{m} = \frac{z-1}{3}$, then $k + m = \dots\dots$

(a) -17 (b) -10 (c) 10 (d) 17

2 Complete :

(1) $\omega + \omega^2 + \cdots + \omega^{100} = \dots\dots$

(2) If a , b , c are the lengths of the sides of the triangle ABC , then the value of $\begin{vmatrix} a & b & c \\ 5 & 7 & 8 \\ \sin A & \sin B & \sin C \end{vmatrix} = \dots\dots$

(3) If $\overrightarrow{A} = (-1, 4, 2)$, $\overrightarrow{B} = (2, 2, 1)$, then the component of $\overrightarrow{A}$ in the direction of $\overrightarrow{B} =$ ....

(4) $x^2 + y^2 + z^2 - 4kx + 4y - 8z + 2k = 0$ is the equation of a sphere, the length of its radius equals $2\sqrt{5}$, then the value of $k =$ ....

(5) If the two planes $3x - y + 2z + 3 = 0$, and $kx - 4y + z - 5 = 0$ are perpendicular, then the value of $k =$ ....

(6) If $C(-1, 6, -5)$ is the midpoint of $\overline{AB}$ where $A(k-2, -1, m+3)$, $B(2, n-7, -2)$, then $k + m - n =$ ....

Second Answer the following questions

3 [a] Find the coefficient of x^5 in the expansion of : $(1 - x + x^2)(1 + x)^{11}$ « 297 »

[b] Prove that the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z}{3}$ intersects the plane $3x + 2y + z - 8 = 0$ at a point and find the measure of the inclination angle of the line with the plane. « 30° »

---

4 [a] Calculate the rank of the matrix $\begin{pmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 3 & -5 & 2 \end{pmatrix}$ hence prove that the equation

$2x - y - 3z = 2$, $x + 2y + z = 1$, $3x - 5y + 2z = 13$ have a unique solution and find this solution using the multiplicative inverse of the matrix. « 3 , (2 , −1 , 1) »

[b] Find the exponential form of the complex number $z = \frac{2+6i}{3-i}$, then find z^{-1} , $\bar{z}$, $\sqrt{z}$ in the trigonometric form.

---

5 [a] Prove that one of the values of the expression : $\sqrt{i} - \sqrt{-i} = \sqrt{2}\, i$

[b] If $(x-2)^2 + (y+4)^2 + (z-2)^2 = 1$, $(x+4)^2 + (y-4)^2 + (z-2)^2 = 4$ are the equations of two spheres, find the distance between the centres of the two spheres and show that the two spheres do not intersect. « 10 »

School Book Examinations

Model 3

First Answer one of the following questions

1 Choose the correct answer :

(1) The sum of the coefficients of the expansion of $(1 + x)^5$ equals ....

(a) zero (b) 5 (c) 32 (d) $\underline{|5}$

(2) If x is a complex number, then the number of different solutions of the equation

$\begin{vmatrix} x^3+1 & x-1 \\ x+1 & x^3-1 \end{vmatrix} = 0$ equals ....

(a) 6 (b) 5 (c) 4 (d) 3

(3) If (x, y, z) is the midpoint of $\overline{AB}$ where $A(-4, 0, 5)$, $B(-2, 4, -13)$, then $x + y + z =$ ....

(a) -5 (b) -6 (c) 3 (d) 4

(4) If $A(-4, -2, 3)$, $B(1, 2, k)$ and the length of $\overline{AB} = \sqrt{77}$ length unit, then one of the values of k is ....

(a) 2 (b) 4 (c) 6 (d) 9

(5) If $\vec{A} = (-1, 3, 4)$, $\vec{B} = (0, -2, 5)$, then $\|\overrightarrow{AB}\| =$ ....

(a) $2\sqrt{3}$ (b) $3\sqrt{3}$ (c) $4\sqrt{3}$ (d) $5\sqrt{3}$

(6) The length of the perpendicular drawn from point $A(3, 0, -5)$ on the plane $2x + \sqrt{5}\,y + 4z - 6 = 0$ equals ....

(a) 4 (b) 5 (c) 6 (d) 7

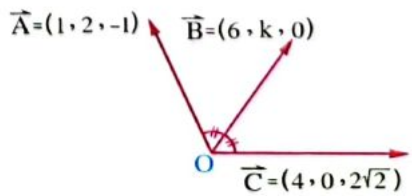
2 Complete each of the following :

(1) If $z = \sin 60° - i \cos 60°$, then the principle amplitude of z = ...

(2) The rank of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 5 & 5 & 5 \end{pmatrix}$ is ....

(3) **In the opposite figure :**

The value of k = ...



(4) The radius length of the sphere

$x^2 + y^2 + z^2 + 4x - 6y + 8z + 4 = 0$ equals ....

(5) If the straight line $\frac{x+3}{2} = \frac{y+1}{-6} = \frac{z-2}{k}$ is parallel to the straight line

$\frac{x+2}{4} = \frac{y-5}{m} = \frac{z-1}{3}$, then $k + m =$ ....

(6) If the straight line $\frac{x+2}{6} = \frac{y-1}{m} = \frac{z-1}{3}$ is perpendicular to the straight line

$\frac{x-9}{-2} = \frac{y+8}{1}$, $z = 3$, then $m =$ ....

Second Answer the following questions

3 [a] If $(m + x)^n = 3a + 6ax + 5ax^2 + \cdots$ where $n \in \mathbb{Z}^+$, find the value of each of m and a

« 3 , 243 »

[b] Prove that the following system of equations has a solution except the non zero solution and write the general form of these solutions :

$2x - y + 3z = 0$, $4x + 5y - z = 0$, $2x + 3y - z = 0$

---

4 [a] If $|z_1| = |z_2| = 1$, and the $\arg(z_1 z_2^3) = 81°$, $\arg\left(\frac{z_1}{z_2}\right) = 33°$, write in the form of $x + yi$ the number $(z_1^{15} z_2^{15})$

« $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$ »

[b] Find the length of the perpendicular drawn from point A (– 2 , 3 , 1) to the line

$\frac{x+2}{2} = \frac{y-3}{4} = \frac{z-1}{4}$

« zero »

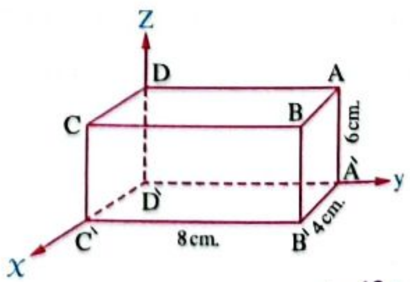
---

5 **[a] Prove that :** $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$

[b] In the opposite figure :

$ABCDA'B'C'D'$ is a cuboid.

Find : $\overrightarrow{BD'} \cdot \overrightarrow{CA'}$



« – 12 »

School Book Examinations

Model 4

First Answer one of the following questions

1 Choose the correct answer :

(1) If ${}^{10}C_{r+1} : {}^{10}C_{r-1} = 21 : 10$, then the value of r = ...

(a) 3 (b) 4 (c) 5 (d) 6

(2) If $\begin{vmatrix} \log_2 3 & 3 & 9 \\ 0 & \log_3 5 & 7 \\ 0 & 0 & \log_5 x \end{vmatrix} = 4$, then $x =$ ....

(a) 16 (b) 32 (c) 64 (d) 128

(3) If $\overrightarrow{A} = (1 , -1 , 2)$, $\overrightarrow{B} = (0 , 2 , -3)$, $\overrightarrow{C} = (-2 , 1 , 0)$

, then $\| 3\overrightarrow{A} - \overrightarrow{B} + \overrightarrow{C} \| =$ ....

(a) $8\sqrt{3}$ (b) 11 (c) 12 (d) $7\sqrt{2}$

(4) If $L_1 : \frac{x+2}{-1} = \frac{y+3}{3} = \frac{z+5}{2}$ is perpendicular to $L_2 : \frac{x}{2} = \frac{y-5}{k} = \frac{z-6}{m}$,

then 3 k + 2 m = ...

(a) – 1 (b) 0 (c) 2 (d) 4

(5) The measure of the smaller angle between the two straight lines :

$x - 1 = \frac{y+2}{\sqrt{2}} = -z + 1$, $-x = z + 3$, $y = 4$ equals ....

(a) 45° (b) 120° (c) 135° (d) 150°

(6) The direction cosines of the vector (2 , – 4 , 4) are ....

(a) (2 , – 4 , 4) (b) (1 , – 2 , 2) (c) $\left(\frac{1}{3} , \frac{-2}{3} , \frac{2}{3}\right)$ (d) $\left(\frac{1}{3} , \frac{2}{3} , \frac{2}{3}\right)$

---

2 Complete :

(1) $(3 + 7\omega + 3\omega^2)(3 - 7\omega^2 + 3\omega) =$ ....

(2) The rank of the matrix $A = \begin{pmatrix} 2 & -6 \\ -3 & 3 \\ 4 & -12 \end{pmatrix}$ equals ....

(3) The centre of the sphere $x^2 + y^2 + z^2 + 8x - 12y + 2z + 1 = 0$ equals ....

(4) ABCD is a square of side length 10 cm. , then $\overrightarrow{AB} \cdot \overrightarrow{AC} =$ ....

(5) The unit vector in the direction of $\overrightarrow{A} = \left(2, 3, 2\sqrt{3}\right)$ equals ....

(6) The length of the perpendicular drawn from point $(-2, -3, 1)$ to x-axis equals .... length unit.

Second Answer the following questions

3 **[a] Find the greatest term in the expansion of :** $(3 + 2x)^6$ according to the ascending powers of x at $x = 1$ « T_3 »

[b] Find the volume of a parallelepiped in which three adjacent sides are represented by the vectors : $\overrightarrow{A} = (1, -1, 2)$, $\overrightarrow{B} = (3, -2, 0)$, $\overrightarrow{C} = (0, 2, 4)$ « 16 »

---

4 **[a] Find the roots of the equation :** $z^4 + 4 = 0$ in the trigonometric form.

[b] If $\overrightarrow{A}, \overrightarrow{B}, \overrightarrow{C}$ are three mutually perpendicular unit vectors

(1) **Find :** $\| 2\overrightarrow{A} - \overrightarrow{B} + 3\overrightarrow{C} \|$ « $\sqrt{14}$ »

(2) If $\overrightarrow{A} = \left(\frac{16}{25}, \frac{-3}{5}, \frac{12}{25}\right)$, $\overrightarrow{B} = \left(\frac{3}{5}, 0, \frac{-4}{5}\right)$ **Find :** $\overrightarrow{C}$ « $\frac{12}{25}\hat{i} + \frac{4}{5}\hat{j} + \frac{9}{25}\hat{k}$ »

---

5 **[a]** Discuss the possibility for solving the following equations and write this solution , if exists : $x + y = 2$, $2x + 3y = 5$ « 1 , 1 »

[b] If $z = \sin\frac{\pi}{9} + i\cos\frac{\pi}{9}$, find $(\bar{z})$ in the trigonometric form and find the cubic roots of the number $(\bar{z})^9$

School Book Examinations

Model 5

First Answer one of the following questions

1 Choose the correct answer :

(1) If $36\ {}^{2n-1}P_{n-1} = 9\ {}^{2n}P_n$, then n = ...

(a) 1 (b) 2 (c) 3 (d) 4

(2) If the two equations $x + y = 2$, $2x + ky = 4$ have more than one solution ,
then k = ...

(a) – 2 (b) – 1 (c) 1 (d) 2

(3) If $\overrightarrow{AB} = -3\hat{i} + 3\hat{j} + 7\hat{k}$, $\overrightarrow{BC} = \hat{j} + 5\hat{k}$, then $\|\overrightarrow{AC}\|$ = ...

(a) 13 (b) 12 (c) 10 (d) 9

(4) If $\vec{A} = (-7 , 3 , 10)$, $\vec{B} = (-4 , -1 , -2)$, then the unit vector in the direction
of $\overrightarrow{AB}$ = ...

(a) $\left(\frac{3}{13} , \frac{4}{13} , \frac{12}{13}\right)$ (b) $\left(\frac{3}{13} , \frac{-4}{13} , \frac{-12}{13}\right)$

(c) $\left(\frac{-3}{13} , \frac{4}{13} , \frac{12}{13}\right)$ (d) $\left(\frac{-3}{13} , \frac{-4}{13} , \frac{-12}{13}\right)$

(5) If $\vec{A} = (1 , -1 , 2)$, $\vec{B} = (3 , -2 , 0)$, $\vec{C} = (0 , 2 , 4)$, then
$\vec{A} \cdot \vec{B} \times \vec{C}$ = ...

(a) 10 (b) 12 (c) 14 (d) 16

(6) The length of the perpendicular drawn from the point A (1 , 0 , 2) to the straight line
$\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-3}{-2}$ equals ....

(a) $\frac{\sqrt{26}}{4}$ (b) $\frac{\sqrt{26}}{5}$ (c) $\frac{\sqrt{26}}{3}$ (d) $\frac{\sqrt{26}}{6}$

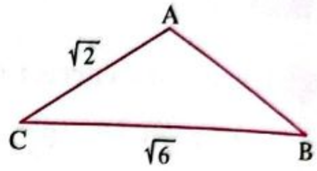
2 Complete :

(1) $\left(2 + \frac{3}{\omega}\right)\left(2 + \frac{3}{\omega^2}\right)\left(3 - \frac{2}{\omega}\right)\left(3 - \frac{2}{\omega^2}\right)$ = ...

(2) If the coefficients of T_6 , T_{16} in the expansion of $(a + b)^n$ according to the descending powers of a are equal, then n = ...

(3) Cosine the measure of the angle between the two lines : $\frac{x}{1}=\frac{y}{-2}=\frac{z+1}{-2}$ and $\frac{x}{1}=\frac{y-2}{-2}=\frac{z}{2}$ equals ....

(4) In the opposite figure :



If $\|\overrightarrow{BC}\|=\sqrt{6}$, $\|\overrightarrow{AC}\|=\sqrt{2}$

, $\overrightarrow{BA}=(-1,0,1)$

, then $\overrightarrow{BA}\cdot\overrightarrow{BC}=$ ....

(5) The general equation of the sphere whose centre is (3 , 4 , – 5) and touches y z plane is ....

(6) The vectors form of the equation of the straight line which passes through the point (2 , – 1 , 4) and its direction vector is $\vec{d}=(4,7,1)$ is ....

Second Answer the following questions

3 [a] In the expansion of $(1+x)^{18}$ according to ascending powers of x , if the coefficients of T_{2r+4} , T_{r-2} are equal, find the value of r « 6 »

[b] If the length of the perpendicular drawn from point A (0 , – 1 , 2) to the plane $\sqrt{2}\,x+y-z+k=0$ equals 2 unit length , find the value of k « 7 or – 1 »

4 **[a] Solve the following equations :**

$2x+y-2z=10$, $x+2y+2z=1$, $5x+4y+3z=6$

using the multiplicative inverse of the matrix. « $\frac{1}{3}$, $\frac{10}{3}$, – 3 »

[b] If $z_1=\frac{6+4i}{1+i}$, $z_2=\frac{26}{5-i}$ If $z=4(z_1-z_2)$, find the cubic roots of z in the exponential form.

5 **[a] Without expanding the determinant , prove that :**

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & cb \\ ac & cb & c^2+1 \end{vmatrix} = a^2+b^2+c^2+1$$

[b] If the plane $2x-y-2z+12=0$ cuts the sphere $(x+3)^2+(y+2)^2+(z-1)^2=15$, find the area of the cross section (trace). « 11 π square unit »

School Book Examinations

Model 6

First **Answer one of the following questions**

1 **Choose the correct answer :**

(1) If ${}^{n}C_3 : {}^{n-1}C_4 = 8 : 5$, then the value of n = ...

(a) 5 (b) 7 (c) 8 (d) 9

(2) The coefficient of the middle term in the expansion of $\left(3x - \frac{1}{6}\right)^{10}$ equals ....

(a) $\frac{-63}{8}$ (b) $\frac{-67}{8}$ (c) $\frac{63}{8}$ (d) $\frac{67}{8}$

(3) The measure of the angle included between the two planes :

$x + y - 1 = 0$, $y + z - 1 = 0$ equals ....

(a) 30° (b) 45° (c) 60° (d) 75°

(4) If $\overrightarrow{A} = (2 , 1 , -2)$, $\overrightarrow{A} + \overrightarrow{B} = \overrightarrow{A} \times \overrightarrow{B}$, then $\overrightarrow{B}$ = ...

(a) (2 , − 1 , − 2) (b) (2 , 1 , − 2) (c) (− 2 , − 1 , 2) (d) (− 2 , − 1 , 3)

(5) If A = (− 2 , 0 , 3) , B = (4 , 2 , − 5) , then $\|\overrightarrow{AB}\|$ = ... length unit.

(a) $\sqrt{12}$ (b) $\sqrt{40}$ (c) $\sqrt{44}$ (d) $\sqrt{104}$

(6) If $\overrightarrow{A} \perp \overrightarrow{B}$, $\overrightarrow{A} \perp \overrightarrow{C}$, $\overrightarrow{B} = (2 , 3 , 2)$, $\overrightarrow{C} = (1 , 2 , 1)$ and $\|\overrightarrow{A}\| = 4\sqrt{2}$,

then $\overrightarrow{A}$ = ...

(a) (2 , 3 , 1) (b) (− 4 , 0 , 4) (c) (4 , 4 , 0) (d) (0 , − 4 , 4)

---

2 **Complete :**

(1) $\left(1 - \frac{1}{\omega}\right)\left(1 - \frac{1}{\omega^2}\right)\left(1 - \frac{1}{\omega^4}\right)\left(1 - \frac{1}{\omega^8}\right)$ ··· to 10 factors = ...

(2) The rank of the matrix $\begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ equals ....

(3) The direction vector of the straight line $\frac{x+2}{3} = \frac{z-1}{2}$, y = p equals ....

(4) If the measure of the angle between the two lines $\frac{x}{a} = \frac{y}{2} = \frac{z}{1}$, $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$

equals 60° , then the value of a = ...

(5) If A (1 , 0 , 0) and B (0 , 1 , 1) lie on the plane $k x + y + m z + 2 = 0$,

then k + m = ...

(6) If $\overrightarrow{A} = (1 , 0 , 2)$, $\overrightarrow{B} = (2 , -1 , -2)$, then $\left(\overrightarrow{A} \times \overrightarrow{B}\right) \cdot \left(\overrightarrow{B} \times \overrightarrow{A}\right) =$ ....

Second Answer the following questions

3 **[a]** If the coefficients of the fourth , fifth and sixth terms in the expansion $(2x + y)^n$ according to the descending powers of x form an arithmetic sequence , find the value of n « 19 or 8 »

[b] A sphere of centre (1 , 2 , 1) touches the plane $x + y + z = 1$, find the equation of the sphere. « $(x-1)^2 + (y-2)^2 + (z-1)^2 = 3$ »

---

4 **[a]** Discuss the possibility of solving the set of the following system equations :

$4x + 3y - 5z = 6$, $3x + 2y + 4z = 12$, $5x - 2y - 7z = 1$, then find the solution set of these equations using the multiplicative inverse of a matrix. « $\{(2 , 1 , 1)\}$ »

[b] If $z_1 = \left(\frac{\sqrt{3} + i}{2}\right)^4$, $z_2 = \sin\frac{\pi}{3} + i\cos\frac{\pi}{3}$, $i^2 = -1$ and $z = \frac{z_1}{z_2}$,

find the square roots of z in the trigonometric form.

---

5 **[a] Without expanding the determinant , prove that :**

$$\begin{vmatrix} x & a & b \\ a & x & b \\ b & a & x \end{vmatrix} = (x + a + b)(x - a)(x - b)$$

[b] Find the different forms of the equation of the straight line passing through (2 , 1 , −3) and parallel to the straight line $\frac{x-1}{5} = \frac{y+3}{2} = \frac{1-z}{3}$

School Book Examinations

Model 7

First Answer one of the following questions

1 Choose the correct answer :

(1) If ${}^{30}C_r = {}^{30}C_{r+10}$, ${}^{n}P_7 = 90 \times {}^{n-2}P_5$, then $\lfloor n - r$ = ...

(a) zero (b) 1 (c) 10 (d) $\lfloor 20$

(2) If the equations : $3x - 2y + z = 0$, $6x - 5y + 2z = 0$, $9x - 6y + kz = 0$ have solutions other than the zero solution , then k = ...

(a) zero (b) 1 (c) 3 (d) 4

(3) The length of the perpendicular drawn between the two planes :
$3x + 12y - 4z = 9$, $3x + 12y - 4z = -17$ equals ....

(a) 2 (b) 3 (c) 4 (d) 5

(4) If $\overrightarrow{A} = (4, -k, 6)$, $\overrightarrow{B} = (2, 2, m)$ and $\overrightarrow{A} // \overrightarrow{B}$, then k + m = ...

(a) – 3 (b) – 2 (c) – 1 (d) zero

(5) If the straight line $x = 3y = az$ is parallel to the plane $x + 3y + 2z + 4 = 0$, then a = ...

(a) 3 (b) 2 (c) 1 (d) – 1

(6) If $\overrightarrow{A} = (1, -2, 1)$, $\overrightarrow{B} = (-2, 1, 2)$, then the vector component of $\overrightarrow{A}$ in the direction of $\overrightarrow{B}$ = ...

(a) $\left(\frac{4}{9}, \frac{-2}{9}, \frac{-4}{9}\right)$ (b) $\left(\frac{4}{9}, \frac{2}{9}, \frac{4}{9}\right)$

(c) $\left(\frac{-4}{9}, \frac{-2}{9}, \frac{-2}{9}\right)$ (d) $\left(\frac{-4}{9}, \frac{2}{9}, \frac{-4}{9}\right)$

---

2 Complete :

(1) $\left(\frac{3 + 5\omega}{5 + 3\omega^2} + \frac{5 + 3\omega^2}{3 + 5\omega}\right)^8$ = ...

(2) The rank of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -1 \\ 1 & -3 & 4 \end{pmatrix}$ equals ....

(3) If the plane $P_1 : x - z + 1 = 0$ and the plane $P_2 : 2x - 2y - z = 0$, then the measure of the smallest angle between the two planes = ...

(4) The radius length of the sphere $(x-2)^2 + (y+4)^2 + (z-5)^2 = 64$ equals ....

(5) If $\overrightarrow{A} = (4, -5, 1)$, $\overrightarrow{B} = (2, -k, -2)$, $\overrightarrow{C} = (-4, 4, m-2)$ and $\overrightarrow{AB} \parallel \overrightarrow{C}$, then $k + m =$ ....

(6) If $\|\overrightarrow{A}\| = 2$, $\|\overrightarrow{B}\| = 3$, $\|\overrightarrow{C}\| = 12$ and $\overrightarrow{A}, \overrightarrow{B}, \overrightarrow{C}$ are mutually orthogonal, then $\|\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C}\| =$ ....

Second Answer the following questions

3 [a] If $z_1 = \left(\sin\frac{\pi}{9} + i\cos\frac{\pi}{9}\right)^5$, $z_2 = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^4$ and $z = \frac{z_1}{z_2}$,

find the square roots of z in its exponential form.

[b] If $\overrightarrow{A} = (2\cos\theta, \log_3 x, \sin\theta)$, $\overrightarrow{B} = (\cos\theta, \log_5 27, 2\sin\theta)$ and $\overrightarrow{A} \cdot \overrightarrow{B} = 11$,

find the value of x « 125 »

---

4 [a] In the expansion of $(1 + x)^n$ according to the ascending power of x if

$T_3 = 17$, $3T_2 \times T_4 = 544$, find the value for each of n and x « $18, \pm\frac{1}{3}$ »

[b] Without expanding the determinant, prove that :

$$\begin{vmatrix} a+b+2 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix} = 2(a+b+1)^3$$

---

5 [a] If $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ and $A^t = A^{-1}$, find the value of each of : x, y, z

« $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}, \pm\frac{1}{\sqrt{3}}$ »

[b] Find the point of intersection of the straight line :

$x = y = z$ and the plane $x + 2y + 3z = 12$ « (2, 2, 2) »

School Book Examinations

Model 8

First Answer one of the following questions

1 **Complete :**

(1) If $\lfloor 1 + \log x = 1$, then $x =$ .... or ....

(2) If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a & b & c \end{vmatrix} = 5$, then the value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a+5 & b+5 & c+5 \end{vmatrix} =$ ....

(3) The measure of the angle between the two lines $\overrightarrow{r_1} = (-2 , 5 , -7) + k(-6 , 6 , 8)$, $\overrightarrow{r_2} = (1 , -2 , 3) + k(4 , 12 , -6)$ equals ....

(4) If $\|\overrightarrow{A}\| = 4$, $\|\overrightarrow{B}\| = 6$ and the measure of the angle between the two vectors $\overrightarrow{A}$, $\overrightarrow{B}$ equals $60°$, then $(2\overrightarrow{A} + \overrightarrow{B}) \cdot (\overrightarrow{A} - \overrightarrow{B}) =$ ....

(5) The equation of the sphere whose diameter is $\overline{AB}$ where $A(7 , 1 , -4)$, $B(3 , -1 , 2)$ is ....

(6) If $\overrightarrow{A} = (1 , 2 , -4)$, $\overrightarrow{B} = (1 , 1 , k-1)$ and $\|\overrightarrow{A} + \overrightarrow{B}\| = 7$ unit of length , then $k =$ ....

---

2 **Choose the correct answer :**

(1) If $\frac{a^2 + b^2}{a + b\,i} = 2 + 3\,i$, then $a \times b =$ .... (where $a , b \in \mathbb{R}$)

(a) -6 (b) -5 (c) 5 (d) 6

(2) The rank of the matrix $A = \begin{pmatrix} 0 & -2 & 3 \\ -2 & 4 & -6 \\ 3 & -6 & 9 \end{pmatrix}$ equals ....

(a) 3 (b) 2 (c) 1 (d) zero

(3) ABCD is a parallelogram in which $\overrightarrow{AB} = (2 , 2 , -1)$, $\overrightarrow{AD} = (-1 , 2 , -3)$, then the surface area of the parallelogram = ... square unit.

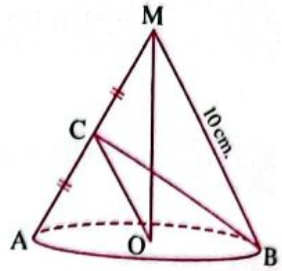
(a) 6 (b) $7\sqrt{2}$ (c) $3\sqrt{11}$ (d) $\sqrt{101}$

(4) **In the opposite figure :**

A right circular cone , the perimeter of its base = 12π cm.

, C is the midpoint of $\overline{AM}$, then

$\overrightarrow{BC} \cdot \overrightarrow{CO} =$ ....



(a) – 43 (b) – 40

(c) – 37 (d) – 33

(5) If $\overrightarrow{A} = \hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{B} = 2\hat{i} - \hat{j} - \hat{k}$, then $\overrightarrow{A} \times \left(\overrightarrow{A} - \overrightarrow{B}\right) =$ ....

(a) $\hat{i} + \hat{k}$ (b) $-3\hat{j} + 3\hat{k}$ (c) $-3\hat{i} - 3\hat{j}$ (d) $3\hat{i} - 2\hat{j}$

(6) If $L_1 : x = 0$, $y = z$, $L_2 : y = 0$, $x = z$ are two straight lines in space , the measure of the angle between them is θ , then $\theta =$ ....

(a) 45° (b) 60° (c) 70° (d) 90°

Second Answer the following questions

3 **[a] Use the multiplicative inverse of a matrix to solve the following equations :**

$2x - y + z = -1$, $x - z = 2$, $x + y = 3$ « (1 , 2 , – 1) »

[b] Find the point of intersection of the planes :

$2x + y - z = -1$, $x + y + z = 2$, $3x - y - z = 6$ « $\left(2 , \frac{-5}{2} , \frac{5}{2}\right)$ »

4 [a] If $z_1 = 1 - \sqrt{3}\,i$, $z_2 = \cos\theta + i\sin\theta$, $z_3 = \left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)^2$ and $z = \frac{z_1 z_2}{z_3}$, find the modulus and the principle amplitude of z , then find the square roots of z in its trigonometric form when $\theta = \frac{\pi}{6}$

[b] Discuss the possibility of existence of a solution except the zero solution for the system of linear equations :

$x + 3y - 2z = 0$, $x - 8y + 8z = 0$, $3x - 2y + 4z = 0$

5 [a] In the expansion of $\left(x^2 + \frac{1}{2x}\right)^{3n}$ according to the descending powers of x :

(1) Prove that the term free of x is of order $(2n + 1)$

(2) Find the ratio between the term free of x and the middle when $n = 4$, $x = 1$

« 15 : 112 »

[b] If the two spheres $(x-3)^2 + y^2 + (z-3)^2 = 16$, $(x+1)^2 + (y-4)^2 + (z-k)^2 = 25$ are tangential. Find the value of k « 10 or – 4 »

School Book Examinations

Model 9

First **Answer one of the following questions**

1 Complete the following :

(1) If ${}^{x+y}P_4 = 360$, $\lfloor 2x+y = 5040$, then ${}^{y}C_{2x} =$ ....

(2) The solution set of the equation $\begin{vmatrix} a+1 & 3 & 2 \\ 0 & a-1 & 5 \\ 0 & 0 & 7 \end{vmatrix} = 21$ is ....

(3) Cosine the angle between the two vectors $\overrightarrow{A} = (1 , -3 , 0)$, $\overrightarrow{B} = (2 , 0 , 1)$ equals ....

(4) The radius length of the sphere : $x^2 + y^2 + z^2 + 2x - 2y - 4z - 3 = 0$ equals ....

(5) If $\overrightarrow{A} = \left(\frac{-1}{2} , \frac{3}{4} , k\right)$ is a unit vector , then the value of $k =$ .... or ....

(6) If $\overrightarrow{A} = (k , -3 , 1)$, $\overrightarrow{B} = (2 , 3 , -k)$ are perpendicular , then the value of $k =$ ....

---

2 Complete :

(1) $(1+\omega)^4 + (1+\omega^2)^4 + (\omega+\omega^2)^4 =$ ....

(2) The rank of the matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ equals ....

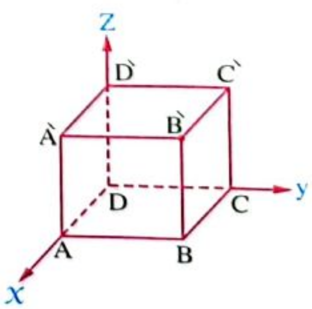
(3) If $\overrightarrow{A} = (3 , -2 , k)$, $\overrightarrow{B} = (1 , m , 2)$ and $\overrightarrow{A} \parallel \overrightarrow{B}$, then $k =$ .... , $m =$ ....

(4) If the measure of the angle which $\overrightarrow{C} = (2 , 4 , k)$ makes with the positive direction of y-axis equals 45° , then $k =$ ....

(5) If the two planes : $x + 2y + kz = 2$, $3x - y + 2z = 0$ are perpendicular , then $k =$ ....

(6) **In the opposite figure :**

ABCDA`B`C`D` is a cube of edge length unity , then $\overrightarrow{AB`} \cdot \overrightarrow{BD} =$ ....



Second Answer the following questions

3 **[a]** If $z_1 = 2\left(\sin\frac{\pi}{3} + i\cos\frac{\pi}{3}\right)$, $z_2 = \sqrt{2}\left(\sin\frac{\pi}{4} - i\cos\frac{\pi}{4}\right)$, $z_3 = 1 + \sqrt{3}\,i$

Find the number $z = \frac{z_1^3 \times z_2^4}{z_3^5}$ in its exponential form ,

then find the square roots of z in its trigonometric form.

[b] If the plane $2a\mathcal{X} - 3ay + 4az + 6 = 0$ passes through the midpoint of the line segment joining the centres of the two spheres $\mathcal{X}^2 + y^2 + z^2 + 6\mathcal{X} - 8y - 2z = 13$, $\mathcal{X}^2 + y^2 + z^2 - 10\mathcal{X} + 4y - 2z = 8$ Find the value of a «-2»

---

4 **[a] Use the multiplicative inverse of a matrix to solve the following equations :**

$\mathcal{X} - 2y + 2z = 2$, $3\mathcal{X} + 4z = 10$, $6z - y = 5$ «(2 , 1 , 1)»

[b] Prove that the term free of $\mathcal{X}$ in the expansion of $\left(\mathcal{X}^2 + \frac{1}{\mathcal{X}^3}\right)^{5n}$ where $n \in \mathbb{Z}^+$

equals $\frac{\lfloor 5n}{\lfloor 2n \lfloor 3n}$

---

5 **[a] Find the value of k which makes the equations :**

$k\mathcal{X} + y + z = 1$, $\mathcal{X} + ky + z = 1$, $\mathcal{X} + y + kz = 1$

have an infinite number of solutions. «1»

[b] Find the length of the perpendicular drawn from point

$(-4 , 1 , 1)$ on the line $\frac{\mathcal{X}+3}{1} = \frac{y-1}{\sqrt{5}} = \frac{z+2}{2}$ «$\frac{\sqrt{30}}{2}$ length unit»

School Book Examinations

Model 10

First Answer one of the following questions

1 Complete :

(1) If $x = \frac{-1-\sqrt{3}\,i}{2}$, $i^2 = -1$, then the numerical value of $x^8 + x^4 + 5 =$ ....

(2) If $\underline{|n}$, $\underline{|n-2}$, n $\underline{|2-n}$ are the side lengths of a triangle , then the numerical value of the perimeter of the triangle = ...

(3) If $\overrightarrow{A} = (-2 , k , -3)$ is parallel to the straight line $\frac{x+2}{4} = \frac{y}{8} = \frac{z-1}{6}$,

then k = ...

(4) The measure of the angle which the vector $\overrightarrow{A} = \left(3 , 4 , \sqrt{11}\right)$ makes with the positive direction of x-axis equals ....

(5) If the two planes $x - 3y + mz = 5$ and $3x + ky + 6z = 10$ are parallel

, then $k \times m =$ ....

(6) The distance between the two parallel planes $4x + 6y + 12z + 18 = 0$

and $4x + 6y + 12z - 10 = 0$ is ....

2 Choose the correct answer :

(1) $1 - 6x + \frac{6 \times 5}{2 \times 1} x^2 - \frac{6 \times 5 \times 4}{3 \times 2 \times 1} x^3 + \cdots + x^6 = 64$, then $x =$ ....

(a) -1 (b) 3 (c) $\{-1 , 3\}$ (d) 2

(2) $\left(\frac{5-3\omega^2}{5\omega-3} - \frac{2-7\omega}{2\omega^2-7}\right)^2 =$ ....

(a) 3 (b) -3 (c) $3i$ (d) $-3i$

(3) If the two straight lines : $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$, $\frac{x}{3} = \frac{y+1}{4} = \frac{z-1}{k}$

are perpendicular , then k = ...

(a) 4 (b) -4 (c) $\frac{9}{2}$ (d) $-\frac{9}{2}$

(4) The equation of the sphere whose centre is $(3 , -2 , 1)$ and its radius length equals 5 length unit is ....

(a) $(x+3)^2 + (y-2)^2 + (z+1)^2 = 5$ (b) $(x+3)^2 + (y-2)^2 + (z+1)^2 = 25$

(c) $(x-3)^2 + (y+2)^2 + (z-1)^2 = 25$ (d) $(x-3)^2 + (y+2)^2 + (z-1)^2 = \sqrt{5}$

(5) The measure of the angle included between the two planes :

$x+\sqrt{2}\,y+z=5$, $x-\sqrt{2}\,y+z=1$ equals ....

(a) $0°$ (b) $45°$ (c) $90°$ (d) $135°$

(6) In the opposite figure :

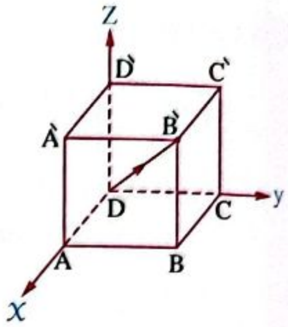
$ABCDA'B'C'D'$ is a cuboid , $A(4, 0, 0)$, $C(0, 9, 0)$

, $D'(0, 0, 7)$

, then $\|\overrightarrow{AC'}\|=$ ....

(a) $\sqrt{146}$ (b) $\sqrt{114}$

(c) 5 (d) $\sqrt{20}$



Second Answer the following questions

3 **[a]** In the expansion of $(2x-3)^{15}$ according to the descending powers of x , find the values of x which makes $13\,T_3+10\,T_4+T_5=0$ «$\frac{9}{2}$ or $\frac{1}{2}$»

[b] Without expanding the determinant , prove that :

$$\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 2\begin{vmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{vmatrix}$$

---

4 **[a] Prove that :** $\left(\frac{1+\sin\varnothing+i\cos\varnothing}{1+\sin\varnothing-i\cos\varnothing}\right)^n=\cos n\left(\frac{\pi}{2}-\varnothing\right)+i\sin n\left(\frac{\pi}{2}-\varnothing\right)$

[b] Find the equation of the straight line passing through the point $(3, -1, 0)$ and intersects the straight line $\vec{r}=(2, 1, 1)+t(1, 2, -1)$ orthogonally.

« $\vec{r}=(3, -1, 0)+t(-1, 1, 1)$ »

---

5 **[a] Use the multiplicative inverse of the matrix to solve the set of the following equations :**

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$, $\frac{1}{x}-\frac{1}{y}+\frac{2}{z}=\frac{1}{2}$, $\frac{2}{x}+\frac{3}{y}-\frac{4}{z}=\frac{4}{3}$

where x , y and z are not equal to zero. « 2 , 3 , 6 »

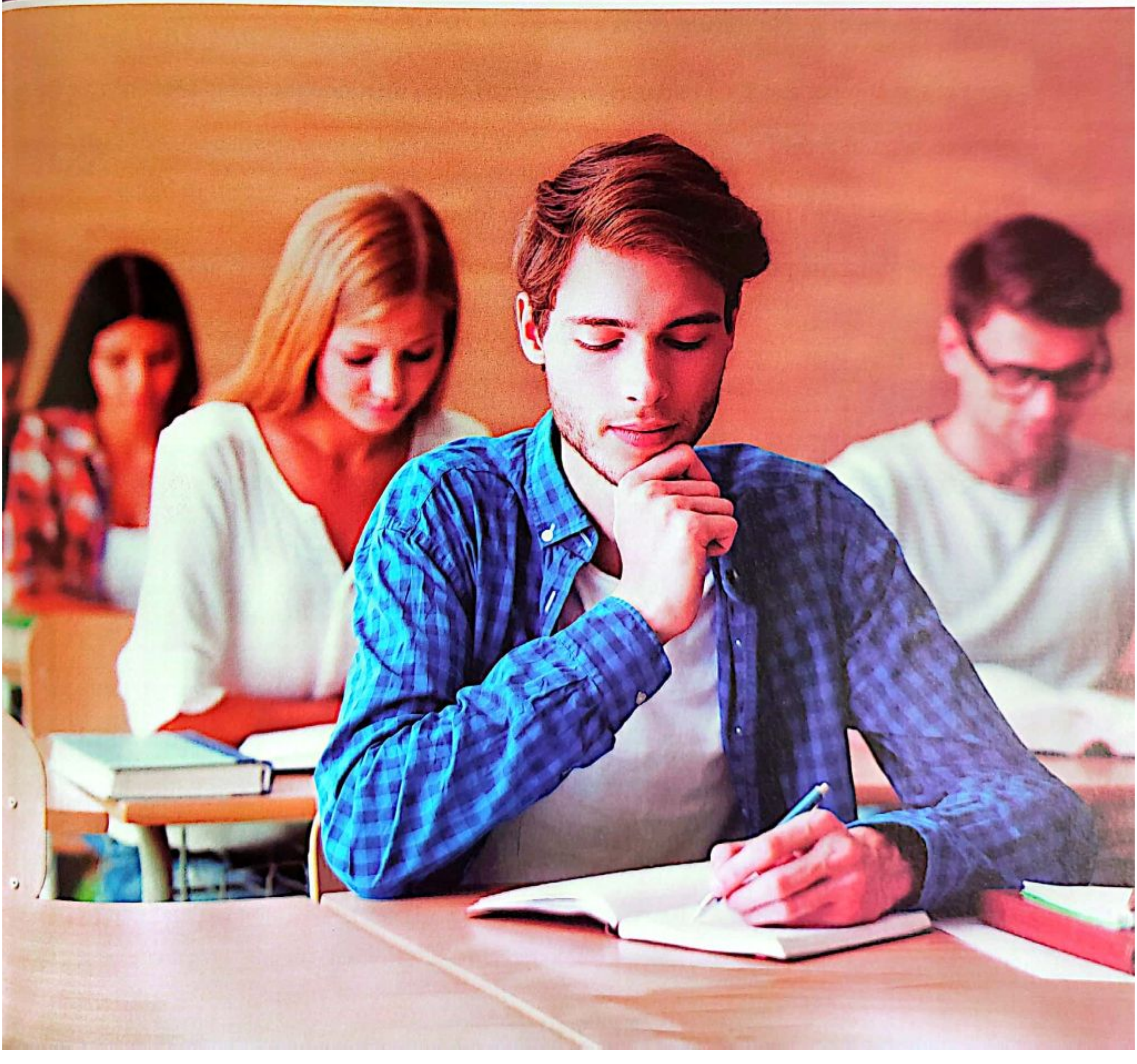
[b] Find the vector component of $\overrightarrow{AB}$ in the direction of $\overrightarrow{M}$ where :

$A(2, 1, 0)$, $B(3, 1, \sqrt{3})$, $\overrightarrow{M}=(3, 2, 2\sqrt{3})$ «$\left(\frac{27}{25}, \frac{18}{25}, \frac{18\sqrt{3}}{25}\right)$»

Egypt Exams
(2017 : 2021 first and second sessions)

in

Algebra & Analytic Solid Geometry



Answer the following questions :

1. If $z = \frac{2-i}{2+i}$, then $|z| =$ ……………

(a) 3 (b) 4 (c) 1 (d) 5

2. The principle amplitude of the complex number $2\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$ is ……………

(a) $\frac{\pi}{4}$ (b) $\frac{-\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $\frac{-3\pi}{4}$

3. *Answer one of the following items :*

[a] Put the number $z = 1 + i$ in the trigonometric form , then find the cubic roots of z in the exponential form.

[b] If $z = 1 - \sqrt{3}\,i$, find $z^{\frac{3}{2}}$ in the trigonometric form.

4. **Without expanding the determinant , prove that :** $\begin{vmatrix} 1 & 1 & 1 \\ x & y & x \\ x & x & -y \end{vmatrix} = x^2 - y^2$

5. The equation of the sphere with centre (2 , 0 , 0) and touches y z-plane is ……………

(a) $x^2 + y^2 + z^2 = 4$ (b) $(x-2)^2 + y^2 + z^2 = 0$

(c) $(x-2)^2 + y^2 + z^2 = 4$ (d) $x^2 + y^2 + z^2 + 4 = 0$

6. **Solve the following system of linear equations using the inverse matrix :**

$2x - 3y - z = 9$, $x + 2y + 3z = 15$, $x - 2z = 12$

7. The midpoint of the line segment $\overline{DE}$ where D (2 , 3 , 3) , E (6 , –1 , –5) is ……………

(a) (4 , 2 , 3) (b) $\left(2 , 1 , \frac{1}{2}\right)$ (c) (4 , 1 , –1) (d) (4 , 1 , 1)

8 The measure of the angle between the two straight lines :

$L_1 : x = 2 - 5k \quad , \quad y = 1 - k \quad , \quad z = 3 + 4k$

$L_2 : \frac{x+1}{3} = \frac{2-y}{4} = \frac{z}{2}$ equals ....

(a) 60° (b) 40° (c) 85° 4 (d) 35.4°

9 The plane $3x + 2y - 4z = 12$ cuts a part from the y-axis of length ....

(a) 3 (b) 2 (c) 4 (d) 6

10 ***Answer one of the following items :***

[a] ABCD is a rectangle in which AB = 6 cm. , BC = 8 cm. , **find :**

(1) $\overrightarrow{AB} \cdot \overrightarrow{AC}$

(2) The component of $\overrightarrow{CD}$ in the direction of $\overrightarrow{BC}$

[b] Find the cartesian form for the vector $\overrightarrow{A}$ whose norm equals $21\sqrt{3}$ and makes angles equal in measures with the positive direction of the coordinates axes.

11 The equation of the plane passes through the point (1 , 2 , 3) and parallel to the coordinate axes x and y is ....

(a) $x + y = 3$ (b) $z = 3$ (c) $x = 1$ (d) $y = 2$

12 The direction cosines of the straight line whose direction ratios (– 1 , 2 , 3) are ....

(a) $\left(\frac{-1}{4}, \frac{1}{2}, \frac{3}{4}\right)$ (b) $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

(c) $\left(\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$ (d) $\left(\frac{-1}{14}, \frac{1}{7}, \frac{3}{14}\right)$

13 Find the equation of the plane that contains the straight line :

$L_1 : \vec{r} = (0, 3, -5) + t_1 (6, -2, -1)$

and parallel to the straight line : $L_2 : \vec{r} = (1, 7, -4) + t_2 (1, -3, 3)$

14 The plane $3x + 2y + 4z = 12$ cuts the coordinate axes at the points A , B and C , calculate the area of ΔABC

15. The number of ways to select 2 different letters together or 3 different letters together from the elements of the set $\{a , b , c , d , e , f\}$ is ....

(a) ${}^6C_2 \times {}^6C_3$ (b) ${}^6P_2 \times {}^6P_3$ (c) ${}^6C_2 + {}^6C_3$ (d) ${}^6P_2 + {}^6P_3$

16. ${}^{n+2}C_4 = n^2 - 1$, then $n =$ ....

(a) 2 (b) 4 (c) 6 (d) 10

17. The term free of x in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ according to the descending powers of x is ....

(a) T_5 (b) T_7 (c) T_6 (d) T_4

18. In the expansion of $(1 + x)^n$ according to the ascending powers of x , if $T_3 = 17$, $3\, T_2 \times T_4 = 544$, then find the value of each of n and x

19. If $1 , \omega , \omega^2$ are the cubic roots of one , then : $\omega + \omega^2 + \omega^3 + \cdots + \omega^{100}$ equals ....

(a) 1 (b) ω (c) ω^2 (d) zero

Answer the following questions :

1. The measure of the angle between the two straight lines whose direction cosines are $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ equals ....

(a) 60° (b) 30° (c) 90° (d) 120°

2. Find the equation of the plane parallel to the plane $2x + y - 4z = 0$ and lies at a distance $\sqrt{21}$ length unit from the point $(1, 2, 0)$

3. **Solve the following matrix equation :** $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix}$

4. If $z = 2 + 2\sqrt{3}\, i$, then the exponential form of z is ....

(a) $4\, e^{\frac{-\pi}{3} i}$ (b) $4\, e^{\frac{\pi}{3} i}$ (c) $4\, e^{\frac{-\pi}{6} i}$ (d) $4\, e^{\frac{\pi}{6} i}$

5. If $x^2 + y^2 + z^2 + 4x - 6y + 8z + 4 = 0$ is the equation of a sphere, then the length of its diameter equals .... length unit.

(a) 5 (b) 10 (c) 15 (d) 20

6. If direction angle of a vector are $(45°, 45°, \theta°)$, then one of the values of θ equals ....

(a) 45° (b) 90° (c) 135° (d) 60°

7. ***Answer one of the following items :***

[a] Find the solution set of the equation : $z^3 = -8\, i$ in the trigonometric form.

[b] If $z = \frac{1}{\sqrt{2}}(1 + i)$, find the square roots of z in the trigonometric form.

8. If ${}^nC_6 : {}^nC_5 = 1 : 3$, then $\underline{|n-3}$ = ...

(a) 24 (b) 11 (c) 120 (d) 6

9. The middle term in the expansion of $\left(2x+\frac{1}{2x^2}\right)^{12}$ equals ....

(a) ${}^{12}C_6\, x^{-6}$ (b) ${}^{12}C_6\, x^{6}$ (c) ${}^{12}C_7\, x^{5}$ (d) ${}^{12}C_6$

10. The midpoint of the line segment whose terminals are $(-3, 2, 4)$, $(-5, 2, 8)$ is ....

(a) $(-2, 2, 4)$ (b) $\left(\frac{-5}{2}, 5, \frac{5}{2}\right)$ (c) $\left(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ (d) $(-4, 2, 6)$

11. Prove that the expansion of $\left(x^2+\frac{2}{x^2}\right)^{11}$ does not include a term free of x

12. Find the area of the parallelogram in which $\overrightarrow{A}$ and $\overrightarrow{B}$ are two adjacent sides such that :

$\overrightarrow{A} = (3, 6, 3)$, $\overrightarrow{B} = (-6, -2, -4)$

13. From the numbers 1, 2, 3, 4 and 5

How many even numbers greater than 300 and smaller than 100000 can be formed with replacement ?

(a) 30 (b) 250 (c) 111 (d) 1530

14. If $z = \sqrt{2}\,(\sin 30° + i \cos 30°)$, then the principle argument (amplitude) of the number z equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

15. The direction cosines of the vector $\overrightarrow{A} = (-2, 1, 2)$ are ....

(a) $(-2, 1, 2)$ (b) $\left(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ (c) $\left(\frac{-5}{2}, 5, \frac{5}{3}\right)$ (d) $(-1, 1, 1)$

16. **Without expanding the determinant, prove that :** $\begin{vmatrix} 3x & 3x & 3x \\ 1 & b & a \\ a+b & a+1 & b+1 \end{vmatrix} = \text{zero}$

17. If $1, \omega, \omega^2$ are the cubic roots of one, then : $1 + \omega + \omega^2 + \omega^3 + \cdots + \omega^{100} =$ ....

(a) zero (b) 1 (c) ω (d) $-\omega^2$

18 ***Answer one of the following items :***

[a] If the two straight lines :

$L_1 : \vec{r} = (2, 3, -4) + k(2, 3, a)$

$L_2 : \frac{x-5}{b} = \frac{y+4}{6} = \frac{z-4}{2}$ are parallel , find the value of each of a and b

[b] **Prove that the two straight lines :**

$L_1 : \vec{r} = (1, 2, 4) + k_1 (4, -2, 2)$

$L_2 : x = 1 - 6k_2 \quad , \quad y = 1 + 21 k_2 \quad , \quad z = 1 + 33 k_2$ are perpendicular.

---

19 If $\vec{A} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{B} = 4\vec{i} - \vec{j}$, then $\vec{A} \cdot \vec{B} =$ ....

(a) 5 (b) 4 (c) 3 (d) 8

Answer the following questions :

1. A four person committee is to be formed out of 9 men and 3 women. The number of committees containing only one woman equals ....

(a) ${}^3C_1 + {}^9C_3$ (b) ${}^3C_1 \times {}^9C_3$ (c) ${}^3P_1 \times {}^9P_3$ (d) ${}^3P_1 + {}^9P_3$

2. $e^{\theta i} + e^{-\theta i} =$ ....

(a) $e^{2\theta i}$ (b) $2 \cos \theta$ (c) $2 \sin \theta$ (d) $e^{-2\theta i}$

3. The equation of the straight line passes through the two points A (2 , 1 , – 3) and B (1 , 2 , – 5) is ....

(a) $\vec{r} = (-1 , 2 , -2) + k (2 , 1 , -3)$ (b) $\vec{r} = (1 , 2 , -5) + k (2 , 1 , -3)$

(c) $\vec{r} = (3 , 2 , 4) + k (-1 , 1 , 2)$ (d) $\vec{r} = (2 , 1 , -3) + k (-1 , 1 , -2)$

4. *Answer one of the following items :*

[a] If $z = \frac{8(\sqrt{3} + i)}{\sqrt{3} - i}$, then find its cubic roots in the exponential form.

[b] If $(x + y i)(1 - 3 i) = 37 \left[\frac{1}{3 - 4\omega^2} + \frac{1}{7 + 4\omega^2}\right]$, find the value of each of the real numbers x and y

5. If ${}^nC_9 : {}^nC_7 = 7 : 9$, then n = ...

(a) 7 (b) 15 (c) 16 (d) 9

6. The equation of the sphere with centre (2 , – 3 , 4) and touches x y-plane is ....

(a) $(x - 2)^2 + (y + 3)^2 + (z - 4)^2 = 4$ (b) $(x - 2)^2 + (y + 3)^2 + (z - 4)^2 = 9$

(c) $(x - 2)^2 + (y + 3)^2 + (z - 4)^2 = 16$ (d) $(x + 2)^2 + (y - 3)^2 + (z + 4)^2 = 16$

7. The equation of the plane passes through the point (3 , 4 , 5) and parallel to the coordinate axes x , y is ....

(a) $x + y = 7$ (b) $z = 5$ (c) $x = 3$ (d) $y = 4$

8 ***Answer one of the following items :***

[a] ABC is a triangle in which A (2 , 3 , 1) , B (3 , 5 , 4) , $\overrightarrow{BC} = (-1 , 4 , 0)$

Find : (1) m (∠ ABC)

(2) The direction component of $\overrightarrow{AC}$ in the direction of $\overrightarrow{AB}$

[b] If $\vec{A}$, $\vec{B}$, $\vec{C}$ are three adjacent edges in a parallelepiped such that :

$\vec{A} = (1 , 4 , 2)$, $\vec{B} = (-3 , 2 , 1)$, $\vec{C} = (-1 , 1 , 4)$

Find : (1) The volume of the parallelepiped.

(2) The height of the parallelepiped drawn on the base determined by the two vectors $\vec{A}$, $\vec{B}$

---

9 In the expansion of $\left(x^2 + \frac{1}{x}\right)^n$, if the coefficient of the fourth term equals the coefficient of T_{13} , then the value of n = ...

(a) 25 (b) 15 (c) 20 (d) 17

---

10 If $\vec{A} = (-2 , 4 , 6)$, $\vec{B} = (0 , k , 3)$ such that $k \in \mathbb{Z}^+$ and $\|\overrightarrow{AB}\| = 7$, then the value of k = ...

(a) 10 (b) 8 (c) 6 (d) 4

---

11 The length of the perpendicular from the point (2 , 3 , 1) to the plane : $2x - 2y + z = 5$ equals .... length unit.

(a) 1 (b) 2 (c) 3 (d) 4

---

12 **Without expansion the determinant , prove that :**

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b - a)(c - a)(c - b)$$

---

13 In $z = \left(1 + \sqrt{3}\, i\right)^n$ and $|z| = 8$, then the principle amplitude for the number z equals ....

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

14 In the cartesian plane $\mathcal{X}$y if θ is the measure of the angle between $\overrightarrow{A}$ and $\overrightarrow{B}$, then $\frac{\|\overrightarrow{A} \times \overrightarrow{B}\|}{\overrightarrow{A} \cdot \overrightarrow{B}} = \ldots\ldots$

(a) sin θ (b) cos θ (c) tan θ (d) cot θ

---

15 Find the term contains $\mathcal{X}^4$ in the expansion of $\left(\mathcal{X}^2 - \frac{1}{\mathcal{X}^2}\right)^{12}$ according to the descending power of $\mathcal{X}$, then find the ratio between the coefficient of this term and the middle term.

---

16 Find the different forms of the equation of the plane passes through the point $(2, -1, 0)$ and the vector $\overrightarrow{u} = 4\hat{i} + 10\hat{j} - 7\hat{k}$ is perpendicular to it.

---

17 If $(1 + \omega)^7 = a + b\,\omega$ such that a and b are two real numbers, then $(a, b) = \ldots\ldots$

(a) $(0, -1)$ (b) $(1, 1)$ (c) $(0, 1)$ (d) $(1, -1)$

---

18 Find the different forms of the equation of the straight line passes through the point $(3, 2, -1)$ and makes equal angles with the positive directions of the coordinate axes.

---

19 **Solve the following system of linear equations using the inverse matrix :**

$2z - 3y = 7$, $y + 5\mathcal{X} = 4$, $\mathcal{X} - 2y - z = 1$

Answer the following questions :

1. The last term in the expansion of $(2 - x)^5 (2 + x)^5$ is ....

(a) x^5 (b) $-x^5$ (c) $-x^{10}$ (d) x^{10}

2. If $\overrightarrow{AB} = -3\hat{i} + 3\hat{j} + 7\hat{k}$, $\overrightarrow{BC} = \hat{j} + 5\hat{k}$, then $\|\overrightarrow{AC}\| =$ .... length unit.

(a) 13 (b) 12 (c) 10 (d) 9

3. If the two planes : $x + 2y + kz = 0$, $2x + y - 2z = 0$ are perpendicular , then k = ...

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) -2 (d) 2

4. **Without expansion the determinant prove that :**

$$\begin{vmatrix} a & b & c \\ b & a & c \\ b & c & a \end{vmatrix} = (a - b)(a - c)(a + b + c)$$

5. If $z = -1 - i$, then the exponential form for the number z is ....

(a) $e^{\frac{3}{4}\pi i}$ (b) $e^{\frac{5}{4}\pi i}$ (c) $\sqrt{2}\, e^{\frac{-3}{4}\pi i}$ (d) $-\sqrt{2}\, e^{\frac{5}{4}\pi i}$

6. If $\overrightarrow{A}, \overrightarrow{B} \in \mathbb{R}^2$, then $\|\overrightarrow{A} \times \overrightarrow{B}\|^2 + (\overrightarrow{A} \cdot \overrightarrow{B})^2 =$ ....

(a) $\|\overrightarrow{A}\|^2 + \|\overrightarrow{B}\|^2$ (b) $\|\overrightarrow{A} - \overrightarrow{B}\|^2$ (c) $\|\overrightarrow{A} + \overrightarrow{B}\|^2$ (d) $\|\overrightarrow{A}\|^2 \|\overrightarrow{B}\|^2$

7. In the expansion of $(1 + x)^8$ according to the ascending power of x , if the fourth term equals 7 , find the value of x , then find the ratio between the sixth term and the middle term in this expansion.

8. Find the different forms of the equation of the straight line passes through the point $(-2, 3, 5)$ and parallel to the straight line : $\frac{x - 1}{2} = \frac{y + 1}{4} = \frac{z - 3}{3}$

9 $\left(1 + 2\,\omega^5 + \frac{1}{\omega^2}\right)\left(1 + 2\,\omega + \frac{1}{\omega^4}\right) = \dots\dots\dots$

(a) 1 (b) -1 (c) 2 (d) 0

10 Find the different forms of the equation of the plane passes through the point $(1, -1, 4)$ and the vector $\vec{u} = (2, -3, 4)$ is perpendicular to it.

11 **Solve the following system of linear equations using the inverse matrix :**

$x - 2y = 5$, $2z + y = x$, $x - 2z = -1$

12 The student should answer 10 questions out of 13 questions on a condition that he should answer 4 questions at least from the first 5 questions , then the number of ways for his answer equals $\dots\dots\dots$

(a) ${}^5C_4 \times {}^8C_6$ (b) ${}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$

(c) ${}^5P_4 \times {}^8P_6$ (d) ${}^5P_4 \times {}^8P_6 + {}^5P_5 \times {}^8P_5$

13 If $1 + 7x + {}^7C_2\,x^2 + \dots + x^7 = 128$, then the value of $x = \dots\dots\dots$

(a) 2 (b) 1 (c) -1 (d) -2

14 The point lies on the straight line $\vec{r} = (2, -1, 3) + k\,(1, 2, -1)$ is $\dots\dots\dots$

(a) $(2, 5, 3)$ (b) $(1, 1, 1)$ (c) $(0, 0, 1)$ (d) $(3, 1, 2)$

15 ***Answer one of the following items :***

[a] If $z = \frac{16}{1 - \sqrt{3}\,i}$, write z in the trigonometric form , then find its cubic roots in the exponential form.

[b] If $\frac{1 + 10\,\omega + 10\,\omega^2}{1 - 3\,\omega - 3\,\omega^2} = (k\,i)^2$, find the value of the real number k

16 If ${}^9C_r > {}^9C_{r-1}$, then $\dots\dots\dots$

(a) $r < 4$ (b) $r > 4$ (c) $r < 5$ (d) $r > 5$

17 The length of the diameter of the sphere : $x^2 + y^2 + z^2 + 4x - 6y + 8z + 4 = 0$ equals .... length unit.

(a) 5 (b) 10 (c) 15 (d) 20

18 The sum of the intercepted parts made by the plane $6x + y + 5z = 30$ with the coordinate axes equals .... length unit.

(a) 12 (b) 30 (c) 31 (d) 41

19 ***Answer one of the following items :***

[a] If A (1 , 2 , – 3) , B (3 , 5 , – 2) , C (m , 1 , – 10 m) , determine the value of m which makes :

(1) A , B , C collinear (on the same straight line).

(2) $\overrightarrow{AB}$, $\overrightarrow{AC}$ perpendicular.

[b] ABCD is a quadrilateral in which A (3 , 0 , 2) , B (6 , 2 , 5) , C (4 , 4 , 5) and D (1 , 2 , 2)

(1) Prove that the figure ABCD is a parallelogram and find its area.

(2) Find the unit vector perpendicular to the plane of the quadrilateral.

Answer the following questions :

1. If $z = \left(1 + \sqrt{3}\, i\right)^n$ and $|z| = 8$, then the principle amplitude of the number z is ....

   (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

---

2. If the two planes : $3x - y + 2z + 4 = 0$, $x + 2y + kz = 2$ are perpendicular , then k = ...

   (a) -4 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

---

3. **Without expanding the determinant , solve the equation :**

$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = 0$$

---

4. **Prove that the two straight lines :**

   $\vec{r}_1 = (3 , -1 , 2) + t_1 (4 , 1 , 3)$ and $\vec{r}_2 = (0 , 4 , -1) + t_2 (1 , -1 , 2)$ are skew.

---

5. The number of ways of selecting a team of 7 members out of 9 girls and 5 boys , if the team has 3 boys only equals ....

   (a) 136 (b) 3084 (c) 1260 (d) 1287

---

6. **The value of :** ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ equals ....

   (a) ${}^{56}C_4$ (b) ${}^{56}C_2$ (c) ${}^{55}C_4$ (d) ${}^{55}C_3$

---

7. If $x^2 + y^2 + z^2 - 2x + 4y = 0$ is the equation of a sphere of centre C and radius r , then ....

   (a) C (1 , −2 , 0) , $r = \sqrt{5}$ length unit. (b) C (−1 , 2 , 0) , $r = \sqrt{5}$ length unit.

   (c) C (1 , −2 , 0) , r = 5 length unit. (d) C (−1 , 2 , 0) , r = 5 length unit.

8 ***Answer only one of the following two questions***

[a] Put the number $z = \frac{8}{1+\sqrt{3}\,i}$ in the trigonometric form , then find its two square roots in the exponential form.

[b] **Solve the following equation in C :** $(x-1)^6 - 9(x-1)^3 + 8 = 0$

---

9 $e^{\pi i} - e^{-\pi i} = \dots\dots\dots$

(a) -2 (b) 0 (c) 1 (d) 2

---

10 Find all the different forms of the equation of the plane passing through the points :

$(1, 0, 0), (0, 2, 0), (0, 0, 3)$

---

11 Investigate the possibility of solving the following system and find the general form of the solution (if it exists).

$$\begin{pmatrix} 2 & -4 & -9 \\ -1 & 2 & 3 \\ -3 & 6 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

---

12 If $z = \omega^x$, where x is a positive integer , then $|z| = \dots\dots\dots$

(a) 1 (b) ω (c) x (d) ω^2

---

13 If the direction angles of a straight line are : θ_x, θ_y and θ_z

, then $\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z = \dots\dots\dots$

(a) -2 (b) -1 (c) 1 (d) 2

---

14 If $L_1 : x = 2t_1 - 1 , y = t_1 + 1 , z = t_1 - 1$, and $L_2 : x = at_2 - 1 , y = 2t_2 + 1 , z = bt_2 - 2$ are parallel , then $a + b = \dots\dots\dots$

(a) 4 (b) 2 (c) 6 (d) -2

---

15 In the expansion of $\left(\frac{1}{x} + x^2\right)^{15}$ according to the ascending powers of x , find the value of the term free of x , then find the value of x which makes the two middle terms equal.

16 The number of terms in the expansion of :

$(x + y)^{2019} + (x - y)^{2019}$ after reduction is ....

(a) 1010 (b) 1009 (c) 2020 (d) 2019

17 If $\overrightarrow{AB} = -3\hat{i} + 3\hat{j} + 7\hat{k}$, $\overrightarrow{BC} = \hat{j} + 5\hat{k}$, then $\|\overrightarrow{AC}\| =$ ....

(a) 13 (b) 12 (c) 10 (d) 9

18 If $\vec{A} \perp \vec{B}$, $\vec{A} \perp \vec{C}$, $\vec{B} = (2, 3, 2)$, $\vec{C} = (1, 2, 1)$, $\|\vec{A}\| = 4\sqrt{2}$, then $\vec{A} =$ ....

(a) $(2, 3, 1)$ (b) $(-4, 0, 4)$ (c) $(4, 4, 0)$ (d) $(0, -4, 4)$

19 ***Answer only one of the following two questions***

[a] If A $(0, 0, 1)$, B $(1, 0, 0)$ and C $(0, 1, 0)$, find the orthogonal unit vector to the plane ABC

[b] If the two spheres :

$(x + 1)^2 + (y - 4)^2 + (z - k)^2 = 25$, $(x - 3)^2 + y^2 + (z - 3)^2 = 16$ are externally tangential , find the value of k

Answer the following questions :

1 The last term in the expansion of $(2 - x)^5 (2 + x)^5$ according to the ascending power of x is ....

ⓐ x^5 ⓑ $-x^5$ ⓒ $-x^{10}$ ⓓ x^{10}

---

2 If $\overrightarrow{A} = (3, -2, m)$, $\|\overrightarrow{A}\| = \sqrt{22}$, then m = ...

ⓐ 21 ⓑ ±9 ⓒ ±3 ⓓ 17

---

3 If ABCD is a parallelogram in which $\overrightarrow{AB} = (2, 2, -1)$, $\overrightarrow{AD} = (-1, 2, -3)$, then the area of the parallelogram = ... square unit.

ⓐ 6 ⓑ $7\sqrt{2}$ ⓒ $3\sqrt{11}$ ⓓ $\sqrt{101}$

---

4 *Answer only one of the following two questions*

[a] Find the vector component of $\overrightarrow{AB}$ in the direction of $\overrightarrow{M}$, where :

$A(2, 1, 0)$, $B(3, 1, \sqrt{3})$ and $\overrightarrow{M} = (3, 2, 2\sqrt{3})$

[b] If the plane : $2x - y - 2z + 12 = 0$ intersects the sphere :

$(x + 3)^2 + (y + 2)^2 + (z - 1)^2 = 15$, find the area of the resulting cross section.

---

5 $\frac{a - d\omega}{a\omega^2 - d} - \omega^2 =$ ....

ⓐ $3i$ ⓑ $\pm\sqrt{3}\,i$ ⓒ -3 ⓓ 3

---

6 The direction cosines of the straight line passing through the two points (10, 9, 1) and (4, 7, −2) are ....

ⓐ (6, 2, 3) ⓑ (2, 4, −13)

ⓒ $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$ ⓓ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right)$

7. If the straight line : $\frac{x+2}{6} = \frac{y-1}{m} = \frac{z-1}{3}$ is perpendicular to the straight line $\frac{x-9}{-2} = \frac{y+8}{1}$, z = 3 , then m = ...

(a) – 12 (b) 12 (c) 6 (d) 0

8. In the expansion of $\left(2x + \frac{3}{x^2}\right)^{20}$ according to the descending power of x if $T_9 = T_{10}$, find the value of x , then find the orders of two consecutive terms such that the ratio between one of them and the term following it equals 8 : 15 and prove that this expansion does not contain a term free of x

9. The number of ways of selecting an even number and two odd numbers out of 4 even numbers and 5 odd numbers equals ....

(a) 40 (b) 14 (c) 80 (d) 70

10. If ${}^9C_r > {}^9C_{r-1}$, then ....

(a) $r < 4$ (b) $r > 4$ (c) $r < 5$ (d) $r > 5$

11. If $x^2 + y^2 + z^2 + 4x - 2y - 6z + 11 = 0$ is the equation of a sphere of centre C and radius length r , then ....

(a) C (– 2 , – 1 , – 3) , r = 3 length units.

(b) C (– 2 , 1 , 3) , r = $\sqrt{3}$ length units.

(c) C (– 2 , 1 , 3) , r = 3 length units.

(d) C (2 , – 1 , – 3) , r = $\sqrt{3}$ length units.

12. ***Answer only one of the following two questions***

[a] Put the number $z = 4 + 4\sqrt{3}\,i$ in the trigonometric form , then find its two square roots in the exponential form.

[b] **Find in the simplest form the value of the expression :**

$\left[k - \frac{k-1}{1+\omega} + (k+1)\,\omega^2\right]^8$, where $k \in \mathbb{R}$

13 $\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} =$ ....

(a) $e^{\frac{\pi}{6} i}$ (b) $e^{\frac{-\pi}{6} i}$ (c) $e^{\frac{5\pi}{6} i}$ (d) $e^{\frac{-5\pi}{6} i}$

14 Find all the different forms of the equation of the plane passing through the point $(-3, 4, 2)$ and the vector $\vec{n} = (1, -1, 3)$ is normal to it.

15 **Show that the following system has an infinite number of solutions and write the general form of the solution :**

$x + 2y + 3z = 0$

$2x + 3y + 5z = 0$

$3x - y + 2z = 0$

16 If $a = \cos\theta + i \sin\theta$, $b = \cos\alpha + i \sin\alpha$, then the value of the expression :

$\frac{1}{2}\left(ab + \frac{1}{ab}\right)$ equals ....

(a) $\sin(\theta + \alpha)$ (b) $\sin(\theta - \alpha)$ (c) $\cos(\theta + \alpha)$ (d) $\cos(\theta - \alpha)$

17 The measure of the angle between the two planes :

$3x - 6y + 6z - 4 = 0$, $x + z - 7 = 0$ is ....

(a) $90°$ (b) $60°$ (c) $45°$ (d) $30°$

18 **Without expanding the determinant , prove that :**

$$\begin{vmatrix} x+a & x+a & 2a \\ a & x & a \\ 0 & a-x & x-a \end{vmatrix} = (x+2a)(x-a)^2$$

19 **Prove that the two straight lines :**

$\vec{r_1} = \vec{j} + t_1 (\vec{i} + 2\vec{j} - \vec{k})$ and $\vec{r_2} = (\vec{i} + \vec{j} + \vec{k}) + t_2 (-2\vec{i} - 2\vec{j})$ intersect at a point

, then find the coordinates of their intersection point.

Answer the following questions :

1 The principal amplitude of the complex number $z = 1 - i$ is ....

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) $-\frac{7\pi}{4}$ (d) $\frac{7\pi}{4}$

2 If the direction cosines of the straight is $\left(\frac{1}{b}, \frac{1}{b}, \frac{1}{b}\right)$ such that $b > 0$, then ....

(a) $b = 1$ (b) $0 < b < 1$ (c) $b = \sqrt{3}$ (d) $b = \sqrt{2}$

3 **Without expanding the determinant , prove that :**

$$\begin{vmatrix} 1 & b & c \\ b & 1+b^2 & bc \\ c & bc & 1+c^2 \end{vmatrix} = 1$$

4 **Find the measure of the angle between the two straight lines :**

$L_1 : \overrightarrow{r_1} = (2, -1, 3) + t_1 (-2, 0, 2)$

$L_2 : x = 1, \frac{y-4}{3} = \frac{z+5}{-3}$

5 $\sqrt{6} - \sqrt{2}\, i =$ ....

(a) $2\sqrt{2}\, e^{\frac{\pi}{6} i}$ (b) $2\sqrt{2}\, e^{\frac{-\pi}{6} i}$ (c) $2\sqrt{2}\, e^{\frac{\pi}{3} i}$ (d) $2\sqrt{2}\, e^{\frac{-\pi}{3} i}$

6 Find the projection of the point A (0, 9, 6) on the straight line that passes through the two points B (1, 2, 3), C (7, −2, 5)

7 Prove that : $\frac{3 + 5\omega + 3\omega^2}{1 - 2\omega - 4\omega^2} + \frac{3 + 5\omega^2 + 3\omega}{1 - 2\omega^2 - 4\omega} = \frac{-2}{19}$

8 ${}^{n}C_r : {}^{n-1}C_r =$ ....

(a) $\frac{n-r}{n}$ (b) $\frac{n}{r}$ (c) $\frac{r}{n}$ (d) $\frac{n}{n-r}$

9 The equation of sphere whose center is the origin point and passes through the point $(3, -1, 2)$ is ....

(a) $x^2 + y^2 + z^2 = \sqrt{14}$ (b) $(x-3)^2 + (y+1)^2 + (z-2)^2 = 14$

(c) $x^2 + y^2 + z^2 = 14$ (d) $(x-3)^2 + (y+1)^2 + (z-2)^2 = \sqrt{14}$

10 In one of the governorates, the license plates of cars consist of 3 different alphabets followed by 4 different digits. If the number of alphabets used is 26 and the digits used are : $1, 2, 3, \cdots, 9$, then the number of plates that can be created in this governorate is equal to ....

(a) ${}^{26}P_3 \times {}^{9}P_4$ (b) ${}^{26}P_3 + {}^{9}P_4$ (c) ${}^{26}C_3 \times {}^{9}C_4$ (d) ${}^{26}C_3 + {}^{9}C_4$

11 ***Answer only one of the following two questions***

[a] Put the number $z = 1 - \sqrt{3}\, i$ in the trigonometric form, then find its two square roots in the exponential form.

[b] If $z_1 = \left(\frac{\sqrt{3} + i}{2}\right)^4$, $z_2 = \sin\frac{\pi}{3} + i\cos\frac{\pi}{3}$, $i^2 = -1$, put the number z in the trigonometric form, then in the exponential form such that : $z = \frac{z_1}{z_2}$

12 The sum of the coefficients of the expansion of $(3x - 2y)^{10}$ equals ....

(a) 1 (b) 1024 (c) 58025 (d) 59049

13 If the point $A(n-1, n+4, 5n)$ lies on the plane $y = 5$, then ....

(a) $A(4, 9, 25)$ (b) $A(5, 0, 5)$ (c) $A(0, 5, 5)$ (d) $A(5, 5, 0)$

14 If $\vec{A}, \vec{B}$ are two unit vectors, then $\vec{A} \cdot \vec{B} \in$ ....

(a) $]0, 1[$ (b) $[-1, 1]$ (c) $]-1, 1[$ (d) $\mathbb{R}^+$

15 *Answer only one of the following two questions*

[a] If the vector $\overrightarrow{A}$ form with the positive directions of the coordinates axes x , y , z the angles whose measure are $45°$, $60°$, $\theta°$ respectively such that θ is an acute angle.

(1) Find the value of θ

(2) The cartesian form of the vector $\overrightarrow{A}$ when $\|\overrightarrow{A}\| = 10$

[b] If $\|\overrightarrow{A}\| = 6$ and the direction cosines of the vector $\overrightarrow{A}$ are $\frac{2}{3}$, $\frac{-2}{3}$, $\frac{1}{3}$ respectively and the vector $\overrightarrow{B} = (-2 , 3 , 5)$, find the area of the parallelogram in which $\overrightarrow{A}$ and $\overrightarrow{B}$ represented two adjacent sides on it.

---

16 **The product of the roots of the equation :** $x^4 - 1 = 0$ equals ....

(a) zero (b) 1 (c) -1 (d) i

---

17 The equation of the straight line passes through the point (1 , 2 , 3) and its direction angles are (30° , 90° , 60°) is ....

(a) $\frac{2x-1}{2} = \frac{y-1}{2} = \frac{2z-\sqrt{3}}{6}$

(b) $\frac{2x-\sqrt{3}}{2} = \frac{y}{2} = \frac{2z-1}{6}$

(c) $\frac{x-1}{2} = \frac{2z-6}{\sqrt{3}}$, $y = 2$

(d) $\frac{2x-2}{\sqrt{3}} = \frac{2z-6}{1}$, $y = 2$

---

18 **If the two straight lines :**

$L_1 : \frac{x+2}{-1} = \frac{y+3}{3} = \frac{z+3}{2}$, $L_2 : \frac{x}{2} = \frac{y-5}{k} = \frac{z-6}{m}$ are perpendicular

, then $3k + 2m =$ ....

(a) -1 (b) 2 (c) 3 (d) 4

---

19 In the expansion of $\left(x^3 - \frac{1}{x}\right)^8$ according to the descending powers of x :

(1) Find the order and the value of the term free of x in this expansion.

(2) Find the ratio between the middle term of this expansion and the term following to it when $x = 1$

Answer the following questions :

1. The coefficient of x^4 in the expansion of $(2x + 1)^8$ according to the descending power of x equals ....

   (a) 16 (b) 256 (c) 1120 (d) 16801

2. If the point A $(1 - k , 2k , 3 + k)$ lies on the x y-plane , then ....

   (a) A (0 , 2 , 5) (b) A (– 6 , 4 , 0)

   (c) A (1 , 0 , 3) (d) A (4 , – 6 , 0)

3. If $\overrightarrow{A}$, $\overrightarrow{B}$ are two unit vectors such that $\overrightarrow{A} \cdot \overrightarrow{B} = \frac{-1}{2}$, then the measure of the angle between them equals ....°

   (a) 150 (b) 120 (c) 60 (d) 30

4. *Answer only one of the following two questions*

   [a] Find the measures of the angles formed by the vector $\overrightarrow{C} = 5\sqrt{2}\,\hat{i} + 5\,\hat{j} + 5\,\hat{k}$ with the positive directions of the coordinates axes.

   [b] If $\overrightarrow{A} \times \overrightarrow{B} = -65\,\hat{c}$ such that $\hat{c}$ is a unit vector perpendicular to the plane contains $\overrightarrow{A}$, $\overrightarrow{B}$, $\|\overrightarrow{A}\| = 5$, $\|\overrightarrow{B}\| = 26$, find the measures of the angle between the two vectors $\overrightarrow{A}$, $\overrightarrow{B}$

5. **The sum of the roots of the equation :** $x^4 - 1 = 0$ equals ....

   (a) zero (b) 1 (c) – 1 (d) i

6. The equation of the straight line passes through the two points (– 4 , 3 , 4) , (6 , – 1 , – 2) is ....

   (a) $\frac{x-4}{10} = \frac{y+3}{-4} = \frac{z+4}{-6}$ (b) $\frac{x-6}{10} = \frac{y+1}{-4} = \frac{z+2}{-6}$

   (c) $\frac{x+6}{10} = \frac{y-1}{-4} = \frac{z-2}{-6}$ (d) $\frac{x}{6} = \frac{y}{-1} = \frac{z}{-2}$

7 **If the two straight lines :**

$L_1 : \vec{r} = (2 , 3 , -4) + t(2 , 3 , a)$, $L_2 : \frac{x-5}{b} = \frac{y+4}{6} = \frac{z-4}{2}$ are parallel

, that a + b = ...

(a) 1 (b) 4 (c) 5 (d) 6

---

8 In the expansion of $\left(x^2 + \frac{2}{x}\right)^8$ according to descending powers of x :

(1) Find the ratio between the sixth term and the fifth term , if this ratio equals 25 : 8

, find the value of x

(2) Prove that there is no term free of x in this expansion.

---

9 If $z = \sqrt{2} (\sin 30° + i \cos 30°)$, then the principal amplitude of z equals ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

---

10 The direction cosines of the straight line passes through the two points

(1 , 2 , 3) , (3 , 2 , 2) is ....

(a) (2 , 0 , – 1) (b) (4 , 4 , 5)

(c) $\left(\frac{2}{\sqrt{5}} , 0 , \frac{-1}{\sqrt{5}}\right)$ (d) $\left(\frac{4}{\sqrt{57}} , \frac{4}{\sqrt{57}} , \frac{5}{\sqrt{57}}\right)$

---

11 **Without expanding the determinant , prove that :**

$$\begin{vmatrix} 3x & 3x & 3x \\ 1 & b & a \\ a+b & a+1 & b+1 \end{vmatrix} = 0$$

---

12 Find the length of the perpendicular drawn from the point A (– 1 , 2 , 5) to the straight line :

$\vec{r} = (3 , 4 , 5) + t(2 , -3 , 6)$

---

13 For all values of x , $y \in \mathbb{R}$, then $e^{x i + y}$ = ...

(a) $e^{x} (\sin y + i \cos y)$ (b) $e^{x} (\cos y + i \sin y)$

(c) $e^{y} (\sin x + i \cos x)$ (d) $e^{y} (\cos x + i \sin x)$

14 **Prove that the two straight lines :**

$\vec{r_1} = \vec{j} + t_1 (\vec{i} + 2\vec{j} - \vec{k})$, $\vec{r_2} = (\vec{i} + \vec{j} + \vec{k}) + t_2 (-2\vec{i} - 2\vec{j})$ are intersected and find their point of intersection.

---

15 **Prove that :** $\left(\frac{1}{1+\omega i} + \frac{\omega + i}{1+\omega^2 i}\right)^8 = 16$

---

16 ${}^{n}C_r : {}^{n-1}C_{r-1} = \dots\dots\dots$

(a) $\frac{r}{n}$ (b) $\frac{n}{n-r}$ (c) $\frac{n-r}{n}$ (d) $\frac{n}{r}$

---

17 The equation of sphere of center $(3 , -2 , 4)$ and radius 5 length unit is ....

(a) $(x+3)^2 + (y-2)^2 + (z+4)^2 = 25$ (b) $x^2 + y^2 + z^2 = 25$

(c) $(x-3)^2 + (y+2)^2 + (z-4)^2 = 25$ (d) $(x-3)^2 + (y+2)^2 + (z-4)^2 = 5$

---

18 In one of the governorates , the license plates of cars consist of 3 different alphabets followed by 3 different digits. If the number of alphabets used is 26 and the digits used are : 1 , 2 , 3 , ... , 9 , then the number of plates that can be created in this governorate is equal to ....

(a) ${}^{26}P_3 + {}^{9}P_3$ (b) ${}^{26}P_3 \times {}^{9}P_3$ (c) ${}^{26}C_3 + {}^{9}C_3$ (d) ${}^{26}C_3 \times {}^{9}C_3$

---

19 ***Answer only one of the following two questions***

[a] Put the number $z = 1 + \sqrt{3}\, i$ in the trigonometric form , then find its two square roots in the exponential form.

[b] If $z_1 = 1 - \sqrt{3}\, i$, $z_2 = e^{\frac{-\pi}{6} i}$, $z_3 = \left(\cos \frac{\pi}{12} - i \sin \frac{\pi}{12}\right)$, $i^2 = -1$

Put the number z in the trigonometric form , then in the exponential form such that : $z = \frac{z_1 z_2}{z_3}$

Answer the following questions :

1. If $z_1 = \cos\theta + i\sin\theta$, $z_2 = \cos 2\theta + i\sin 2\theta$, then the principle amplitude of the complex number $3 z_1 z_2 =$ .... , where $\theta \in]0 , \frac{\pi}{6}[$

ⓐ 9θ ⓑ θ ⓒ 5θ ⓓ 3θ

2. In the expansion of $\left(x^5 - \frac{k}{x^2}\right)^{7n}$ according to the descending powers of x , the term free of x is .... , where k , n $\in \mathbb{Z}^+$

ⓐ T_{5n} ⓑ T_{5n+1} ⓒ T_{6n+1} ⓓ T_{6n-1}

3. If $\begin{vmatrix} a & b & c \\ 1 & -2 & 3 \\ e & f & d \end{vmatrix} = 8$, then $\begin{vmatrix} 2 & -4 & 6 \\ 2a & 2b & 2c \\ e & f & d \end{vmatrix} =$ ....

ⓐ 16 ⓑ 32 ⓒ -32 ⓓ -16

4. If $\overrightarrow{AB} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{CB} = -\hat{j} + \hat{i} - \hat{k}$, then $\overrightarrow{CA} =$ ....

ⓐ $-\hat{i} + 2\hat{j} - \hat{k}$ ⓑ $3\hat{i} + 2\hat{j} - \hat{k}$ ⓒ $3\hat{i} + 4\hat{j} - \hat{k}$ ⓓ $-\hat{i} - 4\hat{j} + \hat{k}$

5. If the two straight lines $L_1 : \vec{r} = t_1(-2 , m , 7)$ and $L_2 : \frac{x-1}{n} = \frac{1-y}{4} = \frac{z-2}{2}$ are perpendicular , then $n + 2m =$ ....

ⓐ 7 ⓑ -7 ⓒ 14 ⓓ -14

6. If the two planes $2x + cy + 4z = 1$ and $(a+2)x + 6y + (b-2)z = 5$ are parallel , then $2a - b =$ ....

ⓐ -6 ⓑ 6 ⓒ -12 ⓓ 12

7. If the cosine of the angle which the vector $\vec{A} = (k , 12 , 4)$ makes with the positive direction of x-axis equals $\frac{3}{13}$, then k = ... where $k \in \mathbb{R}$

ⓐ 4 ⓑ $\sqrt{3}$ ⓒ $-\sqrt{3}$ ⓓ 3

8 If 1 , ω , ω^2 are the cubic roots of unity , and $x = \frac{1}{1+\omega i}$, $y = \frac{\omega + i}{1+\omega^2 i}$, then $x - y =$ ....

(a) $i + 1$ (b) $1 - i$ (c) 1 (d) i

---

9 An office has 9 men and 6 women. It is required to form a committee of 5 persons and the majority of them should be women and contains the two genders , then the number of committees equals ....

(a) 11880 (b) 2871 (c) 3003 (d) 855

---

10 If the coefficient of T_6 in the expansion of $\left(a x + \frac{1}{b x}\right)^{10}$ according to the descending powers of x equals ${}^{10}C_5$, then $\frac{a}{b} =$ .... where $a \in \mathbb{R}^*$, $b \in \mathbb{R}^*$

(a) -1 (b) 1 (c) 10 (d) $\frac{1}{10}$

---

11 If $\begin{vmatrix} x-1 & a & b \\ 0 & x^2+x+1 & c \\ 0 & 0 & 1 \end{vmatrix} = 8$, then the value of $x^9 + 1 =$ ....

(a) 37 (b) 729 (c) 730 (d) 8

---

12 In the triangle XYZ if $\begin{vmatrix} 0 & 0 & x \\ 0 & y & 0 \\ z & 0 & 0 \end{vmatrix} = -100 \text{ cm}^3$ and the area of Δ XYZ $= 6.25 \text{ cm}^2$

, then the length of the diameter of circumcircle of triangle XYZ = ... cm.

(a) 4 (b) 16 (c) 8 (d) 2

---

13 If M_1 , M_2 are two touching internally spheres and $M_1 \left(-3 , 2 , -6\sqrt{2}\right)$, $r_1 = 8$ length unit $M_2 \left(-2 , 1 , -5\sqrt{2}\right)$, then $r_2 =$ .... length unit , where $r_1 > r_2$

(a) 5 (b) 2 (c) 7 (d) 6

---

14 If the perpendicular distance between the point $(-1 , 2 , m)$ and the straight line : $\vec{r} = (-1 , 3 , 0) + t (0 , -3 , 0)$ is 8 length unit , then the value of m equals ....

, where $m \in \mathbb{R}^+$

(a) 4 (b) 16 (c) 8 (d) 2

15 If the measure of the angle between the two planes : $x + y - 1 = 0$, $k\,y + z - 1 = 0$ equals $60°$, then $k =$ .... , where $k > 0$

(a) 4 (b) $\frac{1}{2}$ (c) 2 (d) 1

16 If $z = k\left(\sin \frac{4}{3}\pi - i \cos \frac{4}{3}\pi\right)$, then $z^6 =$ .... , where $k >$ zero

(a) k^6 (b) $6\,k$ (c) $-k^6$ (d) $-6\,k$

17 In the expansion of $(a\,x + b)^n$ according to the descending powers of x , if the two middle terms are equal and n is an odd number , then $x =$ .... , where $a \in \mathbb{R}^*$, $b \in \mathbb{R}^*$

(a) $\frac{b}{a}$ (b) $\frac{a}{b}$ (c) $a\,b$ (d) $-\frac{b}{a}$

18 If the point $(7 , -2 , 2)$ lies on the surface of the sphere whose equation is $(x-4)^2 + (y-1)^2 + (z+1)^2 = k^2$, then $|k| =$ ....

(a) 3 (b) $3\sqrt{3}$ (c) 27 (d) $\sqrt{3}$

19 If z is a complex number and $z + \bar{z} = 2\,e^{\pi i}$, then z could be equal ....

(a) $e^{\pi i}$ (b) $2\,e^{\frac{\pi}{2} i}$ (c) $e^{\frac{-\pi}{2} i}$ (d) $2\,e^{\pi i}$

20 If $\frac{{}^nC_4 + {}^nC_3}{{}^{n+1}C_3} = 1$, then $\underline{|n-6} =$ ....

(a) 6 (b) 1 (c) zero (d) 24

21 In the expansion of $\left(x^2 + 2 + \frac{1}{x^2}\right)^6$ the coefficient of the term containing x^2 is ....

(a) ${}^{12}C_5$ (b) ${}^{12}C_6$ (c) ${}^{12}C_2$ (d) ${}^6C_5$

22 If the matrix (A_{xy}) is of the order 3×3 where $a_{xy} = 2\,x - y$, then the rank of the matrix A is ....

(a) 3 (b) 2 (c) 1 (d) zero

23 If $\|\vec{A}\| = \sqrt{13}$, $\vec{A} \,/\!/\, \overrightarrow{DC}$ and in its direction , such that $D(1 , 3 , -2)$, $C(1 , -1 , 4)$ and $\vec{B} = (-2 , 3 , 5)$, then $\vec{A} \times \vec{B} =$ ....

(a) $-19\,\hat{i} + 6\,\hat{j} + 4\,\hat{k}$ (b) $-28\,\hat{i} - 12\,\hat{j} - 8\,\hat{k}$

(c) $-28\,\hat{i} + 12\,\hat{j} - 8\,\hat{k}$ (d) $-19\,\hat{i} - 6\,\hat{j} - 4\,\hat{k}$

24 ABC is a triangle , where A (1 , 2 , 4) , B (– 2 , 0 , 5) , C (1 , 4 , 0) , if M is the intersection point of its medians , then the equation of straight line $\overleftrightarrow{AM}$ is ....

(a) $\vec{r} = (1, 2, 4) + t(-1, 4, 5)$

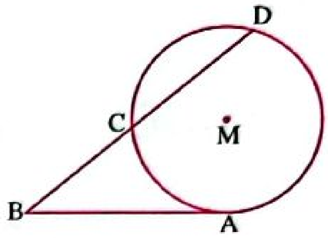
(b) $\vec{r} = (1, 2, 4) + t(-1, 1, 1)$

(c) $\vec{r} = (1, 2, 4) + t(-1, 0, -1)$

(d) $\vec{r} = (0, 2, 3) + t(1, 2, 4)$

---

25 **In the opposite figure :**



If $\overleftrightarrow{AB}$ is a tangent to the circle M at A

, $\overline{DC}$ is a chord in the circle where

$\overleftrightarrow{DC} \cap \overleftrightarrow{AB} = \{B\}$, if $\begin{vmatrix} 1 & 0 & CD \\ -1 & AB & BC \\ 0 & -BC & AB \end{vmatrix} = 32$

, then AB = ... length unit.

(a) 8

(b) 4

(c) 16

(d) 6

Answer the following questions :

1. In the expansion of $\left(x^3 + \frac{5}{x}\right)^n$ according to the descending powers of x , if the term free of x is T_7 , then the value of n = ...

(a) 9 (b) 7 (c) 10 (d) 8

---

2. If $\overrightarrow{A} + \overrightarrow{BC} = 4\hat{i} + 12\hat{j} + 9\hat{k}$ where $\overrightarrow{A} = (0 , -1 , 3)$, B (4 , –2 , 1) , then $\overrightarrow{C}$ = ...

(a) $8\hat{i} + 13\hat{j} + 13\hat{k}$
(b) $8\hat{i} + 11\hat{j} + 7\hat{k}$
(c) $8\hat{i} + 9\hat{j} + 7\hat{k}$
(d) $8\hat{i} + 13\hat{j} - 7\hat{k}$

---

3. The general form of the equation of the plane which passing through the point (–2 , 2 , –1) and parallel to the plane whose equation : $(2 , 3 , -5) \cdot \overrightarrow{r} = 1$ is ....

(a) $2x + 3y - 5z = -7$
(b) $2x + 2y - z = 1$
(c) $2x - 3y + 5z = -7$
(d) $2x + 3y - 5z = 7$

---

4. If $z_1 = 15\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ and $z_2 = 3(\cos\theta + i\sin\theta)$, where $\theta \in]0 , \frac{\pi}{2}[$, then $\frac{z_1}{z_2}$ = ...

(a) $5(\cos\theta + i\sin\theta)$
(b) $12\left(\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right)$
(c) $5\left(\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right)$
(d) $15(\sin\theta + i\cos\theta)$

---

5. The value of determinant :

$$\begin{vmatrix} 7^{2n} & 7^{3n} & 7^{4n} \\ 7^{3n} & 7^{4n} & 7^{5n} \\ 7^{4n} & 7^{5n} & 7^{6n} \end{vmatrix} = \text{..............} \text{ , where } n \in \mathbb{Z}^+$$

(a) zero
(b) 7^{2n}
(c) 7^n
(d) 7^{3n}

6. If the two straight lines :

$L_1 : \overrightarrow{r_1} = (1 , 2 , 3) + t_1 (-1 , 3 , 4)$ and $L_2 : \overrightarrow{r_2} = (-2 , 5 , -1) + t_2 (m , n , 1)$ are perpendicular , then $3n - m =$ ....

(a) -4 (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) 4

---

7. The direction cosines of the vector $\overrightarrow{A} = (-2k , 2k , k)$, where $k \in]0 , 1[$ are ....

(a) $\left(\frac{-2}{3} , \frac{2}{3} , \frac{1}{3}\right)$ (b) $\left(\frac{-2k}{3} , \frac{2k}{3} , \frac{-k}{3}\right)$

(c) $\left(\frac{-2}{3} , \frac{-2}{3} , \frac{-1}{3}\right)$ (d) $\left(\frac{2k}{3} , \frac{2k}{3} , \frac{k}{3}\right)$

---

8. If $1 , \omega , \omega^2$ are the cubic roots of unity , where a , b are two positive real numbers , then the conjugate of the number : $a\omega + b\omega^2$ is ....

(a) $a\omega^2 - b\omega$ (b) $a\omega - b\omega^2$

(c) $a\omega^2 + b\omega$ (d) $a\omega + b\omega^2$

---

9. In the expansion of $(1 + x)^{20}$ according to the ascending powers of x , if the coefficient of T_{r+2} = the coefficient of T_{r+4} , then the value of $r =$ ....

(a) 9 (b) 8 (c) 10 (d) 11

---

10. If $1 , \omega , \omega^2$ are the cubic roots of unity , then the value of the determinant :

$$\begin{vmatrix} 1 & \omega & \omega - 1 \\ 1 & -1 & \omega + 1 \\ 1 & \omega & \omega \end{vmatrix} = \text{..............}$$

(a) $\omega - 1$ (b) ω^2

(c) ω (d) $\omega^2 + 1$

---

11. The perpendicular distance between the point $(2 , 4 , 7)$ and the straight line $2x - 4 = \frac{2y - 8}{3} = \frac{2z - 14}{5}$ equals .... length unit.

(a) zero (b) 1

(c) 2 (d) 5

12 The measure of the angle between the plane $\mathcal{X}$ y and the plane $\mathcal{X}+\sqrt{3}\,z-7=0$ equals .... °

ⓐ 60 ⓑ 90 ⓒ 30 ⓓ 45

---

13 If ABC is a triangle in which D is the midpoint of $\overline{BC}$

, A (3 , 1 , 5) , B (2 , 3 , 7) , C (0 , 3 , 1)

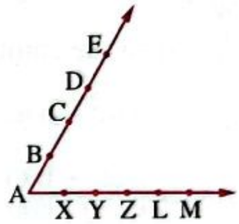
, then the length of $\overline{AD}$ = ... length unit.

ⓐ 9 ⓑ 2 ⓒ 7 ⓓ 3

---

14 **In the opposite figure :**

The ten points lie on the two rays starting from the point A , then the number of different straight lines that can be drawn using these points equals ....



ⓐ 22 ⓑ 45 ⓒ 90 ⓓ 30

---

15 In the triangle ABC ,

$$\text{if} \begin{vmatrix} a+2 & 3 & \sin C \\ 1 & b & 0 \\ 2 & 3 & \sin C \end{vmatrix} = 12$$

, where a , b and c are the side lengths of the triangle ABC , then the surface area of the triangle ABC = ... unit area.

ⓐ 12 ⓑ 6

ⓒ 24 ⓓ 8

---

16 The equation of the sphere whose centre is (–1 , 0 , 5) and its volume 36 π volume unit is ....

ⓐ $(\mathcal{X}+1)^2+y^2+(z-5)^2=36$ ⓑ $(\mathcal{X}-1)^2+y^2+(z+5)^2=6$

ⓒ $(\mathcal{X}+1)^2+y^2+(z-5)^2=27$ ⓓ $(\mathcal{X}+1)^2+y^2+(z-5)^2=9$

17. In the expansion of $\left(\sqrt{x}+\frac{1}{x}\right)^8$ according to the descending powers of x, if $5\,T_4$, T_5, T_7, T_6 are proportional, then the value of $x =$ ....

(a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) $\frac{5}{2}$ (d) $\frac{8}{5}$

---

18. If z_1, z_2 are two complex numbers, $z_1 = e^{5+k\pi i}$, $z_2 = e^{(5+ki)\pi}$, where $-\frac{1}{2} < k < \frac{1}{2}$, then the principle amplitude of the complex number $z_1 + z_2$ could be equal ....

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{2}$ (c) π (d) $-\frac{\pi}{6}$

---

19. If $A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & m & 6 \\ 5 & 7 & 9 \end{vmatrix}$, $RK(A) = 3$, then $m \in$ ....

(a) $\{4\}$ (b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{6\}$

---

20. If the point $A\left(\sqrt{k}, -\sqrt{k}\right)$ represents the complex number z on the Argand's plane, where $k > 1$, then the exponential form of the number z is ....

(a) $\sqrt{2k}\,e^{-\frac{\pi}{4}i}$ (b) $\sqrt{2}\,k\,e^{-\frac{\pi}{4}i}$

(c) $\sqrt{2k}\,e^{\frac{\pi}{4}i}$ (d) $\sqrt{2}\,k\,e^{\frac{\pi}{4}i}$

---

21. If the coefficient of the term containing x^4 in the expansion of $\left(x+\frac{a}{x^2}\right)^7$ equals 49, then the value of the constant $a =$ ....

(a) -7 (b) 49 (c) -49 (d) 7

---

22. If $\vec{u}$ is the perpendicular unit vector to the plane containing the two vectors $\vec{A}$, $\vec{B}$ according to the right-hand rule where $\vec{u} = \left(\frac{3}{5}, 0, \frac{4}{5}\right)$ and $\|\vec{A} \times \vec{B}\| = 5$, then $\left(3\vec{A}+\vec{B}\right) \times \left(4\vec{A}+2\vec{B}\right) =$ ....

(a) $(3, 0, 4)$ (b) $(30, 0, 40)$

(c) $(-30, 0, -40)$ (d) $(6, 0, 8)$

23. The vector equation of the straight line passing through the point A $(2, -1, 4)$ and parallel to the bisector of the angle between $\overrightarrow{oy}$, $\overrightarrow{oz}$ in the plane y z is ....

(a) $\vec{r} = (2, -1, 4) + t(0, 1, 1)$
(b) $\vec{r} = (2, -1, 4) + t(0, -1, 1)$
(c) $\vec{r} = (2, -1, 4) + t(-1, 0, 1)$
(d) $\vec{r} = (2, -1, 4) + t(1, 0, -1)$

---

24. If ${}^{n}C_{r} : {}^{n-1}C_{r} = 3 : 1$, then $\underline{\left|\frac{4n}{r}\right.} =$ ....

(a) 24
(b) 120
(c) 720
(d) 5040

---

25. In the triangle ABC, if $\begin{vmatrix} a^2 & c^2 & -b^2 \\ 1 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix} = ac$

, where a, b and c are the side lengths of triangle ABC, then m $(\angle B) =$ ....°

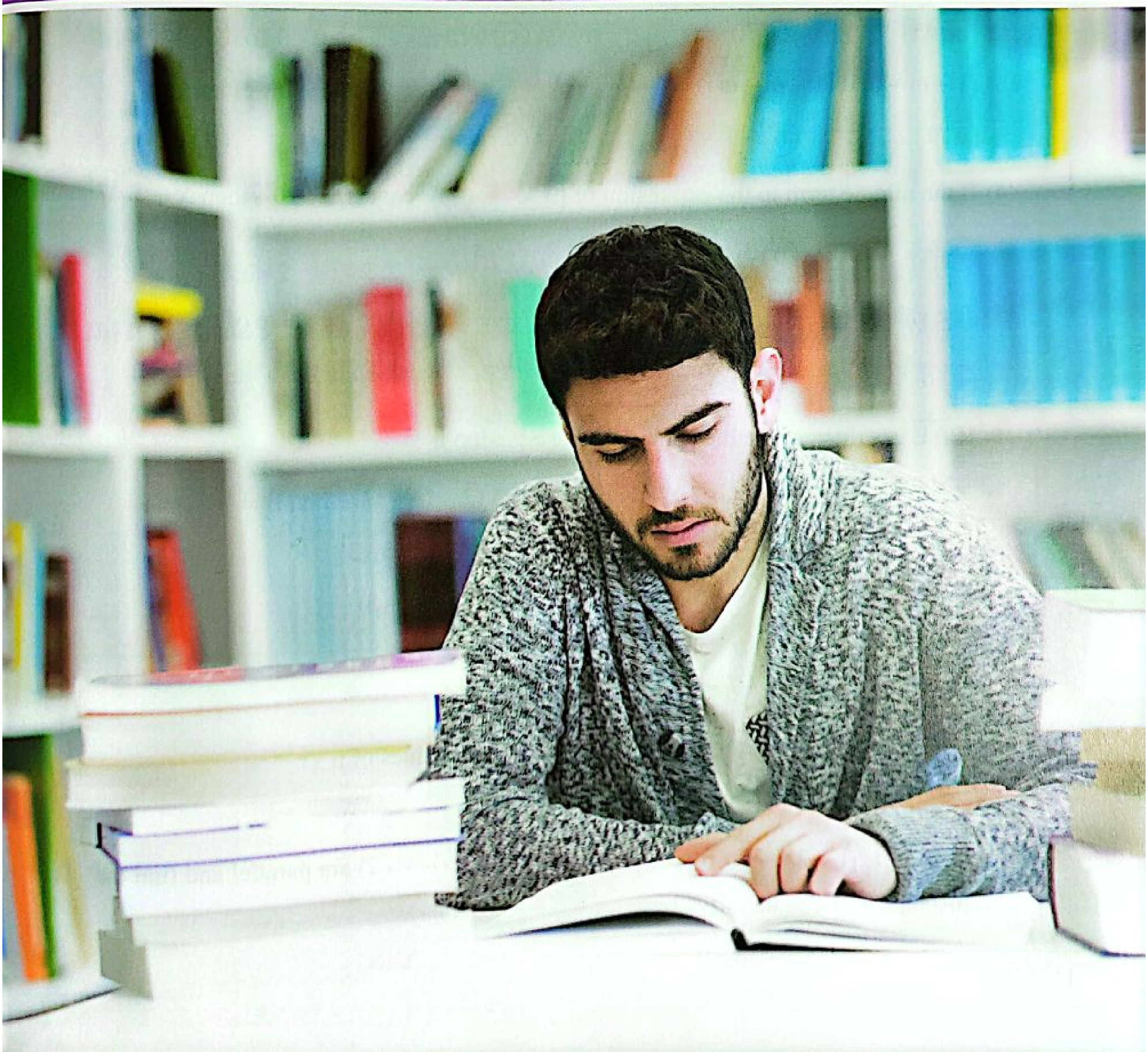
(a) 45
(b) 90
(c) 60
(d) 120

Al-Azhar Exams

(2019 : 2021 first and second sessions)

in

Algebra & Analytic Solid Geometry



Answer the following questions :

1 Choose the correct answer :

(1) In the expansion $(1 + x)^n = 1 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$ if $\frac{a_2 + a_3}{a_2} = 3$, then n = ...

ⓐ 4 ⓑ 6 ⓒ 8 ⓓ 9

(2) The measure of the angle between the two straight lines $L_1 : x - 1 = \frac{y+2}{\sqrt{2}} = -z + 1$ and $L_2 : -x = z + 3$, $y = 4$ equals ....°

ⓐ 45 ⓑ 30 ⓒ 60 ⓓ 135

(3) $(1 + \omega + \omega i)(1 + \omega - \omega i) =$ ....

ⓐ 1 ⓑ -1 ⓒ zero ⓓ 2

(4) If the middle term in the expansion of $\left(3x^2 + \frac{2}{3x}\right)^8$ equals 17920 , then $x =$ ....

ⓐ ± 2 ⓑ 3 ⓒ 4 ⓓ ± 5

(5) The value of $\left(\frac{a+1}{a + (2a+1)\omega + a\omega^2}\right)^6 =$ .... whatever the value of a

ⓐ 6 ⓑ a ⓒ 1 ⓓ ω

(6) If $\overline{AB}$ is a diameter in the sphere whose equation is :

$x^2 + y^2 + z^2 - x + 2y + 3z - 44 = 0$, and A (2 , 4 , – 6) , then B = ...

ⓐ (1 , – 2 , – 3) ⓑ (– 1 , – 6 , 3)

ⓒ (0 , 4 , 1) ⓓ (2 , 3 , – 5)

Answer only three questions from the following :

2 [a] If ${}^{n}C_{r} : {}^{n}C_{r-1} : 5 \times {}^{n-1}C_{r-2} = 7 : 4 : 6$, find the values of n , r

[b] Prove that the two straight lines :

$\vec{r}_1 = \hat{j} + t_1(\hat{i} + 2\hat{j} - \hat{k})$, $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k}) + t_2(-2 , -4 , 2)$ are parallel and find the distance between them.

3 [a] If $z_1 = \frac{6+4i}{1+i}$, $z_2 = \frac{26}{5-i}$, find the number $z = 4(z_1 - z_2)$ in the exponential form , find in the trigonometric form the cubic roots of the number z

[b] Find the equation of the plane which contains the straight line

$L_1 : \vec{r}_1 = (0, 3, -5) + t_1 (6, -2, -1)$ and parallel to the straight line

$L_2 : \vec{r}_2 = (1, 7, -4) + t_2 (1, -3, 3)$

4 [a] **Without expanding the determinant prove that :**

$$\begin{vmatrix} a & -b & -c \\ b & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = a + b^2 + c^3$$

[b] If $\vec{A} = (2\cos\theta, \log_3 x, \sin\theta)$, $\vec{B} = (\cos\theta, \log_5 27, 2\sin\theta)$ and $\vec{A} \cdot \vec{B} = 11$, determine value of x

5 [a] **Use the multiplicative inverse of the matrix to solve the set of the following equations :**

$x + y + 2z = 2$, $-2x - y + z = 5$, $x - y + 3z = 6$

[b] If the plane $8x + 15y + 6z = 120$ intersects the coordinates axes X , Y , Z at A , B , C respectively , calculate the area of the triangle ABC to the nearest unit of area.

Al-Azhar exams

2nd session 2019

Answer the following questions :

1 Choose the correct answer :

(1) If $\overrightarrow{A}$, $\overrightarrow{B}$ are two unit perpendicular vectors , then $(\overrightarrow{A} - 2\overrightarrow{B}) . (3\overrightarrow{A} + 5\overrightarrow{B}) = \ldots\ldots$

(a) -8 (b) -7 (c) 24 (d) 0

(2) If the plane $3x - 2y + 4z - 24 = 0$ intersects x-axis at the point A and intersects z-axis at B , then the perimeter of Δ OAB = …… unit of length where O is the origin point.

(a) 24 (b) 20 (c) 14 (d) 9

(3) The value of $\omega + i^2 + \omega^3 + i^4 + \omega^5 + i^6 + \ldots\ldots$ to 23 terms.

(a) zero (b) -1 (c) 2 (d) 1

(4) In the expansion of $(3a - 2b)^{11}$, if the ratio between the two middle terms according to descending powers of a respectively equals $\frac{-3}{2}$, then a : b = ……

(a) $\frac{4}{9}$ (b) $\frac{-4}{9}$ (c) 1 (d) -1

(5) If $k \in \mathbb{R}$, then $\left(k - \frac{k-1}{1+\omega} + (k+1)\,\omega^2\right)^8 = \ldots\ldots$

(a) 1 (b) 9 (c) 27 (d) 81

(6) In the expansion of $\left(y^2 + \frac{1}{a\,y}\right)^8$, if the coefficient of the middle term equals the coefficient of y^7 , then a = ……

(a) $\frac{5}{4}$ (b) $\frac{-5}{4}$ (c) $\frac{4}{5}$ (d) $\frac{-4}{5}$

Answer only three questions from the following :

2 [a] Put the number $z = \frac{8}{1 + \sqrt{3}\,i}$ in the trigonometric form , then find its two square roots in the exponential form.

[b] Find the length of the perpendicular drown from the point (2 , − 1 , 4) to the plane with the equation $\overrightarrow{r} . (\hat{i} - 2\hat{j} + 4\hat{k}) = 9$

3 [a] If ${}^{13}C_r : {}^{13}C_{r+1} = 9 : 5$, ${}^{n}C_{r-2} + {}^{n}C_{r-1} = 3432$, find the value of each n and r

[b] **Prove that the two straight lines :**

$\vec{r}_1 = (1 , 2 , 4) + t_1 (2 , -1 , 1)$, $\vec{r}_2 = (1 , 1 , 1) + t_2 (-2 , 7 , 11)$ are orthogonal , then show that they are skew.

---

4 [a] Solve the following equations by using the multiplicative inverse of the matrix

$2x + y + 3z = 0$, $x + y = -1$, $y + 2z = 3$

[b] Find the measure of the angle between the two planes

, $\vec{r} \cdot (2 , 1 , -1) = 4$ and $3x - 2y = 7$

---

5 [a] **Without expanding the determinat prove that :**

$$\begin{vmatrix} a & b & c \\ b & a & c \\ b & c & a \end{vmatrix} = (a-b)(a-c)(a+b+c)$$

[b] If the x-axis intersects the sphere $(x-2)^2 + (y+3)^2 + (z-1)^2 = 14$ at the two points A , B , then find the length of $\overline{AB}$

Answer the following questions :

1 Choose the correct answer :

(1) $(1+\omega)^4+(1+\omega^2)^4+(\omega+\omega^2)^4=$ ....

(a) -2 (b) -1 (c) zero (d) 3

(2) The two spheres $(x-2)^2+(y-1)^2+(z-3)^2=25$

, $(x+2)^2+(y+3)^2+(z+4)^2=16$ are ....

(a) distant. (b) touches each other internally.

(c) touches each other externally. (d) intersected.

(3) The length of perpendicular drawn from the point A (1 , 0 , 2) to the straight line

$\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z-3}{-2}$ equals .... unit of length.

(a) $\frac{\sqrt{26}}{4}$ (b) $\frac{\sqrt{26}}{5}$ (c) $\frac{\sqrt{26}}{3}$ (d) $\frac{\sqrt{26}}{6}$

(4) $\left(\frac{1+i}{1+2\omega}\right)^4+\left(\frac{1-i}{1+2\omega^2}\right)^4=$ ....

(a) $\frac{8}{9}$ (b) $\frac{-8}{9}$ (c) 8 (d) 9

(5) If ${}^{14}C_r={}^{14}C_{3r+2}$, ${}^{n}P_r=720$, then $\underline{|n-2r}=$ ....

(a) 6 (b) 24 (c) 120 (d) 720

(6) If $\underline{|n}$, $\underline{|2-n}$, n $\underline{|2-n}$ are side lengths of a triangle , then the numerical value of its perimeter = ... unit of length.

(a) 3 (b) 4 (c) 5 (d) 6

Answer only three questions from the following :

2 **[a]** If the coefficients of three consecutive terms of the expansion of $(1+x)^n$ are 35 , 21 , 7 according to the ascending power of x , find the value for each of n and the orders of these three terms.

[b] If $\overrightarrow{A}=(1,6,2)$, $\overrightarrow{B}=(k,3,m)$, $\overrightarrow{C}=(k,m,k+m)$ and $\overrightarrow{A}\,/\!/\,\overrightarrow{B}$ find $\|\overrightarrow{C}\|$

3 [a] Solve by using the properties of determinant the following equation.

$$\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = x - 1$$ where x is an integer.

[b] Find the value of n which makes the two staight lines

$L_1 : \vec{r} = (3 , -1 , n) + t_1 (4 , 1 , 3) , L_2 : x = \frac{y-4}{-1} = \frac{z+1}{2}$

intersect at a point , then find the point of their intersection.

---

4 [a] If $z_1 = \left(\frac{\sqrt{3}+i}{2}\right)^4 , z_2 = \sin\frac{\pi}{3} + i\cos\frac{\pi}{3} , i^2 = -1$ and $z = \frac{z_1}{z_2}$

Find the square roots of the number z in the trigonometric form.

[b] Find the component of the force $\vec{F} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ in the direction of the vector $\overrightarrow{AB}$ where $A(1 , 4 , 0) , B(-1 , 2 , 3)$

---

5 [a] **Without expanding the detrminant , prove that :**

$$\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$$

[b] **Find the measure of the angle between the two straight lines :**

$L_1 : \vec{r_1} = (2 , -1 , 3) + k_1 (-2 , 0 , 2) , L_2 : x = 1 , \frac{y-4}{3} = \frac{z+5}{-3}$

Answer the following questions :

1 Choose the correct answer :

(1) $\frac{30}{2+\omega} = \dots\dots$

(a) $10\ \omega$ (b) $20 + 10\ \omega^2$ (c) $10 + 20\ \omega^2$ (d) $10\ \omega^2$

(2) $\omega + \omega^2 + \omega^3 + \cdots + \omega^{101} = \dots\dots$

(a) -1 (b) 1 (c) 0 (d) ω

(3) If ${}^{30}C_r = {}^{30}C_{r+20}$, ${}^{n}P_4 = 90\ {}^{n-2}P_2$, then $\underline{|n-r}$ $\dots\dots$

(a) 0 (b) 10 (c) 1 (d) 120

(4) If ${}^{n-2}P_6 = \underline{|7}$, then $n + 1 = \dots\dots$

(a) 8 (b) 10 (c) 5 (d) 9

(5) If the equation $x^2 + y^2 + z^2 - 4x - 6y - 8z + 4 = 0$ is an equation of a sphere , then the surface area of this sphere = $\dots\dots$ π cm^2.

(a) 25 (b) 15 (c) 20 (d) 100

(6) If the measure of the angle between the two straight lines $\frac{x}{A} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$ equals 60° , then $A \in \dots\dots$

(a) $\{\pm 1\}$ (b) $\{1 , \frac{-13}{5}\}$ (c) $\{\frac{2}{5}\}$ (d) $\{-1\}$

Answer only three questions from the following :

2 [a] Find the roots of the equation $z^4 = 1$, and then represent these roots on the Argand plane.

[b] Find the cartesian form of the vector $\overrightarrow{A}$ if its magnitude is $7\sqrt{3}$ and makes equal angles with the positive directions of the cartesian axes.

3 [a] **Prove that :** $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$

[b] **Find the measure of the angle between the two straight lines :**

Whose direction ratios are : $(1 , 1 , 2)$, $(\sqrt{3} - 1 , -\sqrt{3} - 1 , 4)$

---

4 [a] In the expansion of $\left(2x - \frac{1}{2x^2}\right)^9$ according to descending powers of x find :

(1) Coefficient of x^3

(2) The value of x which makes the two middle terms in this expansion are equal.

[b] If $\overrightarrow{A} = (-3 , 1 , 2)$, $\overrightarrow{B} = (3 , 4 , -1)$

Find the area of the parallelogram in which $\overrightarrow{A}$ and $\overrightarrow{B}$ are two adjacent sides.

---

5 [a] If $(x - 1)$ is one of the factors of the determinant.

$\begin{vmatrix} 2-x & 2 & 2 \\ x & x & x^2 \\ 3 & 4 & k \end{vmatrix}$, then find the value of k

[b] Find all different forms of the straight line which passes through the point A $(1 , -1 , 0)$ and parallel to the striaght line which passing through the two points B $(-3 , 2 , 1)$ and C $(2 , 1 , 0)$, and then show that the point D $(-14 , 2 , 3)$ lies on this straight line.

Answer the following questions :

1 Choose the correct answer :

(1) If $k \in \mathbb{R}$, then $\left(k - \frac{k-1}{1+\omega} + (k+1)\,\omega^2\right)^4 = \ldots\ldots\ldots\ldots$

(a) -9 (b) 9 (c) 81 (d) -81

(2) If ${}^nC_1$, ${}^nC_2$, ${}^nC_3 + 28$ are in geometric sequence, then $n = \ldots\ldots\ldots\ldots$

(a) 7 (b) 6 (c) 8 (d) 9

(3) If $\overrightarrow{A} = (2, 1, -2)$, $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{A} + \overrightarrow{B}$, then $\overrightarrow{B} = \ldots\ldots\ldots\ldots$

(a) $(2, -1, -2)$ (b) $(2, 1, -2)$

(c) $(-2, -1, 2)$ (d) $(-2, -1, 3)$

(4) If ${}^nP_d + {}^dP_n = 240$, then ${}^nC_d = \ldots\ldots\ldots\ldots$

(a) 10 (b) 1 (c) 45 (d) 5

(5) The point which lying on the straight line : $\overrightarrow{r} = (2, -1, 3) + t\,(1, 2, -1)$ is $\ldots\ldots\ldots\ldots$

(a) $(1, 1, 1)$ (b) $(0, 2, -2)$

(c) $(3, 1, 2)$ (d) $(4, -3, 0)$

(6) $\left(2 + \omega^2 - \frac{2}{\omega}\right)\left(2 - \omega^2 + \frac{2}{\omega}\right) + \frac{1}{\omega^2} = \ldots\ldots\ldots\ldots$

(a) 2 (b) -2 (c) -4 (d) 4

Answer only three questions from the following :

2 [a] If $z_1 = 1 + \sqrt{3}\,i$, $z_2 = 1 + i$, find $z = \frac{z_1}{z_2}$

in the trigonometric form and then find the square roots of z in the exponential form.

[b] Find the equation of the sphere with centre $(1, 2, 1)$

and touches the plane $x + y + z = 1$

---

3 [a] In the expansion of $\left(x^2 + \frac{1}{x}\right)^9$, find the term free of x, and if the ratio between

the term free of x and the sixth term is $9 : 4$, find the real value of x

[b] Find the measure of the angle of inclination of the straight line

$L : \frac{x-3}{\sqrt{2}} = \frac{y-1}{1} = \frac{-z-2}{1}$ with the plane $\sqrt{2}\,x - y - z + 5 = 0$

---

4 **[a]** Solve the following system of linear equations using the inverse matrix :

$x + y + 2z = 2$, $-2x - y + z = 5$, $x - y + 3z = 6$

[b] Find the different forms of the equation of straight line passing through the point $(2 , 1 , -3)$ and parallel to the straight line $L : \frac{x-2}{3} = \frac{y+3}{1} = \frac{1-z}{3}$

---

5 **[a]** **Without expanding the determinant , prove that :**

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

[b] Find the Cartesian forms of a vector $\vec{A}$ whose norm is $7\sqrt{3}$ and makes equal measured angles with the positive directions of the coordinate axes.

Answer the following questions :

1 Choose the correct answer :

(1) The two straight lines : $L_1 : \frac{x+1}{-1} = \frac{y-2}{-1} = \frac{z-1}{3}$, $L_2 : \frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{-1}$

lie on the plane ....

(a) $2x - 5y + z - 1 = 0$ (b) $5x - 4y + 2z - 7 = 0$

(c) $7x - 5y - z - 4 = 0$ (d) $7x + 2y + 3z = 0$

(2) If $n \underline{|n+2} = 60 \underline{|n}$, then n = ...

(a) 5 (b) 2

(c) 3 (d) 4

(3) If $\|\overrightarrow{F}\| = 12$ netwon , $\overrightarrow{F} \parallel \overrightarrow{A}$ and $\overrightarrow{A} = 2\hat{i} - \hat{j} + 2\hat{k}$, then $\overrightarrow{F}$ may be equal ....

(a) $(-8 , 4 , 8)$ (b) $(8 , -4 , 8)$

(c) $(-4 , 8 , 8)$ (d) $(-8 , -4 , 8)$

(4) If ${}^nP_r = 5040$, ${}^nC_r = 210$, then ${}^{n+1}C_{2r-8} =$ ....

(a) 1 (b) 24

(c) 45 (d) 15

(5) $\frac{a - c\,\omega}{a\,\omega^2 - c} - \omega^2 =$ ....

(a) $2i$ (b) $\pm\sqrt{3}\,i$

(c) -3 (d) 3

(6) If $x = \frac{-1 + \sqrt{3}\,i}{2}$, then $x^{10} + x^5 + 1 =$ ....

(a) 2 (b) 1

(c) 0 (d) $\sqrt{3}\,i$

Answer only three questions from the following :

2 **[a]** **Solve the following system of linear equations using the inverse matrix :**

$2x - 3y - z = 9$, $x + 2y + 3z = 15$, $x - 2z = 12$

[b] Find all different forms of the equation of the plane passing through the point $(2, -1, 0)$ and vector $\vec{a} = 4\hat{i} + 10\hat{j} - 7\hat{k}$ is normal to it.

---

3 **[a]** Find the different forms of the number $z = \dfrac{-\sqrt{3} - i}{\sqrt{3} - i}$

, then find the square roots of the number z in the exponential form.

[b] If the plane $x + y + z = 4$ intersects the coordinate axes at the points A , B , C

Calculate the perimeter of Δ ABC

---

4 **[a]** In the expansion of $\left(x^2 + \dfrac{1}{a x}\right)^8$, if the coefficient of the middle term equals the coefficient of the term containing x^{10} , find the value of a

[b] If the x-axis cuts sphere $(x - 2)^2 + (y + 3)^2 + (z - 1)^2 = 14$ at the two points A and B , find the length of $\overline{AB}$

---

5 **[a]** **Without expanding the determinant , prove that :**

$$\begin{vmatrix} x + a & x + a & 2a \\ a & x & a \\ 0 & a - x & x - a \end{vmatrix} = (x + 2a)(x - a)^2$$

[b] **Find the measure of the angle between the two straight lines :**

$L_1 : \dfrac{x - 3}{2} = \dfrac{z + 1}{-2}$, $y = 1$ and $L_2 : \vec{r} = (-1, 2, -1) + t(1, 2, -2)$

Notes

Notes

Notes

يُصرف مجانًا مـع هـذا الكتاب الجـزء الخـاص بالإجـابات

Question Bank
& Practice Exams

3rd Sec
2022

NOW At All Bookstores

EL-MOASSER

Question Bank
& Practice Exams in :

- Differential & Integral Calculus
- Statics
- Dynamics
- Physics
- Biology
- Chemistry
- Hello English
- French

GPS

AL TALABA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
e-mail: info@elmoasserbooks.com
15014 www.elmoasserbooks.com

/ElMoasser.eg

6 223007 310949